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BOOK OF ABSTRACTS AND PROGRAMME

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## Welcome

**Dear Participants,**

Welcome to the International Conference on Pure and Applied Mathematics, ICPAM 2015, Van, Turkey. The conference is organized and is to be held at Yüzüncü Yıl University from August 25th to August 28th. We are happy to have you here in Van.

The main aim of the conference is to provide participants with an opportunity to exchange the latest information and ideas, and to encourage debate on many issues in international mathematical researches. During the conference you will certainly meet old and new colleagues, exchange ideas, develop new projects. You will also feel and enjoy the special atmosphere of Van and Turkey.

With 76 sessions and 230 presentations and more than 250 participants from 24 countries, Algeria, Armenia, Azerbaijan, Canada, Egypt, France, Georgia, India, Iran, Iraq, Isle of Man, Japan, Jordan, Kosovo, Kuwait, Macedonia, Mauritania, Nigeria, Pakistan, Russian Federation, Saudi Arabia, South Africa, Turkey, United Arab Emirates, USA, as well as people from 55 different university from Turkey, ICPAM 2015 will provide a stimulating opportunity for a global interchange of ideas on recent advances in mathematics.

I would like to express my deep gratitude to Prof. Dr. Peyami BATTAL, President of Yüzüncü Yıl University, for his encouragement and support in all stages of this conference.

I am grateful to all the participants in the International Conference on Pure and Applied Mathematics, particularly the members of the Scientific and Organizing Committees, the referees and the authors for producing such a high standard conference.

The conference is almost entirely from the registration support of participants. Behind this, we are grateful to Rectorate of YYU, Faculty of Sciences of YYU and Administrative Coordination Office of Research Project (BAP) for their financial support. We would also like to thank to the sponsor, Mayor of Tuşba Municipality for their generous support. Have a pleasant stay in Van.



Professor Cemil Tunç  
Chair Organizer of ICPAM Organizing Committee



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## Programme

Tuesday, August 25

**Room Amphi** Chair: Hakan TOR

10:30-11:00 Opening Ceremony

**Room Amphi** Chair: Hüseyin MERDAN

11:00-11:30 **Marat AKHMET**

Extension of chaos, control and applications

11:30-11:45 Coffee Break

**Room Amphi** Chair: Hüseyin MERDAN

11:45-12:15 **Mahmoud ABDEL-ATY**

Prospective of quantum information

12:15-13:45 Lunch Break

**Room Amphi** Chair: Bülent KARAKAŞ

14:00-14:30 **Hidenori OGATA**, Hiroshi Hirayama

Hyperfunction method for numerical integrations

14:30-15:00 **Feyzi BAŞAR**

Recent trends related to four dimensional matrix transformations

15:00-15:15 Coffee Break

**Room 101** Chair: Ercan TUNÇ

15:15-15:35 **Nihal YOKUŞ**, Nimet Coskun

Jost solution and spectrum of the discrete Sturm-Liouville equations with hyperbolic eigenparameter

15:35-15:55 **Şerifenur CEBESÖY**, Elgiz Bairamov, Yelda Aygar

Matrix-valued difference operators with polynomial type Jost solutions on the whole axis

15:55-16:15 **Hilmi ERGÖREN**

Impulsive neutral fractional differential inclusions at variable times

16:15-16:30 Coffee Break

**Room 101** Chair: Zeynep KAYAR

16:30-16:50 **Serkan ASLIYÜCE**, Feza Güvenilir

Grüss inequality on discrete fractional calculus with delta operator

16:50-17:10 **Sibel DOĞRU AKGÖL**, Ağacık Zafer

A general result on asymptotic integration of impulsive differential equations

17:10-17:30 Cemil Tunç, **Yener ALTUN**

On the asymptotic behaviors of solutions certain non-linear neutral equations with multiple deviating arguments

**Room 102** Chair: Marat AKHMET

- 15:15-15:35 **Mehtap LAFCI**, Gizem S. Öztepe  
Oscillation of mixed type third order nonlinear differential equation with piecewise constant arguments
- 15:35-15:55 **Mohamed Vall OULD MOUSTAPHA**  
Laplace equation with triple-inverse square potentials on euclidean space and applications
- 15:55-16:15 Allaberen Ashyralyev, **Sema AKTÜRK**  
Fractional spaces generated by the positive differential operator in the half-space
- 16:15-16:30 Coffee Break

**Room 102** Chair: Zineb ARAB

- 16:30-16:50 **Matallah ATIKA**  
Multiple solutions to p-Kirchhoff type problems in  $\mathbb{R}^n$  critical p-Kirchhoff equations
- 16:50-17:10 **Benmansour SAFIA**  
Multiple solutions to nonhomogeneous elliptic Kirchhoff equations in  $\mathbb{R}^n$
- 17:10-17:30 **Abdullahi Rashid ADEM**  
Solutions and conservation laws of a two-dimensional integrable generalization of the Kaup-Kupershmidt equation

**Room 103** Chair: Semra SARAÇOĞLU ÇELİK

- 15:15-15:35 **Erhan GÜLER**, Yusuf Yaylı  
Generalized Bour's theorem in Minkowski space form
- 15:35-15:55 Erhan Güler, **Vahit ZAMBAK**  
Algebraic surfaces of Henneberg in Minkowski 3-space
- 15:55-16:15 **Muhammed Talat SARIAYDIN**, Vedat Asıl  
Characterizations of quaternionic some surfaces in Minkowski 3-space
- 16:15-16:30 Coffee Break

**Room 103** Chair: Mehmet ATÇEKEN

- 16:30-16:50 **Semra SARAÇOĞLU ÇELİK**, Yusuf Yaylı, Erhan Güler  
Euler spirals in space forms
- 16:50-17:10 **Muhammed Talat SARIAYDIN**, Vedat Asıl  
New parametric representation of a surface family with common smarandache asymptotic by using Bishop frame
- 17:10-17:30 **Mahrouz TAYEB**, Chailli Rachid  
Comparison of differential operators in Gevrey space

**Room 104** Chair: Bülent KARAKAŞ

- 15:15-15:35 **Sahar MOAYERI RAHNI**  
The skew inverse semigroup ring
- 15:35-15:55 **Okan ARSLAN**, Hatice Kandamar  
 $\Gamma$ -radicals of gamma rings
- 15:55-16:15 Alev Firat, **Şule AYAR ÖZBAL**  
Symmetric bi-multipliers on incline algebras

16:15-16:30 Coffee Break

**Room 104** Chair: Şenay BAYDAŞ

16:30-16:50 **Sahar MOAYERI RAHNI**

When the universal inverse semigroup  $Pr(s)$  of inverse semigroup  $S$  is  $E^*$ -unitary

16:50-17:10 **Tevfik BİLGİN**, İsmail Gökhan Kelebek

Characterization of  $U_1(\mathbb{Z}[C_n \times C_4])$

17:10-17:30 **Ahmad MOUSSAVI**, Alireza Moussavi

On nilpotent elements in Ore extensions

**Room 105** Chair: Musa ÇAKIR

15:15-15:35 **Hatice KARABENLİ**, Alaattin Esen, E. Nesligül Aksan

Collocation finite element solutions for Stefan problem with Neumann boundary condition

15:35-15:55 Musa Cakir, **Derya ARSLAN**

A numerical method for nonlinear singularly perturbed multi-point boundary value problem

15:55-16:15 Hakki Duru, **Akbar BARATI CHIYANEH**

Difference schemes on adapted mesh for the initial boundary value Sobolev problems with boundary layers

16:15-16:30 Coffee Break

**Room 105** Chair: Nagehan AKGÜN

16:30-16:50 **Alper KORKMAZ**, Hakan Kasım Akmaz

Exponential cubic B-spline based solutions of advection-diffusion equation

16:50-17:10 **Reza ABAZARI**, Mehrdad Lakestani

Numerical solution of the Rosenau-Burgers equation using quintic B-spline collocation method

17:10-17:30 **Dashti AHMED ALI**

A comparison of methods for computing the matrix exponential appears in systems of differential equations

**Room 106** Chair: Cesim TEMEL

15:15-15:35 Ümit Totur, **İbrahim ÇANAK**

Tauberian conditions for the  $(C, \alpha)$  integrability of functions

15:35-15:55 **Mehmet Ali AKTÜRK**, Alexey Lukashov

Sharp rusak-type inequalities for rational functions on several intervals

15:55-16:15 **Abdallah BENAÏSSA**

Asymptotic expansion of double oscillatory integrals: Contribution of non stationary critical points of the second kind

16:15-16:30 Coffee Break

**Room 106** Chair: M. KARAKUŞ

- 16:30-16:50 **Samet ERDEN**, Mehmet Zeki Sarıkaya  
Generalized Bullen type inequalities for local fractional integrals and its applications
- 16:50-17:10 **Emre DENİZ**, Gülsüm Ulusoy, Ali Aral  
Convergence properties on Jain-Durrmeyer operators
- 17:10-17:30 **Gülsüm ULUSOY**, Tuncer Acar  
Generalized Szasz-Mirakyan-Durrmeyer operators and their approximation properties

**Room 107** Chair: Ishak ALTUN

- 15:15-15:35 **Gülhan MINAK**, Ishak Altun  
F-contractions on metric spaces and some related fixed point results
- 15:35-15:55 **Abdurrahman BÜYÜKKAYA**, Mahpeyker Öztürk  
Some fixed point theorems satisfying Meir-Keeler type contractions via rational expression in 2-metric spaces
- 15:55-16:15 **Gonca DURMAZ**, Ishak Altun  
Some fixed point theorems for a weak partial metric space
- 16:15-16:30 Coffee Break

**Room 107** Chair: Esra DALAN YILDIRIM

- 16:30-16:50 **Ayşegül ÇAKSU GÜLER**, Esra Dalan Yıldırım  
Some fixed point theorems on soft G-metric spaces
- 16:50-17:10 **Yağmur KARAKOÇ**  
Cone metric spaces and cone two metric spaces
- 17:10-17:30 **Süleyman GÜLER**, Yücel Özdaş  
On weak continuity of soft topological spaces

**Room 108** Chair: Feyzi BAŞAR

- 15:15-15:35 Uğur Ulusu, **Ömer KİŞİ**  
I-Cesro summability of sequences of sets
- 15:35-15:55 **Medine YEŞİLKAYAGİL**, Feyzi Başar  
Some topological properties of the spaces of almost null and almost convergent double sequences
- 15:55-16:15 **Alireza KHALILI GOLMANKHANEH**  
Fractal calculus and application
- 16:15-16:30 Coffee Break

**Room 108** Chair: Hakan TOR

- 16:30-16:50 **A. HOMID**, Mahmoud Abdel-Aty  
Implementing quantum search algorithm in the presence of dissipation
- 16:50-17:10 **Ahmad SHAYGANMANESH**, Ahmad Saeedi  
Stability and accuracy of RBF direct method for solving a dynamic investment model
- 17:10-17:30 **Seda İĞRET ARAZ**, Murat Subaşı, Hakkı Güngör, Hülya Durur  
On obtaining stable solution for a hyperbolic coefficient control problem

August 25, 2015 Tuesday									
Time	Amphi	Room 101	Room 102	Room 103	Room 104	Room 105	Room 106	Room 107	Room 108
9:00-10:15									
10:15-10:30					Registration Coffee Break				
10:30-11:00	Opening Ceremony								
<b>Chair</b>	<b>H. MERDAN</b>								
11:00-11:30	Marat AKHMET				Invited Speaker				
11:30-11:45	<b>H.</b>				Coffee Break				
<b>Chair</b>	<b>MERDAN</b>								
11:45-12:15	Mahmoud ABDEL-ATY				Invited Speaker				
12:15-13:45	<b>B. KARAKAŞ</b>				Lunch Break				
14:00-14:30	Hidenori OGATA				Invited Speaker				
14:30-15:00	Feyzi BAŞAR				Invited Speaker				
15:00-15:15					Coffee Break				
<b>Chair</b>	<b>E. TUNÇ</b>	<b>M. AKHMET</b>	<b>S. SARAÇOĞLU ÇELİK</b>	<b>B. KARAKAŞ</b>	<b>M. ÇAKIR</b>	<b>C. TEMEL</b>	<b>I. ALTUN</b>	<b>H. TOR</b>	
15:15-15:35	N.YOKUS	M. LAFCI	E. GÜLER	S. MOAYERI RAHNI	H. KARABENLİ	İ. ÇANAK	G. MINAK	Ö. KIŞI	
15:35-15:55	Ş. CEBESÖY	M. V. OULD MOUSTAPHA	V. ZAMBAK	O. ARSLAN	D. ARSLAN	M. A. AKTÜRK	A. BÜYÜKKAYA	M. YEŞİLKAYAGİL	
15:55-16:15	H. ERGÖREN	S. AKTÜRK	M. T. SARIAY-DİN	Ş. ÖZBAL AYAR	A. BARATI CHIYANEH	A. BENAİSSA	G. DURMAZ	A. KHALILI GOLMANKHANEH	
16:15-16:30					Coffee Break				
<b>Chair</b>	<b>Z. KAYAR</b>	<b>Z. ARAB</b>	<b>M. ATÇEKEN</b>	<b>Ş. BAYDAŞ</b>	<b>N. AKGÜN</b>	<b>KARAKUŞ</b>	<b>M. DALAN YILDIRIM</b>	<b>F. BAŞAR</b>	
16:30-16:50	S. ASLIYÜCE	M. ATİKA	S. SARAÇOĞLU ÇELİK	S. MOAYERI RAHNI	A. KORKMAZ	S. ERDEN	A. ÇAKSU GÜLER	A. HOMİD	
16:50-17:10	S. DOĞRU AKGÖL	B. SAFİA	M. T. SARIAY-DİN	T. BİLGİN	R. ABAZARI	E. DENİZ	Y. KARAKOÇ	A. SHAYGANMANESH	
17:10-17:30	Y. ALTUN	A. R. ADEM	M. TAYEB	A. MOUSSAVI	D. AHMED ALI	G. ULUSOY	S. GÜLER	S. İGRET ARAZ	

Wednesday, August 26

**Room Amphi** Chair: Murat SUBAŞI

09:00-09:30 **Ersan AKYILDIZ**

An overview of discrete Log and Trace based public key cryptography on finite fields

09:30-10:00 **Vatan KARAKAYA**

10:00-10:15 Coffee Break

**Room Amphi** Chair: Ersan AKYILDIZ

14:00-14:30 **Hüseyin Merdan**

Asset flow differential equations

14:30-15:00 *Poster Presentations*

**Room 101** Chair: Hilmi ERGÖREN

10:15-10:35 Khanlar R. Mamedov, **F. Ayca CETINKAYA**, Ozge Akcay

Boundary value problem for a Sturm-Liouville operator with piecewise continuous coefficient

10:35-10:55 **Asghar AHMADKHANLU**

Existence and uniqueness results for a class of fractional boundary value problem

10:55-11:15 **Zeynep KAYAR**

Fractional boundary value problems (BVPs) and Lyapunov type inequality

11:15-11:30 Coffee Break

**Room 101** Chair: Alireza KHALILI GOLMANKHANEH

11:30-11:50 Khanlar R. Mamedov, **Özge AKÇAY**, F. Ayca Cetinkaya

On the inverse problem for a class of Dirac operators

11:50-12:10 Shahram Mehry, **Saeid MOMENALI**

Heat source using the conjugate gradient method with adjoint problem

12:10-12:30 **Ahmad JAFARIAN**

Artificial neural network method for solving fractional Fredholm integral equations

12:30-13:45 Lunch Break

**Room 101** Chair: Erdal KORKMAZ

15:15-15:35 **Derya ALTINTAN**, Vilda Purutçuoğlu, Ömür Uğur

Functional impulses in exact stochastic simulation

15:35-15:55 **Sebaheddin ŞEVGİN**, Pınar Yurdakul

Ulam stability of some Volterra equations

15:55-16:15 Cemil Tunç, **Sizar Abid MOHAMMED**

On the stability and boundedness of differential equations of third order with retarded argument

16:15-16:30 Coffee Break



**Room 101** Chair: Derya ALTINTAN

16:30-16:50 Ayşe Feza Güvenilir, Billur Kaymakçalan, **Neslihan Nesliye PELEN**

Some results on predator-prey dynamic systems with Beddington-Deangelis type functional response

16:50-17:10 **Erdal KORKMAZ**, Cemil Tunç

Inequalities and exponential decay of certain differential equations of first order in time varying delay

17:10-17:30 **Abderrahim HOUMAT**

Coupling of finite and hierarchical infinite elements: Application to a homogeneous isotropic half-space

**Room 102** Chair: Sebaheddin ŞEVGİN

10:15-10:35 **Özgür AYDOĞMUŞ**

Extinction in a generalized chain binomial epidemic model

10:35-10:55 **Gamzegül AYDIN**, Hüseyin Merdan, Abdessamad Tridane

Stability analysis of HIV infection model with tumor

10:55-11:15 **Azam AZIMI**, Mahmood Ghaffari, Gholam Hossein Riazi, Mohammad Mehdi Tavako

Theoretical and computational modeling studies on the  $\alpha$ -cyperone-tubulin interaction

11:15-11:30 Coffee Break

**Room 102** Chair: Hidenori OGATA

11:30-11:50 **Hakan TOR**

Hyperbolic smoothing method for sum-max problems

11:50-12:10 **Sameera Abdulsalam OTHMAN**

Forecasting by adaptive double exponential smoothing

12:10-12:30 **Mahmoud MAHDIAN**

Phonon spectral density of the FMO light-harvesting complex with associated and generalized Jacobi polynomials

12:30-13:45 Lunch Break

**Room 102** Chair: Özgür AYDOĞMUŞ

15:15-15:35 **Haci Mehmet BASKONUS**, Hasan Bulut, Mirac Kayhan

Regarding analytical prototype studies for the generalized nonlinear Pochhammer-Chree equation

15:35-15:55 **Marat U. AKHMET**, Duygu Aruğaslan, Nur Cengiz

Exponential stability of periodic solutions of recurrent neural networks with functional dependence on piecewise constant argument

15:55-16:15 Nesir Huseyin, Anar Huseyin, **Khalik GUSEINOV**, Vladimir Ushakov

Approximation of the set of trajectories of control system described by an affine Volterra type integral equation

16:15-16:30 Coffee Break

**Room 102** Chair: Nagehan AKGÜN

16:30-16:50 **Amir BAKLOUTI**

Jordan superalgebras with some homogeneous structures

16:50-17:10 Shahram Mehry, **Reza SAFAKISH**, Amir Saeidi

Nilpotent Lie algebras and Betti number behavior

17:10-17:30 **Mehmet KIR**, Hukmi Kızıltunç

The concept of weak  $(\psi, \alpha, \beta)$  contractions in partially ordered metric spaces

**Room 103** Chair: Şenay BAYDAŞ

10:15-10:35 Morteza Faghfour, **Sahar MASHMOULI**

Anti-invariant semi-Riemannian submersions admitting vertical from Lorentzian Sasakian and para Sasakian manifolds

10:35-10:55 **Süleyman DİRİK**, Mehmet Atçeken, Ümit Yıldırım

Pseudo-slant submanifold in Sasakian space forms

10:55-11:15 **Ali UÇUM**, Kazım İlarıslan, Makoto Sakaki

On generalized null Bertrand curves in  $E_2^4$

11:15-11:30 Coffee Break

**Room 103** Chair: Süleyman DİRİK

11:30-11:50 **Mohammad ILMAKCHI**

Real hypersurfaces in complex projective space with weakly constant holomorphic curvature

11:50-12:10 **Mehmet ATÇEKEN**, Umit Yıldırım, Süleyman Dirik

On almost  $C(\alpha)$ -manifold satisfying certain conditions on quasi-conformal curvature tensor

12:10-12:30 **Umit YILDIRIM**, Mehmet Atçeken, Süleyman Dirik

On almost  $C(\alpha)$ -manifold satisfying some conditions on the weyl projective curvature tensor

12:30-13:45 Lunch Break

**Room 103** Chair: Süleyman EDİZ

15:15-15:35 **İbrahim Halil GUMUS**, Omar Hirzallah

Schatten p-norm inequalities for accretive-dissipative 2x2 operator matrices

15:35-15:55 **Shahram MEHRY**, Reza Safakish

A classification of Ramanujan complements of unitary Cayley graphs

15:55-16:15 **Abdelkader LAKMECHE**, Mohamed Helal, Ahmed Lakmeche

Pulsed chemotherapy model

16:15-16:30 Coffee Break

**Room 103** Chair: Güzide ŞENEL

16:30-16:50 **Aysegul BAYRAM**, Vedat Siap

Graph-theoretic approach to the ideal structure of a family of non-chain rings

16:50-17:10 **Ömer KÜSMÜŞ**

Another description of units of integral group ring of dicyclic group of order 12

17:10-17:30 Nazmiye Alemdar, **Sedat TEMEL**

Group-2-groupoids and 2g-crossed modules

**Room 104** Chair: Turgut HANOYMAK

10:15-10:35 **Özge ÇOLAKOĞLU**, Hamza Menken  
On the q-extension of the p-adic Beta function

10:35-10:55 **Mirac CETİN FİRENGİZ**, Naim Tuğlu  
Some incomplete q-polynomials

10:55-11:15 **Murat BEKAR**, Yusuf Yaylı  
Involutions of dual split-quaternions

11:15-11:30 Coffee Break

**Room 104** Chair: Turgut HANOYMAK

11:30-11:50 **Emel A. UGURLU**, Kürsat Hakan Oral, Unsal Tekir  
Extension of uniformly primary ideals

11:50-12:10 **Emine Serap KARACAN**, Emel A.Ugurlu, Unsal Tekir  
On almost prime ideals

12:10-12:30 **Ismail Hakkı DENİZLER**  
Nakayama's lemma for artinian modules and generalized matlis duality

12:30-13:45 Lunch Break

**Room 104** Chair: Afshin BEHMARAM

15:15-15:35 **Gülüstan Kaya GÖK**, Nursah Mutlu, Serife Büyükköse  
Kirchoff index of weighted graphs

15:35-15:55 **Mehdi ELIASI**  
On ordering of trees by multiplicative version of Zagreb indices

15:55-16:15 **Adnan MELEKOĞLU**  
Mirrors on Hurwitz surfaces

16:15-16:30 Coffee Break

**Room 104** Chair: Adnan MELEKOĞLU

16:30-16:50 **Gülüstan Kaya GÖK**, Nursah Mutlu, Serife Büyükköse  
Wiener index of weighted graphs

16:50-17:10 **Murat CANCAN**, Süleyman Ediz  
Inverted distance and inverted Wiener index

17:10-17:30 **Süleyman EDİZ**, Murat Cancan  
Reverse Zagreb indices of cartesian product of graphs

**Room 105** Chair: Mahmut KARAKUŞ

10:15-10:35 **Gülsüm ULUSOY**, Emre Deniz, Ali Aral  
On generalized Durrmeyer operators

10:35-10:55 **Mehmet Zeki SARIKAYA**, Samet Erden, Hüseyin Budak  
Some generalized Ostrowski type inequalities involving local fractional integrals and applications

10:55-11:15 Mehmet Zeki Sarıkaya, **Tuba TUNÇ**, Samet Erden  
Generalized Steffensen inequalities for local fractional integrals

11:15-11:30 Coffee Break

**Room 105** Chair: Hakki DURU

- 11:30-11:50 **Ahmad SAMAILA**, Basant Jha  
Effects of transpiration on g-gitter fully developed mixed convection flow in a vertical channel
- 11:50-12:10 **Youssef S. HASSAN**, Amr Mamdouh, Kareemeldien Maklad, Ahmed A. Elghannam, Mahmoud Abdelaty  
Adaptive step size numerical solution to first order ODEs, a refinement of Eulers and RK methods
- 12:10-12:30 **Sunnie JOSHI**, Benjamin Seibold, Pak-Wing Fok  
A computational model for the simulation of atherosclerotic plaques
- 12:30-13:45 Lunch Break

**Room 105** Chair: Ali SIRMA

- 15:15-15:35 **Abdelaziz MENNOUNI**  
A modified Galerkin method for solving integral equations of the second kind
- 15:35-15:55 **Mehdi JALALVAND**, Zeinab Moeini Rad, Ameneh Sayahi  
Numerical solution for some weakly singular nonlinear Volterra integral equations
- 15:55-16:15 **Sedigheh TOUBAEI**  
A finite difference method for smooth solution of system of linear weakly singular Volterra integral equations
- 16:15-16:30 Coffee Break

**Room 105** Chair: Mehmet Giyas SAKAR

- 16:30-16:50 **Shabnam JAMSHIDZADEH**, Nasrin Eghbali  
Stability of Cauchy functional equation and quadratic equation by generalized operations
- 16:50-17:10 **Nadia Amel MESSAOUDI**, Salah Manseur, Mustapha Blidia  
Revisit scheme of adomian decomposition method for nonhomogeneous heat equation
- 17:10-17:30 **Mustafa KUDU**, Gabil M. Amiraliyev  
Fourth order approximate method for a time-delayed pseudo-parabolic equation

**Room 106** Chair: İbrahim ÇANAK

- 10:15-10:35 **Ümit Totur, Muhammet Ali OKUR**  
On Tauberian remainder theorems for Cesàro summability method of noninteger order
- 10:35-10:55 **Ümit TOTUR**, İbrahim Çanak  
The  $(C, \alpha, \beta)$  integrability of functions and a Tauberian theorem
- 10:55-11:15 **Abasalt BODAGHI**  
Various notions of module amenability
- 11:15-11:30 Coffee Break

**Room 106** Chair: Zeynep KAYAR

- 11:30-11:50 **Ali AL-KARALY**, Karim Kholy  
Aspects of analitical solutions and simulation of high order ODE
- 11:50-12:10 **Boukemara IBTISSEM**  
Structure of basin and bifurcation phenomena in two-dimensional  
piecewise maps
- 12:10-12:30 Fatemeh Fatahi, **Gholamreza SAFAKISH**, Rohollah Piri  
Rings over which Monoid rings are semicommutative
- 12:30-13:45 Lunch Break

**Room 106** Chair: Mehmet Zeki SARIKAYA

- 15:15-15:35 **Mehmet Eyüp KİRİŞ**, Naki Çaltıner  
On generalized some inequalities for  $s$ -convex functions
- 15:35-15:55 **Mohammad Reza JABBARZADEH**  
Moore-penrose inverse of weighted composition operators on  
 $L^p$ -spaces
- 16:15-16:30 Coffee Break

**Room 106** Chair: İsmail KÜÇÜK

- 16:30-16:50 **Gopal DATT**  
Operator equations generalizing the notions of Hankel and Toeplitz  
operators
- 16:50-17:10 **Abdulhamit KUCUKASLAN**  
Generalized fractional maximal operator on generalized local Morrey  
spaces
- 17:10-17:30 **Alev MERAL**, Ömür Uğur  
Optimal portfolio strategies under various risk measures

**Room 107** Chair: Hamza MENKEN

- 10:15-10:35 **Hacer BOZKURT**, Yılmaz Yılmaz  
Some new theorems in Hilbert quasilinear spaces
- 10:35-10:55 **Arife ATAY**, H. İlhan Tutalar  
Regular local functions in ideal topological spaces
- 10:55-11:15 **İzzettin DEMİR**, Oya Bedre Özbakır, İsmet Yıldız  
Some properties of soft proximity spaces
- 11:15-11:30 Coffee Break

**Room 107** Chair: Ayşegül ÇAKSU GÜLER

- 11:30-11:50 **Hamza MENKEN**, Özge Çolakoğlu  
On the  $p$ -adic log beta function
- 11:50-12:10 **Ramazan EKMEKÇİ**, Rıza Ertürk  
 $Q$ -convergence of graded difilters
- 12:10-12:30 Engin Ozkan, **Aykut GÖÇER**, İpek Altun  
The relationship between  $n$ th lucas number and a sequence defined  
by  $m$ -sequences
- 12:30-13:45 Lunch Break

**Room 107** Chair: Hatice TAŞKESEN

- 15:15-15:35 **Hadeel Salim ALKUTUBI**  
Regression analysis for breast cancer patients
- 15:35-15:55 **Enes Abdurrahman BİLGİN**, Sıddık Keskin  
Regression analysis algorithm for circular data
- 15:55-16:15 **Hossein JABBARI KHAMNEI**, Roghaye Makouyi  
Recurrence relation for the moments of order statistics from a beta-pareto distribution
- 16:15-16:30 Coffee Break

**Room 107** Chair: Sebaheddin ŞEVGİN

- 16:30-16:50 **Qais Mustafa ABDULQADER**  
Comparison between discriminant analysis and logistic regression analysis: An application on caesarean births and natural births data
- 16:50-17:10 Mehmet Güngör, Ahmet Demiralp, Yunus Bulut, M.Şamil Şık, **Yusuf KIRAÇ**  
On comparisons of coherent systems via dynamic system signature
- 17:10-17:30 **Mahmoud MAHDIAN**  
General non-Markovian dynamics of open quantum systems and spectral density of complex systems using exceptional orthogonal polynomials

**Room 108** Chair: Hatice TAŞKESEN

- 10:15-10:35 **Nazish SHAHID**  
The influence of thermal radiation, mass diffusion and fractional parameters on mhd flow over a vertical plate that applies time dependent shear to the fluid
- 10:35-10:55 **Alex PIJYAN**  
On estimation of unknown parameters of exponential-logarithmic distribution by censored
- 10:55-11:15 **Seçil YALAZ TOPRAK**, Mujgan Tez, H.Ilhan Tutalar  
Asymptotic normality of parametric part in partially linear models in the presence of measurement error
- 11:15-11:30 Coffee Break

**Room 108** Chair: Gopal DATT

- 11:30-11:50 **İlker GENÇTÜRK**, Kerim Koca  
Dirichlet boundary value problem for a nth order complex differential equation
- 11:50-12:10 **Tugba YAVUZ**  
Coefficient estimates for a new subclass of close-to-convex functions
- 12:10-12:30 **Serhan ULUSAN**, Adnan Melekoğlu  
Symmetry groups of petrie polygons
- 12:30-13:45 Lunch Break

**Room 108** Chair: Ümit TOTUR

15:15-15:35 M. Emin Özdemir, **Havva KAVURMACI ÖNALAN**  
 $(g, (h - m))$ -convex dominated functions

15:35-15:55 **Hatice KUSAK SAMANCI**, Serpil Kaya  
The dual-variable Bernstein polynomials

15:55-16:15 **Alaa El-Din EL-OZEIRI**  
Research on using uranium-lead radioactive decay in detecting the  
age of earth

16:15-16:30 Coffee Break

**Room 108** Chair: Musa ÇAKIR

16:30-16:50 **Feda İLHAN**, Zahir Muradoğlu  
Elasto-plastic deformation of an incompressible bending plate with  
clamped boundary

16:50-17:10 Musa Cakır, **Derya ARSLAN**  
Reduced differential transform method for sixth-order singularly  
perturbed Boussinesq equation

17:10-17:30 **Rukiye ÖZTÜRK**, Ali Aydogdu, Engin Ozkan  
An alternative proof for a lemma used in the trace formula for  $GL(2)$   
over a number field

Time	Room 101	Room 102	Room 103	Room 104	Room 105	Room 106	Room 107	Room 108
9:00-09:30	Amphi <b>M. SUBAŞI</b> Ersan AKYILDIZ Vatan KARAKAYA							
9:30-10:00					Invited Speaker			
10:00-10:15					Invited Speaker			
					Coffee Break			
<b>Chair</b>	<b>H. ERGÖREN</b>	<b>S. ŞEVGIN</b>	<b>Ş. BAYDAŞ</b>	<b>T. HANOYMAK</b>	<b>H. KAVURMACI ONALAN</b>	<b>İ. ÇANAK</b>	<b>H. MENKEN</b>	<b>H. TAŞKESEN</b>
10:15-10:35	F. A. CETINKAYA	O. AYDOĞMUS	S. MASHMOULI	Ö. ÇOLAKOĞLU	G. ULUSOY	M. A. OKUR	H. BOZKURT	N. SHAHID
10:35-10:55	A. AHMADKHANLU	G. AYDIN	S. DİRİK	M. CETİN FİRENGİZ	M. Z. SARIKAYA	Ü. TOTUR	A. ATAY	A. PIJYAN
10:55-11:15	Z. KAYAR	A. AZIMI	A. UÇUM	M. BEKAR	T. TUNÇ	A. BODAGHI	İ. DEMİR	S. YALAZ TOPRAK
11:15-11:30					Coffee Break			
<b>Chair</b>	<b>A. KHALILİ GOL-MANKHANEH</b>	<b>H. OGATA</b>	<b>S. DİRİK</b>	<b>T. HANOYMAK</b>	<b>H. DURU</b>	<b>Z. KAYAR</b>	<b>A. ÇAKSU GÜLER</b>	<b>G. DATT</b>
11:30-10:50	Ö. AKÇAY	H. TOR	M. ILMAKCHI	E. A. UGURLU	A. SAMAILA	A. AL-KARALY	H. MENKEN	İ. GENÇTÜRK
11:50-12:10	S. MOMENALI	S. A. OTHMAN	M. ATÇEKEN	E. S. KARACAN	Y. S. HASSAN	B. İBTİSSEM	R. EKMEKÇİ	T. YAVUZ
12:10-12:30	A. JAFARIAN	M. MAHDIAN	U. YILDIRIM	İ. Hakkı DENİZLER	S. JOSHI	G. SAFAKISH	A. GÖÇER	S. ULUSAN
12:30-13:45					Lunch Break			
<b>Chair</b>	<b>E. AKYILDIZ</b>							
14:00-14:30	Hüseyin MERDAN				Invited Speaker			
14:30-15:00					Poster Presentations			
15:00-15:15					Coffee Break			
<b>Chair</b>	<b>E. KORKMAZ</b>	<b>Ö. AYDOĞMUŞ</b>	<b>S. EDİZ</b>	<b>A. BEHMARAM</b>	<b>A. SİRMA</b>	<b>M. Z. SARIKAYA</b>	<b>H. TAŞKESEN</b>	<b>Ü. TOTUR</b>
15:15-15:35	D. ALTINTAN	H. M. BASKONUS	İ. H. GUMUS	G. Kaya GÖK	A. MENNOUNI	M. E. KIRIŞ	H. S. ALKUTUBI	H. KAVURMACI ONALAN
15:35-15:55	S. ŞEVGIN	M. U. AKHMET	S. MEHRY	M. ELIASI	M. JALALVAND	M. R. JABBARZADEH	E. A. BİLGİN	H. KUSAK SAMANCI
15:55-16:15	S. A. MOHAMMED	K. GUSEINOV	A. LAKMECHE	A. MELEKOĞLU	S. TOUBAEI	H. JABBARI KHAMNEI	A. EL-D. EL-OZEIRI	
16:15-16:30					Coffee Break			
<b>Chair</b>	<b>D. ALTINTAN</b>	<b>N. AKGÜN</b>	<b>G. ŞENEL</b>	<b>A. MELEKOĞLU</b>	<b>G. SAKAR</b>	<b>İ. KÜÇÜK</b>	<b>S. ŞEVGIN</b>	<b>M. ÇAKIR</b>
16:30-16:50	N. N. PELEN	A. BAKLOUTI	A. BAYRAM	G. Kaya GÖK	S. JAMSHIDZADEH	G. DATT	Q. M. AB-DULOQADER	F. İLHAN
16:50-17:10	E. KORKMAZ	R. SAFAKISH	Ö. KÜSMÜS	M. CANGAN	N. A. MESSAOUDI	A. KUCUKASLAN	Y. KIRIŞ	D. ARSLAN
17:10-17:30	A. HOUMAT	M. KIR	S. TEMEL	S. EDİZ	M. KUDU	A. MERAL	M. MAHDIAN	R. ÖZTÜRK
17:30-18:30					Visit to the Van cat house (Social Program)			



Thursday, August 27

**Room Amphi** Chair: Abasalt BODAGHI

10:30-11:00 **Kenzu ABDELLA**

Solving boundary value problems using the sinc collocation method with derivative interpolation

11:00-11:15 Coffee Break

12:15-13:45 Lunch Break

**Room Amphi** Chair: Şenay BAYDAŞ

14:00-14:30 **İsmail KÜÇÜK**

Some aspects of optimal control

14:30-15:00 **Heybetkulu MUSTAFAYEV**

Some ergodic properties of measures

**Room 101** Chair: Ercan TUNÇ

11:15-11:35 **Cemil TUNÇ**

Stability and boundedness of solutions of volterra integro-differential equations

11:35-11:55 Cemil Tunç, **Melek GÖZEN**

On exponential stability of solutions of neutral differential system with multiple variable delays

11:55-12:15 **Timur AYHAN**, Cemil Tunç

Global existence and boundedness results for solutions of specific third order nonlinear vector differential equations

12:15-13:45 Lunch Break

**Room 101** Chair: Zeynep KAYAR

15:15-15:35 **Ercan TUNÇ**, Osman Tunç

On the oscillation of a class of damped fractional differential equations

15:35-15:55 **Hilmi ERGÖREN**

Neutral fractional differential equations with impulses at variable times

15:55-16:15 **Nihan ALIEV**, Ahmad Pashavand

Multipoint boundary value problem for a fractional order ordinary linear differential equation with variable coefficient

16:15-16:30 Coffee Break

**Room 101** Chair: Hilmi ERGÖREN

16:30-16:50 Cemil Tunç, **Ramazan YAZGAN**

On the existence of pseudo almost periodic solutions to a class of Lasota-Ważewska model differential equation

16:50-17:10 **Vahid ROOMI**

Asymptotic behavior of solutions of generalized Lienard system

**Room 102** Chair: Derya ALTINTAN

- 11:15-11:35 **Benseridi HAMID**  
Asymptotic analysis of a dynamical problem of non-isothermal linear elasticity with friction
- 11:35-11:55 **Dilmi MOURAD**  
Existence and regularity of the solution for nonlinear and oblique problems with friction
- 11:55-12:15 **Belaidi MOHAMED**  
Generalized Markov processes
- 12:15-13:45 Lunch Break

**Room 102** Chair: Mehmet Giyas SAKAR

- 15:15-15:35 Yakup Haci, **Muhammet CANDAN**  
Optimal control problem for processes given by multi-parameter linear stochastic dynamic system
- 15:35-15:55 Latifa Debbi, **Zineb ARAB**  
Numerical approximations for some fractional stochastic partial differential equations
- 15:55-16:15 **Aykut OR**, Yakup Haci  
Graphical method for interval valued bimatrix games
- 16:15-16:30 Coffee Break

**Room 102** Chair: Hakan TOR

- 16:30-16:50 **Reza DANAEI**, Aliasghar Jodayree Akbarfam  
Dynamical stability and mathematical modeling of heroin epidemic in Urmia
- 16:50-17:10 **Tarini Kumar DUTTA**  
Analysis on stability of limit cycles and hopf bifurcation in Van der pol nonlinear differential equation

**Room 103** Chair: Bülent KARAKAŞ

- 11:15-11:35 **Ahmet MOLLAOĞULLARI**, Mehmet Tekkoyun  
Euler-Lagrange and Hamilton-Jacobi equations on a Riemann almost contact model of a Cartan space of order  $k$
- 11:35-11:55 **Abeidallah MOHAMMED**  
Generalized Newton transformation and its application to transversal submanifolds
- 11:55-12:15 **Mustafa BUYUKARSLAN**, Oguzhan Bahadır  
On statistical manifold with dual connection and its applications
- 12:15-13:45 Lunch Break

**Room 103** Chair: Ali BAJRAVANI

- 15:15-15:35 **Hatice KUSAK SAMANCI**, Ali Caliskan  
A new approach to one parameter motion
- 15:35-15:55 Bülent Karakaş, **Şenay BAYDAŞ**  
Group structure of Markov polygons
- 15:55-16:15 Şenay Baydaş, **Bülent KARAKAŞ**  
Lie group structure on  $n \times n$  Markov matrix
- 16:15-16:30 Coffee Break

**Room 103** Chair: Şenay BAYDAŞ

- 16:30-16:50 **Hatice KUSAK SAMANCI**, Ali Caliskan  
The level curves and surfaces on time scales
- 16:50-17:10 **Ali BAJRAVANI**  
Projective geometry related to the secant loci in symmetric product of smooth algebraic curves
- 17:10-17:30 **Asra REZAFADAEI**  
Beautiful number 6174

**Room 104** Chair: Kenzu ABDELLA

- 11:15-11:35 **Kamil ARI**  
On the generalized k-Pell (p,i)-numbers
- 11:35-11:55 **Ali ZALNEZHAD**, Ghasem Shabani, Hossein Zalnezhad, Mehdi Zalnezhad  
Achieving the largest primes: Algorithm and relations in order
- 11:55-12:15 **Murat KARAKAS**, Hasan Karabudak  
A new regular matrix defined by Fibonacci numbers and its applications
- 12:15-13:45 Lunch Break

**Room 104** Chair: Turgut HANOYMAK

- 15:15-15:35 **Ayşın ERKAN GÜRSOY**, Kürşat Aker  
Murnaghan-Nakayama rule for Jack polynomials
- 15:35-15:55 **Essa AGHDASSI**  
Exact constants for best approximation on the group  $SU(2)$
- 15:55-16:15 **Tiachachat MERIEM**, Miloud Mihoubi  
The r-Whitney numbers linked to generalised Bernoulli polynomials
- 16:15-16:30 Coffee Break

**Room 104** Chair: Süleyman EDİZ

- 16:30-16:50 **Ayşın ERKAN GÜRSOY**, Kürşat Aker  
A combinatorial approach to Catalan numbers
- 16:50-17:10 **Afshin BEHMARAM**  
On some graph invariants of generalized Fullerene
- 17:10-17:30 Shahram Mehry, Reza Safakish, **Sadegh SADEGHI**  
The problem of clothseller's son

**Room 105** Chair: Hakkı DURU

- 11:15-11:35 **Fevzi ERDOGAN**, Kerem Yamac, Mehmet Gıyas Sakar  
An exponential fitted method for singularly perturbed reaction-diffusion equations
- 11:35-11:55 Fevzi Erdogan, **Kerem YAMAC**, Mehmet Gıyas Sakar  
A fitted Numerov method for singularly perturbed reaction-diffusion equations
- 11:55-12:15 **Onur SALDIR**, Mehmet Gıyas Sakar  
A combination of VIM and asymptotic expansion for singularly perturbed convection-diffusion problem
- 12:15-13:45 Lunch Break

**Room 105** Chair: Fevzi ERDOGAN

- 15:15-15:35 **Nagehan AKGÜN**  
DQM solution of natural convection flow of water-based nanofluid
- 15:35-15:55 **Ali SIRMA**  
Approximating the Riemann-Stieltjes integral in terms of Simpson's rule
- 15:55-16:15 **Opheyemi Oluwole ENOCH**  
The eigenvalues (energy levels) of the Riemann Zeta function
- 16:15-16:30 Coffee Break

**Room 105** Chair: Fevzi ERDOGAN

- 16:30-16:50 **Nagehan AKGÜN**  
DRBEM solution of natural convection flow of water-based nanofluids in an inclined angle
- 16:50-17:10 Hakki Duru, **Akbar BARATI CHIYANEH**  
Finite difference schemes on Shishkin mesh for singularly perturbed initial-boundary value Sobolev type problems
- 17:10-17:30 **Mehmet Giyas SAKAR**, Onur Saldır  
A new numerical approach for solving time-fractional partial differential equations

**Room 106** Chair: İ. Hakkı DENİZLER

- 11:15-11:35 **Houas AMRANE**, Mokhtari Zouhir  
A novel binary image encryption algorithm based on diffuse representation
- 11:35-11:55 Ömer Küsmüş, **Turgut HANOYMAK**  
A possible key exchange protocol over group rings
- 11:55-12:15 Fatemeh Fatahi, **Gholamreza SAFAKISH**  
On generalization of the strongly irreducible submodules
- 12:15-13:45 Lunch Break

**Room 106** Chair: İ. Hakkı DENİZLER

- 15:15-15:35 **Gulsen ULUCAK**, Unsal Tekir, Kursat Hakan Oral  
A note on lattice module
- 15:35-15:55 **Selçuk TOPAL**  
Algorithms in minimal ferrer graph constructions
- 15:55-16:15 **Selçuk TOPAL**  
A computational approach to syllogistic English sentences with ditransitive verbs in formal semantics
- 16:15-16:30 Coffee Break

**Room 106** Chair: Ali SIRMA

- 16:30-16:50 **Enes Abdurrahman BİLGİN**  
Development of an educational software for basic statistics
- 16:50-17:10 **Mustafa GÖK**, Erdal Beyde  
Analysis of the reasoning skills of students in solving a non-routine problem
- 17:10-17:30 **Barış KESLER**, Necat Görentaş  
On  $S_1$ ,  $B_1$  near rings

**Room 107** Chair: Nagehan AKGÜN

- 11:15-11:35 **Akindele Michael OKEDOYE**  
Heat transfer in hydro magnetic oscillatory flow past an impulsively started porous limiting surface with heat generation/absorption
- 11:35-11:55 **Hatice TAŞKESEN**  
Blow up of solutions for a nonlinear Timoshenko equation with positive initial energy
- 11:55-12:15 Mahmoud Abdel-Aty, **Mohamed KAMEL**  
Prospectives of geometric phase
- 12:15-13:45 Lunch Break

**Room 107** Chair: Özlem AK GÜMÜŞ

- 15:15-15:35 **Esra KARAOĞLU**, Hüseyin Merdan  
Hopf bifurcations of a ratio-dependent predator-prey model involving two discrete maturation time delays
- 15:35-15:55 **Mehmet Nuri ALMALI**, Zinnur Dikici, Özkan Atan  
The simulation of sound signal masking with sprout chaotic oscillation
- 15:55-16:15 **Omar EL-BASHA**, Ahmed El-Shahat, Hussin Fayed, Mahmoud Abdel-Aty  
Chaos theory and Lorenz attractors
- 16:15-16:30 Coffee Break

**Room 107** Chair: Mahmut KARAKUŞ

- 16:30-16:50 Ercan Tunç, **Orhan ÖZDEMİR**  
Oscillation theorems for second-order nonlinear differential equations with nonlinear damping
- 16:50-17:10 Hüseyin Merdan, **Özlem AK GÜMÜŞ**, Gamzegül Aydın  
Global stability analysis of a general scalar difference equation
- 17:10-17:30 Heybetkulu Mustafayev, **Cesim TEMEL**  
Mixing type theorem for power bounded measures

**Room 108** Chair: Cesim TEMEL

- 11:15-11:35 **Güzide ŞENEL**  
Matrix representation of soft points and its application
- 11:35-11:55 **Mahmut KARAKUŞ**  
On  $\lambda$ - semiconservative FK spaces
- 11:55-12:15 **Fatih KUTLU**, Tunay Bilgin  
Distance measures for temporal intuitionistic fuzzy sets
- 12:15-13:45 Lunch Break

**Room 108** Chair: Khalik GUSEINOV

- 15:15-15:35 **İclal GÖR**, Rifat Aşlıyan, Ömer Kalfa  
Textile image classification using naive bayes and multi-layer perceptron
- 15:35-15:55 **G. Selin SAVAŞKAN**, Aykut Or, Yakup Hacı  
Lemke-Howson algorithm for two-person non-zero games
- 15:55-16:15 **Reza KEYKHAEI**  
A generalized static mean-variance portfolio optimization

16:15-16:30 Coffee Break

**Room 108** Chair: Özgür AYDOĞMUŞ

16:30-16:50 **İclal GÖR**, Korhan Günel

Solving systems of linear differential equations by using artificial neural networks

16:50-17:10 **Ghanbary FATEMEH**, Ahmad Jafarian

Artificial intelligence based modeling for water treatment

17:10-17:30 **Hacer ŞENGÜL**, Mikail Et

On Wijsman  $I$ -lacunary statistical convergence of order  $\alpha$  of sequences

August 27, 2015 Thursday											
Time	Amphi	Room 101	Room 102	Room 103	Room 104	Room 105	Room 106	Room 107	Room 108		
8:00-10:00					Room 104						
<b>Chair</b>	<b>A. BODAGHI</b>				Breakfast (Social Program)						
10:30-11:00	Kenzu ABDELLA										
11:00-11:15											
<b>Chair</b>											
11:15-11:35		<b>E. TUNÇ</b>	<b>D. ALTINTAN</b>	<b>B. KARAKAŞ</b>	<b>K. ABDELLA</b>	<b>H. DURU</b>	<b>İ. H. DENİZLER</b>	<b>N. AKGÜN</b>	<b>C. TEMEL</b>		
11:35-11:55		C. TUNÇ	B. HAMID	A. MOL-LAOGÜLLARI	K. ARI	F. ERDOGAN	H. AMRANE	A. M. OKE-DOYE	G. ŞENEL		
11:55-12:15		M. GÖZEN	D. MOURAD	A. MOHAMMED NEZHAD	A. ZAL-	K. YAMAC	T. HANOY-MAK	H. TAŞKESEN	M. KARAKUŞ		
12:15-13:45		T. AYHAN	B. MOHAMED	M. BUYUKARSLAN	M. KARAKAS	O. SALDIR	G. SAFAKISH	M. KAMEL	F. KUTLU		
<b>Chair</b>	<b>Ş. BAYDAŞ</b>										
14:00-14:30	İsmail KUÇUK										
14:30-15:00	Heybetkulu MUSTAFAYEV										
15:00-15:15											
<b>Chair</b>		<b>Z. KAYAR</b>	<b>M. G. SAKAR</b>	<b>A. BAJRAVANI</b>	<b>T. HANOYMAK</b>	<b>F. ERDOGAN</b>	<b>İ. DENİZLER</b>	<b>Ö. AK GÜMÜŞ</b>	<b>K. GUSEINOV</b>		
15:15-15:35		E. TUNÇ	M. CANDAN	H. KUSAK SAMANCI	A. ERKAN GÜRSOY	N. AKGÜN	G. ULUCAK	E. KARAOĞLU	İ. GÖR		
15:35-15:55		H. ERGÖREN	Z. ARAB	Ş. BAYDAŞ	E. AGHDASSI	A. SIRMA	S. TOPAL	M. N. ALMALI	G. S. SAVAŞKAN		
15:55-16:15		N. ALIEV	A. OR	B. KARAKAŞ	T. MERIEM	O. O. ENOCH	S. TOPAL	O. EL-BASHA	R. KEYKHAEI		
16:15-16:30											
<b>Chair</b>		<b>H. ERGÖREN</b>	<b>H. TOR</b>	<b>Ş. BAYDAŞ</b>	<b>S. EDİZ</b>	<b>F. ERDOGAN</b>	<b>A. SIRMA</b>	<b>M. KARAKUŞ</b>	<b>Ö. AYDOĞMUŞ</b>		
16:30-16:50		R. YAZGAN	R. DANAELI	H. KUSAK SAMANCI	A. ERKAN GÜRSOY	N. AKGÜN	E. A. BÜLGİN	O. ÖZDEMİR	İ. GÖR		
16:50-17:10		V. ROOMI	T. K. DUTTA	A. BAJRAVANI	A. BEHMARAM	A. BARATI CHIYANEH	M. GÖK	Ö. AK GÜMÜS	G. FATEMEH		
17:10-17:30				A. REZAFADAELI	S. SADEGHI	M. G. SAKAR	B. KESLER	C. TEMEL	H. ŞENGÜL		

**Social Programme**

Wednesday, August 26

17:30-18:30 Visit to the Van cat House

19:00-21:00 Conference Dinner

Thursday, August 27

8:00-10:00 Van Breakfast

Friday, August 28

All day excursion: Visit to Van Fortress and Akdamar Island and Church



## **Abstracts of Invited Speakers**

PROSPECTIVE OF QUANTUM INFORMATION

Mahmoud ABDEL-ATY<sup>1</sup>, Lyazzat Atymtayeva

<sup>1</sup>*Zewail City of Science and Technology, Egypt*

<sup>2</sup>*Kazakh-British Technical University, Kazakhstan*

**Abstract**

In this communication we discuss different aspects of Bioinformatics models and its application quantum information and quantum computer. We focus on the dynamics of charge qubits coupled to a nanomechanical resonator under influence of both a phonon bath in contact with the resonator and irreversible decay of the qubits. Even in the presence of environment, the inherent entanglement is found to be rather robust. Due to this fact, together with control of system parameters, the system may therefore be especially suited for quantum computer. Our findings also shed light on the evolution of open quantum many-body systems.

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SOLVING BOUNDARY VALUE PROBLEMS USING THE SINC  
COLLOCATION METHOD WITH DERIVATIVE INTERPOLATION

**Kenzu ABDELLA**<sup>1</sup>

*Trent University, Peterborough, Canada*

**MSC 2000:** 34B05, 34B15, 34B40, 34B60, 65L10, 65Z05

**Abstract**

We consider the application of a Sinc-Collocation approach based on first derivative to solve boundary value problems (BVPs) arising from fluid dynamics related models. Even in the presence of singularities that are often present in fluid dynamics problems involving boundary layers, the Sinc-collocation technique provides exponentially convergent approximations including those posed on unbounded domains. The typical Sinc strategy is to start with the Sinc interpolation of the unknown function and to obtain its first and higher derivatives through successive differentiation in order to transform the BVP into discrete system which has a basic drawback as it is well-known that numerical differentiation process is highly sensitive to numerical errors. However, the first derivative interpolation approach presented in this paper uses Sinc-based integration to approximate the unknown has advantages over the customary Sinc method since integration has the effect of damping out numerical errors that are inherently present in numerical approximations. Moreover, the approach presented in this paper preserves the appropriate endpoints behaviors of the Sinc bases, resulting in a highly accurate and computationally efficient method [1]. The accuracy and stability of the proposed method is demonstrated through several fluid dynamics model problems including a hydrodynamic model of wind-driven currents and the Blasius and nonlinear BVPs [2]. It is further shown that the proposed approach is more accurate and computationally efficient than those obtained by other approaches.

**Keywords:** Boundary value problems, sinc-collocation, fluid dynamics.

**References**

- [1] K. Abdella, J. Comp. Methods in Sci. and Eng. **15** (2015) 1-11.
- [2] Y. Mohseniahouei, K. Abdella, M. Pollanen, J. of Comp. Sci. **7** (2015) 13-26.

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EXTENSION OF CHAOS, CONTROL AND APPLICATIONS

**Marat U. AKHMET**<sup>1</sup>

<sup>1</sup>*Department of Mathematics, Middle East Technical University, 06800 Ankara,  
Turkey*

**Abstract**

A new method of chaos extension introduced and developed in several our papers and summarized in the book [1] will be discussed. We concern theoretical aspects of the method as well as application opportunities in physics, economics, robotics, neural networks and meteorology.

**References**

- [1] M. U. Akhmet, M. O. Fen, Replication of chaos in neural networks, Physics and Economy, Springer, 2015.

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AN OVERVIEW OF DISCRETE LOG AND TRACE BASED PUBLIC KEY  
CRYPTOGRAPHY ON FINITE FIELDS

**Ersan AKYILDIZ**<sup>1</sup>

<sup>1</sup> *Institute of Applied Mathematics and Department of Mathematics, METU,  
Ankara, TURKEY*

**Abstract**

The Discrete Logarithm Problem (DLP), that is computing  $x$ , given  $y = \alpha^x$  and  $(\alpha) = G \subset \mathbb{F}_q^*$ , based Public Key Cryptosystem (PKC) have been studied since the late 1970s. Such development of PKC was possible because of the trapdoor function  $f : \mathbb{Z}_l \rightarrow G = (\alpha) \subset \mathbb{F}_q^*$ ,  $f(m) = \alpha^m$  is a group homomorphism. Due to this fact we have: Diffie Hellman (DH) type key exchange, ElGamal type message encryption, and Nyberg-Rueppel type digital signature protocols. The cryptosystems based on the trapdoor  $f(m) = \alpha^m$  are well understood and complete. However, there is another trapdoor function  $f : \mathbb{Z}_l \rightarrow G$ ,  $f(m) \rightarrow Tr(\alpha^m)$ , where  $G = (\alpha) \subset \mathbb{F}_{q^k}^*$ ,  $k \geq 2$ , which needs more attention from researchers from a cryptographic protocols point of view. In the above mentioned case, although  $f$  is computable, it is not clear how to produce protocols such as Diffie Hellman type key exchange, ElGamal type message encryption, and Nyberg-Rueppel type digital signature algorithm, in general. It would be better, of course if we can find a more efficient algorithm than repeated squaring and trace to compute  $f(m) = Tr(\alpha^m)$  together with these protocols. In the literature we see some works for a more efficient algorithm to compute  $f(m) = Tr(\alpha^m)$  and not wondering about the protocols. We also see some works dealing with an efficient algorithm to compute  $Tr(\alpha^m)$  as well as discussing the cryptographic protocols. In this review paper, we are going to discuss the state of art on the subject.

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RECENT TRENDS RELATED TO FOUR DIMENSIONAL MATRIX  
TRANSFORMATIONSFeyzi BAŞAR<sup>1</sup>,<sup>1</sup>*Fatih University, İstanbul, Turkey***MSC 2000:** 46A45, 40C05**Abstract**

Our main goal is to present a short survey on the spaces of double sequences and four dimensional matrix transformations. In Section 2, we give the corresponding results for four dimensional dual summability methods of the new sort to the results obtained by Altay and Başar in [*Some paranormed Riesz sequence spaces of non-absolute type*, Southeast Asian Bull. Math. **30** (5) (2006), 591–608] for two dimensional dual summability methods of the new sort. In Section 3, we present multidimensional analogues of Petersen' theorem "The necessary and sufficient conditions for the regular matrix  $A = (a_{mn})$  to be stronger than the regular Riesz mean  $(R, p_n)$ , where  $p_n > 0$  for all  $n \in \mathbb{N}$ " for double sequences. In Section 4, as the domain of four dimensional Riesz mean  $R^{qt}$  associated with the sequences  $q = (q_k)$  and  $t = (t_l)$  of non-negative real numbers in the spaces  $\mathcal{M}_u$ ,  $\mathcal{C}_p$ ,  $\mathcal{C}_{bp}$  and  $\mathcal{C}_r$ , we introduce the double sequence spaces  $R^{qt}(\mathcal{M}_u)$ ,  $R^{qt}(\mathcal{C}_p)$ ,  $R^{qt}(\mathcal{C}_{bp})$  and  $R^{qt}(\mathcal{C}_r)$ , and also examine some properties of those sequence spaces. Furthermore, we show that these sequence spaces are Banach spaces. Let  $\vartheta \in \{p, bp, r\}$ . We determine the  $\alpha$ -dual and  $\beta(\vartheta)$ -dual of the space  $R^{qt}(\mathcal{M}_u)$  and  $\beta(\vartheta)$ -duals of the spaces  $R^{qt}(\mathcal{C}_{bp})$  and  $R^{qt}(\mathcal{C}_r)$  of double sequences. Finally, we characterize the classes  $(R^{qt}(\mathcal{C}_r) : \mathcal{C}_\vartheta)$ ,  $(\mu : (R^{qt}(\mathcal{C}_\vartheta)))$  and  $(R^{qt}(\mathcal{C}_\vartheta) : \mathcal{C}_f)$  of four-dimensional matrix transformations, where  $\mu$  and  $\mathcal{C}_f$  denote any given double sequence space and the space of almost convergent double sequences, respectively. Section 5 is devoted to Steinhaus type theorems together with the definitions of four dimensional conull and coregular matrices, and the characterizations of the classes  $(\mathcal{M}_u : \mathcal{C}_\vartheta)$ , where  $\vartheta \in \{p, p0, f\}$ . In Section 6, we state and prove the Mercerian theorem for a four dimensional matrix and the space of convergent double sequences in the Pringsheim's sense.

**Keywords:** Double sequence space, paranormed sequence space, alpha-, beta-duals and matrix transformations.

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## SOME ASPECTS OF OPTIMAL CONTROL

**Ismail KUCUK<sup>1</sup>***Yildiz Technical University, Istanbul, Turkey***Abstract**

Applications of the optimal control problems arise in many fields of engineering and science. This talk presents a brief overview of the main ideas and concepts of optimal control problems. The discussion will take place in terms of the distributed parameter systems and on its applications to active control of smart mechanical systems and possible other applications will be introduced. The behavior of smart mechanical systems involving the control of vibrations are modeled through partial differential equations that involve unit step functions and their derivatives due to patches. Engineering applications of the patches can be seen in beams, plates, etc. The solution of the problem necessitates the implementation of numerical or approximate methods. The applications of these methods to piezolaminated smart beams using actuators will be discussed to illustrate the main ideas [1].

$$\mathcal{L}[w] = Kf(t) \left( \mathcal{H}''(x - x_1) - \mathcal{H}''(x - x_2) \right), \quad 0 < x < L, 0 < t < t_f, \quad (1)$$

Optimal control of nonlinear applications will also be presented briefly [2] along with possible future projects.

**Keywords:** Optimal control, maximum principle, variational methods.

**References**

- [1] I. Kucuk, I. S. Sadek, E. Zeini, S. Adali, Optimal vibration control of piezolaminated smart beams by the maximum principle, *Computers and Structures* **89** (2011) 744-749.
- [2] I. Kucuk, I. Sadek, A robust technique for solving optimal control of coupled Burgers equations, *IMA Journal of Mathematical Control and Information* **28** (2011) 239-250.

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## ASSET FLOW DIFFERENTIAL EQUATIONS

Hüseyin MERDAN<sup>1</sup>

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Ankara, Turkey*

**MSC 2010:** 91B25, 91B50, 91G99

### Abstract

In this talk, I will give an overview on deterministic asset pricing models. I will present asset flow differential equations used for modeling a single asset market involving a group of investors. Derivation of models are based on the assumption of the finiteness of assets (rather than assuming unbounded arbitrage) in addition to investment strategies that are based on either price momentum (trend) or valuation considerations.

**Keywords:** Ordinary differential equations for asset pricing, price dynamics, asset flow, dynamical system approach to mathematical finance.

### References

- [1] H. Merdan, G. Caginalp, W. C. Troy, Bifurcation analysis a single-group asset flow model, *Quarterly Applied Mathematics*, in press, 2015.
- [2] H. Merdan, M. Alisen, A mathematical model for asset pricing, *Applied Mathematics and Computation* **218** (2011) 449-1456.
- [3] G. Caginalp, H. Merdan, Asset price dynamics with heterogeneous groups, *Physica D* **225** (2007) 43-54.
- [4] G. Caginalp, G. B. Ermentrout, Numerical studies of differential equation related to theoretical financial markets, *Appl. Math. Lett.* **4** (1991) 35-38.
- [5] G. Caginalp, D. Balenovich, Market oscillations induced by the competition between value-based and trend-based investment strategies, *Appl. Math. Finance* **1** (1994) 129-164.
- [6] G. Caginalp, D. Balenovich, Asset flow and momentum: deterministic and stochastic equations, *Phil. Trans. R. Soc. Lond. A* **357** (1999) 2119-2133.

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## SOME ERGODIC PROPERTIES OF MEASURES

Heybetkulu MUSTAFAYEV<sup>1</sup>

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MSC 2000: 16S34, 16U60

**Abstract**

Let  $G$  be a locally compact abelian group with the dual group  $\Gamma$  and let  $M(G)$  be the convolution measure algebra of  $G$ . By  $\widehat{\mu}$  we denote the Fourier-Stieltjes transform of  $\mu \in M(G)$  :

$$\widehat{\mu}(\gamma) = \int_G \overline{\gamma}(g) d\mu(g), \quad \gamma \in \Gamma.$$

For  $n \in \mathbb{N}$ , by  $\mu^n$  we denote  $n$ -times convolution power of  $\mu \in M(G)$ . A measure  $\mu \in M(G)$  which satisfies  $\sup_{n \in \mathbb{N}} \|\mu^n\| < \infty$  is called *power bounded*.

In the case when  $1 < p \leq 2$ , by  $\widehat{f}$  we will denote the Hausdorff-Young-Plancherel transform of  $f \in L^p(G)$ . For a closed subset  $F$  of  $\Gamma$ , by  $L^p(F)$  we denote the set of all  $f \in L^p(G)$  such that  $\widehat{f} = 0$  almost everywhere on  $F$  ( $\widehat{f}$  is only defined up to sets of Haar measure zero).

We have the following.

**Theorem.** Let  $G$  be a locally compact abelian group and let  $\mu$  be a power bounded measure on  $G$ . If  $1 < p \leq 2$ , then the following conditions are equivalent for a closed subset  $F$  of  $\Gamma$ :

- (a)  $\lim_{n \rightarrow \infty} \left\| \frac{1}{n} \sum_{k=0}^{n-1} \mu^k * f \right\|_p = 0$ , for all  $f \in L^p(F)$ .  
 (b)  $\widehat{\mu}(\gamma) \neq 1$ , for all  $\gamma \in \Gamma \setminus F$ .

**References**

- [1] U. Krengel, Ergodic Theorems, Walter de Gruyter, Berlin New York, 1985.  
 [2] B. M. Schreiber, Measures with bounded convolution powers, Trans. Amer. Math. Soc. **151** (1970) 405-431.  
 [3] R. Jones, J. Rosenblatt, A. Tempelman, Ergodic theorems for convolutions of a measure on a group, Illinois J. Math., **38** (1994) 521-553.  
 [4] E. Kaniuth, A.T. Lau, A. Ülger, Power boundedness in Fourier and Fourier-Stieltjes algebras and other commutative Banach algebras, J. Funct. Anal. **260** (2011) 2366-2386.

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## HYPERFUNCTION METHOD FOR NUMERICAL INTEGRATIONS

**Hidenori OGATA<sup>1</sup>, Hiroshi HIRAYAMA<sup>2</sup>***<sup>1</sup>The University of Electro-Communications, Chofu, Japan**<sup>2</sup>Kanagawa Institute of Technology, Atsugi, Japan***MSC 2000:** 65D30, 65D32**Abstract**

In this paper, we examine a numerical integration method proposed by Hirayama [1]. In his method, an integral  $I = \int_a^b f(x)w(x)dx$  ( $-\infty < a < b < +\infty$ ), where  $f(x)$  is a given real analytic function and  $w(x)$  is a weight function, is transformed into the complex integral on a closed contour

$$I = \frac{1}{2\pi i} \oint_C f(z)\Psi(z)dz \quad \text{with} \quad \Psi(z) = \int_a^b \frac{w(x)}{z-x} dx, \quad (1)$$

where  $C$  is a closed contour surrounding the interval  $[a, b]$  and included in a complex domain  $D$  such that  $f(z)$  is analytic in it, and is approximated by the trapezoidal rule. We here call this method the “hyperfunction method” since (1) is the definition of the integral  $I$  when the integrand  $f(x)w(x)$  is regarded as a hyperfunction [2]. The hyperfunction method gives good approximations especially for integrals with so strong end-point singularities that the DE rule [3] does not work for them.

**Keywords:** Numerical integration, analytic function, hyperfunction.

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## **Abstracts of Speakers**

EXTENSION OF UNIFORMLY PRIMARY IDEALS

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**MSC 2000:** 13A15

**Abstract**

In this study, we combine the concept of *strongly primary* ideal and the concept of *uniformly primary* ideal. So, to define *strongly uniformly primary* ideal, the ring  $R$  must be integral domain with the quotient field  $K$ . We study the basic properties of *strongly uniformly primary* ideals. Moreover we examine the relation between *strongly uniformly primary* ideals and *strongly prime* ideals. Finally, we define *Noetherian Strongly  $s$ -primary ideals* and *Mori Strongly  $s$ -primary* ideals. Thus, we obtain the relation between *strongly uniformly primary* ideals, *strongly primary* ideals, *Noetherian Strongly  $s$ -primary ideals* and *Mori Strongly  $s$ -primary* ideals.

**Keywords:** strongly primary, uniformly primary, strongly uniformly primary

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NUMERICAL SOLUTION OF THE ROSENAU-BURGERS EQUATION USING  
QUINTIC B-SPLINE COLLOCATION METHOD

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**MSC 2000:** 65M06, 65M12

### Abstract

Consider the initial-boundary value problem of Rosenau-Burgers equation as follow

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial^5 u}{\partial x^4 \partial t} + \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = 0, \\ u(x, t) = u_{xx}(x, t) = 0, \quad x \in \partial\Omega, t \in (0, T], \\ u(x, 0) = u_0(x), \quad x \in \bar{\Omega}, \end{cases} \quad (1)$$

where  $x \in \Omega = [0, L]$ ,  $t \in (0, T]$ ,  $L > 0$ ,  $T > 0$ . Since, the exact solution of the Rosenau–Burgers equation is unknown, and on the other side, this equation has important role in the mechanical engineering, therefore, there are some good works that has been devoted to approximate the numerical solutions to the initial–boundary value problem of Rosenau–Burgers equation [3, 5]. In this paper, a B-spline algorithm based on the collocation method with trial functions taken as quintic B-spline functions over the elements will be constructed. This scheme is based on the Crank-Nicolson formulation for time integration and quintic B-spline functions for space integration. The unconditional stability of the method is proved using Von-Neumann approach. A prior bound and the error estimates of the approximate solutions are discussed with a numerical example.

**Keywords:** Rosenau-Burgers equation, Quintic B-spline method, Crank-Nicolson scheme, Thomas algorithm, error estimate.

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PROSPECTIVES OF GEOMETRIC PHASE

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**Abstract**

In this paper we discuss some new models of geometric phase and its applications. Numerical treatments have been done to compare between the strong coupling and weak coupling.

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COUPLING OF FINITE AND HIERARCHICAL INFINITE ELEMENTS:  
APPLICATION TO A HOMOGENEOUS ISOTROPIC HALF-SPACE

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**Abstract**

Boundary value problems of half-spaces often find applications in structure-soil interaction where the soil bounding the structure extends to infinity. Such problems are generally difficult if not impossible to solve analytically and resort to numerical methods is usually made. Houmat [1] developed a method for coupling cubic-order quadrilateral finite elements with the finite side of a new coordinate ascent hierarchical infinite element using the method of least squares [2]. The method was applied to a non-homogeneous cross-anisotropic half-space subjected to a non-uniform circular loading.

In this work, the method developed in [1] is extended to a homogeneous isotropic half-space subjected to a uniform circular load applied on the surface. A fixed mesh constructed from coupled finite and hierarchical infinite elements is used and convergence is sought simply by increasing the degree of the interpolating polynomial. The vertical displacement and vertical stress below the center of the load are obtained. The efficiency of the technique is demonstrated through convergence study as a function of the degree of the interpolating polynomial and comparison with analytical solutions.

**Keywords:** Coupling of finite and hierarchical infinite elements, least squares, homogeneous isotropic half-space.

**References**

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COMPARISON BETWEEN DISCRIMINANT ANALYSIS AND LOGISTIC  
REGRESSION ANALYSIS:AN APPLICATION ON CAESAREAN BIRTHS AND  
NATURAL BIRTHS DATA

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**MSC 2000:** 62-06

### **Abstract**

The Discriminant Analysis (DA) and the Logistic Regression Analysis (LRA) are two statistical methods used for analyzing data and predicting group membership from a set of predictors. Many applications have been done in this area such as the recent works of [1,2]. In [3] Shaheen focused through application on Leukemia data for the comparison between three forms for classification data belongs two groups when the response variable has two categories only. In this paper we shall apply both (DA) and (LRA) for the caesarean births and natural births data using stepwise method. We also make a comparison between the two analysis and then we choose the best one for classifying the type of birth depending on the results of the analysis.

**Keywords:** Discriminant analysis, logistic regression analysis, caesarean births

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SOLUTIONS AND CONSERVATION LAWS OF A TWO-DIMENSIONAL  
INTEGRABLE GENERALIZATION OF THE KAUP-KUPERSHMIDT  
EQUATION

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**Abstract**

A two-dimensional integrable generalization of the Kaup-Kupershmidt equation, which arises in various problems in mathematical physics. Exact solutions are obtained using the Lie symmetry method along with the extended tanh method and the extended Jacobi elliptic function method. More over we present conservation laws which are derived using the multiplier approach.

**Keywords:** Lie symmetry method, extended tanh method, Jacobi elliptic function method.

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EXACT CONSTANTS FOR BEST APPROXIMATION ON THE GROUP  $SU(2)$

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**Abstract**

In the present paper we study the properties of least upper bound of the best approximation by algebraic polynomials in metrics  $L_1$  and  $L_\infty$  for classes of convolutions defined on group  $SU(2)$ . The exact constants for best approximation by trigonometric polynomials in  $L_\infty(-\pi, \pi)$  is studied by many authors. Finally in this paper we proved that for group  $SU(2)$  analog of the Favard-Akheizer-Krein theorem does not hold.

**Keywords:** Group  $SU(2)$ , the Favard-Akheizer-Krein theorem, the best approximation.

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EXISTENCE AND UNIQUENESS RESULTS FOR A CLASS OF FRACTIONAL  
BOUNDARY VALUE PROBLEMAsghar AHMADKHANLU<sup>1</sup><sup>1</sup>*Azərbaycan Şahid Mədani Universiteti, Təbriz, İran*

MSC 2000: 34BXX

**Abstract**

In this work a class of boundary value problem including fractional differential equation is studied. The existence and uniqueness of solution for a nonlinear fractional boundary value problem are discussed. This problem includes a nonlinear fractional differential equation of order  $\alpha \in (0, 1]$  and fractional integral boundary conditions. In fact we consider the following boundary value problem of fractional differential equation

$$\begin{aligned} {}^c\mathcal{D}^\alpha y(t) &= f(t, y(t)) & 0 < \alpha < 1, & \quad t \in J := [0, T] \\ y(0) + \mu \int_0^T y(s) ds &= y(T), \end{aligned} \tag{1}$$

where  ${}^c\mathcal{D}^\alpha$  denotes the Caputo fractional derivative of order  $\alpha$ ,  $f : J \times \mathbb{R} \rightarrow \mathbb{R}$  is given function will be specified later and  $\mu \in \mathbb{R}$ .

Banach contraction principle and Browder-Poter fixed point theorem will be used for proving existence and uniqueness of solution for that problem.

**Keywords:** Fractional differential equations, fractional integral condition, boundary value problem, fixed point.

**References**

- [1] R. P. Agarwal, B. Andrade, C. Cuevas, Weighted pseudo-almost periodic solutions of a class of semilinear fractional differential equations, *Nonlinear Anal. Real World Appl.* **11** (2010) 3532-3554.
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A COMPARISON OF METHODS FOR COMPUTING THE MATRIX  
EXPONENTIAL APPEARS IN SYSTEMS OF DIFFERENTIAL EQUATIONS

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**MSC 2000:** 65F99, 15A15

**Abstract**

The matrix exponential commonly arises in the applications of various scientific fields due to the fact that it can provide the solution of the systems of linear differential equations arising in the mathematical modelling of scientific problems. There are a number of methods to compute the matrix exponential  $e^A$  for any given square matrix  $A$ . However, none of them are completely satisfactory. This paper aims to investigate and analyse a certain number of these methods, in terms of accuracy and efficiency, such as Taylor series method, Padé approximant, the scaling and squaring algorithm, and the spectral decomposition technique. All of the methods have been implemented in MATLAB environment, and then a number of experiments have been carried out on these methods. As a result of the accuracy and efficiency tests, we have found that the scaling and squaring algorithm, is the most accurate and cost-efficient method.

**Keywords:** Matrix exponential, Taylor series, Padé approximant, accuracy, efficiency.

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EXPONENTIAL STABILITY OF PERIODIC SOLUTIONS OF RECURRENT  
NEURAL NETWORKS WITH FUNCTIONAL DEPENDENCE ON PIECEWISE  
CONSTANT ARGUMENT

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**MSC 2010:** 34K13,34K20,92B20

### Abstract

Akhmet [1] generalized differential equations with piecewise constant argument by taking any piecewise constant functions as arguments, and recently he introduced functional dependence on piecewise constant argument [2]. These equations play an important role in applications such as neural networks [3]. In this study, we develop a model of recurrent neural network with functional dependence on piecewise constant argument of generalized type given by

$$x'(t) = -Ax(t) + Ex(\gamma(t)) + Bh(x_t) + Cg(x_{\gamma(t)}) + D. \quad (1)$$

Using the theoretical results obtained by Akhmet [2], we investigate conditions for exponential stability of periodic solutions for (1).

**Keywords:** Differential equations with functional dependence on piecewise constant argument, recurrent neural networks, stability, periodic solutions.

### References

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SHARP RUSAK-TYPE INEQUALITIES FOR RATIONAL FUNCTIONS ON  
SEVERAL INTERVALS**Mehmet Ali AKTÜRK**<sup>1</sup>, **Alexey LUKASHOV**<sup>2</sup><sup>1</sup>*Istanbul University, Istanbul, Turkey*<sup>2</sup>*Fatih University and Saratov State University, Istanbul and Saratov, Russia***MSC 2000:** 41A17, 41A20**Abstract**

We consider sharp Rusak-type inequalities for rational functions on several intervals when the system of intervals is a “rational function inverse image” of an interval and those functions are large in gaps.

Let  $\mathfrak{R}(\xi_1, \dots, \xi_{2n})$  be the set of all “rational functions” of the form

$$r(x) = \frac{b_0 x^n + b_1 x^{n-1} + \dots + b_n}{\sqrt{\rho_\nu(x)}}, \quad b_0, \dots, b_n \in \mathbb{C} \text{ and } \rho_\nu(x) = \prod_{j=1}^{2n} (x - \xi_j)$$

is a real polynomial of degree  $\nu$  which is positive on  $E = \bigcup_{j=1}^l [a_{2j-1}, a_{2j}]$ ,  $-1 = a_1 < a_2 < \dots < a_{2l} = 1$ . ( $\xi_j$  might be equal to  $\infty$ , then  $(x - \xi_j)$  should be omitted) Consider also the set  $\mathfrak{R}^*(\xi_1, \dots, \xi_{2n})$  which consists of those functions  $r \in \mathfrak{R}(\xi_1, \dots, \xi_{2n})$ , which satisfy  $|r(x)| > \|r\|_{C(E)}$  for all  $x \in [-1, 1] \setminus E$ . The last condition can not omit.

**Theorem.** Suppose that  $\sum_{j=1}^{2n} \omega_k(\xi_j) = 2q_k$ ,  $q_k \in \mathbb{N}$ ,  $k = 1, \dots, l$ , and  $|\xi_j| > 1, j = 1, \dots, 2n$ . Then for any  $r \in \mathfrak{R}^*(\xi_1, \dots, \xi_{2n}), \|r\|_{C(E)} = 1$  the inequality

$$|r'(x)| \leq \begin{cases} \gamma'_n(x), & x \in \tilde{E}_n, \\ |m'_n(x)|, & x \in E \setminus \tilde{E}_n \end{cases} \quad (1)$$

is valid, where

$$m_n(x) = \cos(\gamma_n(x)), \quad \gamma_n(x) = \frac{\pi}{2} \int_{a_1}^x \sum_{j=1}^{2n} \varpi_E(x, \xi_j) dx,$$

$$\tilde{E}_n = [x_1, x_{q_1}] \cup [x_{q_1}, x_{q_1+q_2}] \cup \dots \cup [x_{q_1+\dots+q_{l-1}}, x_n],$$

and  $x_1 < \dots < x_n$  are zeros of  $m_n$  (there are  $q_k$  zeros on  $[a_{2k-1}, a_{2k}], k = 1, \dots, l$ ).

For  $r(x) \equiv \varepsilon m_n(x)$ ,  $|\varepsilon| = 1$ , inequality in (1) is attained.

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**Keywords:** Inequalities in approximation, approximation by rational functions.

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ASPECTS OF ANALITICAL SOLUTIONS AND SIMULATION OF HIGH  
ORDER ODE

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**MSC 2000:** 34C10

**Abstract**

This article will introduce the concept of Laplace transformation and how it improved the process of obtaining the exact solution of ODE and its application. This article will focus on the one and two degree of freedom systems, and how can the system be solved by even easier models such as state space modeling, and also how the system can be simulated by different methods. Finally there will be a comparative study to see which method is the least complex and more accurate to find the solution.

**Keywords:** Laplace transformation, ODE, state space, simulink

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GROUP-2-GROUPOIDS AND 2G-CROSSED MODULES

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**MSC 2000:** 18D05, 18D35, 20J15, 20L05

**Abstract**

The main idea of this paper is to construct the group structure on a 2-groupoid which we call *group-2-groupoid*. As an algebraic structure corresponding to a group-2-groupoid, a *2G-crossed module* is obtained on the structure of crossed modules. Then we prove the categorical equivalence between group-2-groupoids and 2G-crossed modules.

**Keywords:** 2-groupoid, group-2-groupoid, 2G-crossed module.

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MULTIPOINT BOUNDARY VALUE PROBLEM FOR A FRACTIONAL  
ORDER ORDINARY LINEAR DIFFERENTIAL EQUATION WITH VARIABLE  
COEFFICIENT

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**Abstract**

In problems stated for ordinary linear differential equations with fractional order derivative, the number of boundary conditions is determined by the step changed by the derivative in the equation. In this paper we'll consider a multipoint boundary value problem for a variable coefficient equation and the domain of the problem will be taken arbitrary (i.e. that can contain zero). For that factorials are determined in a new sense and the independent solutions are constructed with its help.

**Keywords:** fractional derivative, a factorial in a new sense, multipoint boundary value problem, a new expansion formula for a function.

**References**

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## REGRESSION ANALYSIS FOR BREST CANCER PATIENTS

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### **Abstract**

In this study, Breast cancer in Al-Sader Medical city was introduced . Regression analysis is used to analyzed the data to get the mathematical model and the effect between all variables. In the results, there exist a positive relationship between Y (tumor levels) and X1 (education) in the years 2013, 2014.

**Keywords:** Linear regression, correlation coefficient, brest cancer.

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THE SIMULATION OF SOUND SIGNAL MASKING WITH SPROUT  
CHAOTIC OSCILLATION

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**MSC 2000:** 34C28

**Abstract**

Chaotic masking, which is one of the subjects aimed to supply the information security in communication medium, is addressed in this work. The system of Sprout used in chaotic masking and how to make chaotic masking in system are introduced. Then, PID control method providing synchronization in the system is defined. The masking process on sound signals using chaotic oscillation is simulated with Matlab/Simulink registered program.

**Keywords:** Chaos, chaotic masking, synchronization, PID, chaotic oscillator.

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DQM SOLUTION OF NATURAL CONVECTION FLOW OF WATER-BASED  
NANOFLUIDS

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**Abstract**

In this study, unsteady natural convection heat transfer of water-based nanofluid in a square cavity with heat source at the left vertical wall is studied by solving the equation of conservation of mass, momentum and energy. Stream function-vorticity form of the governing equations are solved by using the differential quadrature method (DQM). Vorticity transport and energy equations are transformed to the form of modified Helmholtz equations by discretizing the time derivative terms first. This procedure eliminates the need of another time integration scheme in vorticity transport and energy equations, and has the advantage of using large time increments. The computational results are obtained for Rayleigh number values between  $10^3$  and  $10^6$ , volume fraction of nanoparticles changing from 0 to 0.2 and the length of the heater varying from 0.25 to 1.0. Also, two types of nanoparticles ( $Al_2O_3$  and Cu) are tested. The results are show that the type of the nanoparticles and the length of the heat source affect the flow and temperature flow.

**Keywords:** DQM, natural convection, nanofluid

**References**

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DRBEM SOLUTION OF NATURAL CONVECTION FLOW OF  
WATER-BASED NANOFLUIDS IN AN INCLINED ANGLE

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**Abstract**

In this study, heat transfer and fluid flow due to buoyancy forces in a partially heated and an inclined square enclosure is carried out using two types of nanoparticles ( $Al_2O_3$  and Cu). Stream function-vorticity form of the governing equations are solved by using dual reciprocity boundary element method (DRBEM) with the fundamental solution of modified Helmholtz equation. By using the form of modified Helmholtz equation for the governing equations, the need of another time integration scheme is eliminated. Results are given in terms of streamlines, isotherms and vorticity contours for inclined angle from  $0^\circ$  to  $90^\circ$ , Rayleigh number values between  $10^3$  and  $10^6$ , and volume fraction of nanoparticles changing from 0 to 0.2. Also, the length of the heater is taken 0.25, 0.5 and 1.0 which is placed at the center of the left wall. The results are show that the type of the nanoparticles, the length of the heat source and the inclined angle affect the heat transfer of the fluid.

**Keywords:** DRBEM, natural convection, nanofluid, inclined angle.

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FUNCTIONAL IMPULSES IN EXACT STOCHASTIC SIMULATION

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**Abstract**

Jumps which are observed in many population models give rise to fluctuations in the dynamics of systems. Deterministic model which is based on the Impulsive Differential Equations (IDEs) considers these jumps as impulses and defines the dynamics of the system between successive jump times with the Ordinary Differential Equations (ODEs). From our previous studies, we have proposed a model which is the complement of IDEs in the sense that both studies consider the jumps as impulses. The main difference between these two approaches is that the former implements ODEs to model the dynamics of system between successive jump times while the latter applies the Chemical Master Equation (CME). From the analyses we have shown that such impulses can be added to the system under the two main scenarios, namely, impulses at fixed time and impulses at fixed states. Hereby as the novelty in this work, we extend our model in such a way that if the jump function and the realization of the model intersect, we update the time to the intersection time point and update the state vector according to the jump function. We insert this idea in the exact Gillespie algorithm and assess the performance of our extended model in different epidemic modellings.

**Acknowledgement:** This work is supported by the AGEF grant (No: BAP-08-11-2014-007) of the Middle East Technical University.

**Keywords:** Impulsive differential equations, ordinary differential equations, stochastic simulation, Gillespie algorithm, epidemic models.

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A NOVEL BINARY IMAGE ENCRYPTION ALGORITHM BASED ON  
DIFUSE REPRESENTATION

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**Abstract**

Despite the fact that binary images are very common in our life, only a few research works have been done to protect this type of images. In this work, we propose a new algorithm to encrypt binary images. In the first step, we present a new basis to reduce the amount of data required to represent the image. In the second stage, the image is split into  $d$  blocks, which use in new images of the same size as the original, and represent them in the new basis to obtain a key-image and encrypted images, the parameters obtained by this transformation are considered as key-image for the encryption and decryption algorithm. The decryption step is made by subtraction between each encrypted image and key-image, then summing them in an image to obtain the original one. Experimental results introduced at the end of this article demonstrate the effectiveness of the proposed strategy.

**Keywords:** Cryptography, binary image encryption, key-image.

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ON THE GENERALIZED K-PELL (P,I)-NUMBERS

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**MSC 2010:** 11B39; 65Q30; 05A15

**Abstract**

This study focus on the generalized  $k$ -Pell  $(p, i)$ -numbers for  $k = 1, 2, \dots$  and  $0 \leq i \leq p$ . It introduces the generalized  $k$ -Pell  $(p, i)$ -numbers and their generating matrices and generating functions. Some interesting identities are established. The basic properties of Fibonacci and Fibonacci-like numbers are well known and are outlined, for example in [1] and generalizations of Pell numbers can be found in the literature. In [2] P. Catarino consider a generalization of Pell numbers, which the author calls the  $k$ -Pell numbers.

**Keywords:** Fibonacci numbers, Pell numbers, Binet's formula.

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$\gamma$ -RADICALS OF GAMMA RINGS

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**MSC 2000:** 16N60, 16W25, 16Y99

**Abstract**

Let  $M$  be a weak Nobusawa  $\Gamma$ -ring and  $\gamma$  be a nonzero element of  $\Gamma$ . The main focus of this work is to find out new properties for the structure of  $\Gamma$ -ring  $M$ . For this reason, we define  $\gamma$ -Lie ideals of  $\Gamma$ -ring  $M$  and investigate commutativity conditions for  $M$  with derivation. We also define some  $\gamma$ -radicals of the  $\Gamma$ -ring  $M$  and show these radicals are strictly weaker than the radicals of  $M$  in the literature.

**Keywords:** Gamma ring, prime  $\Gamma$ -ring,  $k$ -derivation, commutativity,  $\gamma$ -radical.

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FRACTIONAL SPACES GENERATED BY THE POSITIVE DIFFERENTIAL  
OPERATOR IN THE HALF-SPACE

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**MSC 2000:** 35J25, 47E05, and 34B27

### Abstract

In the study, we consider the positivity of multi-dimensional differential operator in the half-space. We investigate the structure of fractional spaces generated by differential operators in the half-space. We establish the equivalence of the norms of these fractional spaces and Hölder spaces.

We also discuss its applications to theory of partial differential equations.

**Keywords:** Positive operator, fractional spaces, Green's function, Hölder spaces.

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GRÜSS INEQUALITY ON DISCRETE FRACTIONAL CALCULUS WITH  
DELTA OPERATOR**Serkan ASLIYÜCE<sup>1</sup>, A. Feza GÜVENİLİR<sup>2</sup>**<sup>1,2</sup>Ankara University, Ankara, Turkey**MSC 2000:** 39A12, 34A25, 26A33**Abstract**

In this talk, firstly we will give basic definitions and theorems of discrete fractional calculus with delta operator. After that, using fractional delta operators we shall introduce the inequality given by G. Grüss in 1935:

If  $f$  and  $g$  are continuous functions on  $[a, b]$  satisfying

$$\phi \leq f(t) \leq \Phi \text{ and } \gamma \leq g(t) \leq \Gamma \text{ for all } t \in [a, b],$$

then

$$\left| \frac{1}{b-a} \int_a^b f(x)g(x)dx - \frac{1}{(b-a)^2} \int_a^b f(x)dx \int_a^b g(x)dx \right| \leq \frac{1}{4}(\Phi - \phi)(\Gamma - \gamma).$$

**Keywords:** Discrete fractional calculus, Grüss type inequality.

**References**

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REGULAR LOCAL FUNCTIONS IN IDEAL TOPOLOGICAL SPACES

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**MSC 2000:** 54A05,54C10

**Abstract**

This paper deals with a space in which topology is replaced by its generalized open sets. We define an operator  $A^{*r}(I, \text{RO}(X, \tau))$  called the regular local function of  $A$  with respect to  $I$  and  $\text{RO}(X, \tau)$  as follows:  $A^{*r}(I, \text{RO}(X, \tau)) = \{x \in X : A \cap U \notin I \text{ for every } U \in \text{RO}(X, \tau)\}$ . We investigate properties of  $A^{*r}(I, \text{RO}(X, \tau))$ .

**Keywords:** Regular open set, regular closed set, ideal topological space, local function, regular local function.

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ON ALMOST  $C(\alpha)$ -MANIFOLDS SATISFYING CERTAIN CONDITIONS ON  
QUASI-CONFORMAL CURVATURE TENSOR

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**MSC 2000:** 53C15, 53C44, 53D10.

### Abstract

In the present paper, we have studied the curvature tensors of almost  $C(\alpha)$ -manifolds satisfying the conditions  $\tilde{C}(\xi, X)\tilde{Z} = 0$ ,  $\tilde{C}(\xi, X)P = 0$ ,  $\tilde{C}(\xi, X)S = 0$ ,  $\tilde{Z}(\xi, X)\tilde{C} = 0$  and  $\tilde{C}(\xi, X)\tilde{C} = 0$ . According these cases, we classified almost  $C(\alpha)$ -manifolds, where  $P$  is the Weyl projective curvature tensor,  $\tilde{Z}$  is the concircular curvature tensor,  $S$  is the Ricci tensor and  $\tilde{C}$  is quasi-conformal curvature tensor.

**Keywords:** Almost  $C(\alpha)$ -manifold, quasi-conformal curvature tensor, Einstein manifold.

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MULTIPLE SOLUTIONS TO P-KIRCHHOFF TYPE PROBLEMS IN  $\mathbb{R}^N$   
CRITICAL P-KIRCHHOFF EQUATIONS

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**Abstract**

In this paper, we use variational methods to study the existence and multiplicity of non negative solutions for a p-Kirchhoff equation involving the critical Sobolev exponent.

**Keywords:** p-Kirchhoff equation, the critical Sobolev exponent, non negative solutions.

**References**

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STABILITY ANALYSIS OF HIV INFECTION MODEL WITH TUMOR

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**MSC 2010:** 34D20, 37C75, 92B99

### Abstract

In this study, we propose a mathematical model of HIV infection with tumor cells. We model the interaction between tumor cells, helper T cells, infected helper T cells and virus cells by using a nonlinear dynamical system approach which gives rates of change of the four cell populations in the body. First, we prove the positivity of the solution, as desired in any population dynamics. Then, we analyze the local asymptotic stability of equilibrium points of the HIV infection model. In the end, we support our theoretical results by some numerical simulations.

**Keywords:** HIV infection, nonlinear dynamical system, stability analysis.

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EXTINCTION IN A GENERALIZED CHAIN BINOMIAL EPIDEMIC MODEL

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**Abstract**

Here, our aim is to study extinction times in a stochastic epidemic model. First we consider the mean dynamics of the stochastic model. Since we are interested in a nonoverlapping population, our meanfield equations are difference equations. We give conditons for existence and global stability of endemic equilibrium. We show that the stochastic model stays close to the deterministic model for finite time. Using this approximation, we also give exponential lower bounds for mean time to extinction. In addition, we also calculate mean time to reach endemic equilibrium for large populations.

**Keywords:** Chain binomial epidemic model, difference equations, extinction time.

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GLOBAL EXISTENCE AND BOUNDEDNESS RESULTS FOR SOLUTIONS OF  
SPECIFIC THIRD ORDER NONLINEAR VECTOR DIFFERENTIAL  
EQUATIONS

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**Abstract**

We give criteria for the global existence and boundedness of all solutions of a kind of third order nonlinear ordinary vector differential equations of the form:

$$\left( q(t) \left( p(t) X' \right)' \right)' + F(X, X') X'' + G(X') X' + cX = P(t)$$

By means of the Lyapunov second (direct) method, we obtain a new result on the subject and give an example for the illustration of the topic. Our result includes and generalizes some earlier results in the literature.

**Keywords:** Global existence, Lyapunov function, boundedness, third order.

**References**

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THEORETICAL AND COMPUTATIONAL MODELING STUDIES ON THE  
-CYPERONE - TUBULIN INTERACTION

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**Abstract**

In order to determine the binding site of -cyperone, the main sesquiterpenoid of *Cyperus rotundus* L. on tubulin, one of the main protein in the eukaryotic cytoskeleton, we performed molecular dynamics (MD) simulation and docking studies. Simulation was done using the GROMACS package with G43a1 force field. The equilibrium geometries of the tubulin with heteroatoms GTP and GDP (PDB: 1tub) were achieved using MD simulation at 300 K. The entire system was minimized using the steepest descent of 1000 steps followed by the conjugate gradients of 9000 steps. To demonstrate the quality of the simulation data, we determined the root mean square deviation (RMSD) of protein backbone. The final structure was employed as the main configuration for -cyperone binding studies. Autodock 3.0 package was used for molecular blind docking, and pre-set calculation parameters in Autodock 3.0 were considered as default. Effective surface hydrophobicity influences the intermolecular interactions, such as binding of small hydrophobic ligands with macromolecules, suggesting a hydrophobic interaction between -cyperone and -tubulin. Understanding the association site and energy of binding, that underlies -cyperone action to tubulin, is critical in the rational design of new drugs. The tubulin RMSD increased within 4 ns and then fluctuated in nano-scale around 0.3 nm till 10 ns, indicating that after elimination of the unfavorable strains from X-ray geometry, tubulin acquired a rather stable form, following an energy minimization (steepest descent of 1000 steps followed by the conjugate gradients of 9000 steps) for maintaining the better structure. We found an energy score of 9.61 kcal/mol upon simulation of -cyperone interaction with GDP tubulin.

**Keywords:** Molecular dynamics, simulation, tubulin RMSD, -cyperone, computational modeling.

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PROJECTIVE GEOMETRY RELATED TO THE SECANT LOCI IN  
SYMMETRIC PRODUCT OF SMOOTH ALGEBRAIC CURVES

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**MSC 2000:** 14H99; 14H51.

**Abstract**

We describe the tangent space of the secant loci associated to a line bundle on a projective smooth algebraic curve. Denoting by  $V_d^r(L)$  the  $(d-r)$ -th secant loci of  $C$  associated to the line bundle  $L$  on  $C$ , we obtain:

**Theorem .1** (a) If  $D$  belongs to  $V_d^r(L) \setminus V_d^{r+1}(L)$ , the tangent space to  $V_d^r(L)$  at  $D$  is  $T_D(V_d^r(L)) = (\text{Im}(\alpha_L \mu_0^L))^\perp$  where  $\mu_0^L$  is the cup product map

$$\mu_0^L : H^0(C, \mathcal{O}(D)) \otimes H^0(C, L(-D)) \rightarrow H^0(C, L).$$

(b) If  $D \in V_d^{r+1}(L)$  then  $T_D(V_d^r(L)) = H^0(C, L \otimes \mathcal{O}_D)$ . In particular, if  $V_d^r(L)$  has the expected dimension and  $d < s + 1 + r$ , then  $D \in \text{Sing}(V_d^r(L))$ .

**Theorem .2** The scheme  $V_d^r(L)$  is smooth at  $D \in V_d^r(L) \setminus V_d^{r+1}(L)$  and has the expected dimension  $d - r \cdot (s + 1 - (d - r))$  if and only if  $\mu_0^L$  is injective.

**Lemma .3** For a very ample line bundle  $L$  on  $C$  and an integer  $d$  with  $d \geq 4$ , if  $V_d^r(L) \neq \emptyset$ , then no irreducible component of  $V_d^r(L)$  is contained in  $V_d^{r+1}(L)$ .

**Theorem .4** Let  $C$  be a hyper-elliptic curve and  $L$  a line bundle on  $C$  whose space of global sections has dimension  $s + 1$ . Assume moreover that  $d \leq s + 1$ . Then  $V_d^r(L)$  is empty or irreducible of dimension  $d - r$  according to whether  $d < 2r$  or  $2r \leq d$ , respectively.

**Theorem .5** If  $C$  is non hyper-elliptic and  $L$  a very ample line bundle on  $C$  with  $d \leq h^0(L) - 1$ , then every component of  $V_d^r(L)$  has dimension at most equal to  $d - r - 1$ .

**Corollary .6** Assume that  $L$  is a very ample line bundle on  $C$  with  $h^0(L) = d + 1 \geq 4$ . Then  $V_d^1(L)$ , if non empty, is of dimension  $d - 2$ .

**Keywords:** Symmetric products, very ample line bundle.

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JORDAN SUPERALGEBRAS WITH SOME HOMOGENEOUS STRUCTURES

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**MSC 2000:** 17C50, 17B60, 17B20, 17B05

**Abstract**

A symplectic pseudo-Euclidean Jordan superalgebra is a Jordan superalgebra endowed with both a homogeneous symplectic form and a homogeneous associative non-degenerate supersymmetric bilinear form. We give a necessary and sufficient condition to construct a symplectic Lie superalgebra given a symplectic Jordan superalgebra. We also give an inductive description of symplectic pseudo-Euclidean Jordan superalgebras. Next, we establish in this paper the equivalence between the existence of solutions of the Yang Baxter equation from Jordan superalgebras (YBE) and that of symplectic forms on Jordan superalgebras.

**Keywords:** Jordan superalgebras, quadratic Lie superalgebras, TKK construction, double extensions, Yang Baxter equation.

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REGARDING ANALYTICAL PROTOTYPE STUDIES FOR THE  
GENERALIZED NONLINEAR POCHHAMMER-CHREE EQUATION**Haci Mehmet BASKONUS<sup>1</sup>, Hasan BULUT<sup>2</sup>, Mirac KAYHAN<sup>3</sup>**<sup>1</sup> *Faculty of Engineering, Tunceli University, Tunceli, Turkey*<sup>2</sup> *Faculty of Science, Firat University, Elazig, Turkey*<sup>3</sup> *Faculty of Science, Inonu University, Malatya, Turkey***Abstract**

In this study, we have applied the Bernoulli sub-equation function method to obtain some new analytical solutions for the generalized nonlinear Pochhammer-Chree equation. We have submitted the general structure of Bernoulli sub-equation function method in section 2. In Section 3, as an application, we have obtained some new analytical solutions of the generalized nonlinear Pochhammer-Chree equation defined by [1];

$$u_{tt} - u_{ttxx} + \gamma u_{xxt} - (\lambda_1 u + \lambda_2 u^p + \lambda_3 u^{2p-1})_{xx} = 0, \quad (1)$$

where  $\gamma, \lambda_1, \lambda_2, \lambda_3$  are constants and they are not zero. Then, we have plotted two and three dimensional surfaces of analytical solutions by the help of wolfram Mathematica 9.

**Keywords:** The Bernoulli sub-equation function method, generalized nonlinear Pochhammer-Chree equation, exponential function solution, trigonometric function solutions, hyperbolic function solutions, complex function solution.

**References**

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LIE GROUP STRUCTURE ON  $N \times N$  MARKOV MATRIX

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**MSC 2000:** 51B25

**Abstract**

This paper presents a Lie group structure on the set of all  $n \times n$  Markov matrices.

**Keywords:** Lie groups, Markov matrices.

**References**

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GRAPH-THEORETIC APPROACH TO THE IDEAL STRUCTURE OF A  
FAMILY OF NON-CHAIN RINGS

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**MSC 2000:** 05C99, 94B05

### Abstract

For finite  $k \geq 1$

$$R_k = F_2[u_1, u_2, \dots, u_k] / \langle u_i^2 = 0, u_i u_j = u_j u_i \rangle. \quad (1)$$

and  $R_0 = F_2$  (finite field with two elements) are commutative rings with characteristic two. In [1], Yildiz *et.al* studied some special codes, called self-dual codes, over this ring. This family of rings has been studied as a new source for building linear codes. As pointed out in these studies, it is not easy to determine all ideals of  $R_k$  where  $k > 1$ , since it is not a principal ideal ring. In this study, we determine the ideal structure of  $R_3$  by using the zero-divisor graph [2] of  $R_3$  ( $k = 3$ ).

**Keywords:** Zero-divisor graphs, non-chain rings.

### References

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## ON SOME GRAPH INVARIANTS OF GENERALIZED FULLERENE

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Fullerene graph is cubic planar graphs which faces are pentagon or Hexagon. In chemistry, Fullerenes are allotropes of carbon with a spherical nanostructure and study as one of important molecule structure in nano and carbon structure.

A connected 3-regular planar graph is called  $m$ -generalized fullerene if it has the following types of faces: two  $m$ -gons and all other pentagons and hexagons. note that for  $m=5,6$  an  $m$ -generalized fullerene is a classical fullerene graph. One of the important class of  $m$ -generalized Fullerene is elongated barrel and denoted by  $F(m,k)$ . In  $F(m,k)$ , The first circle is an  $m$ -gon. Then  $m$ -gon is bounded by  $m$  pentagons. After that we have additional  $k$  layers of hexagon. At the last circle  $m$ -pentagons connected to the second  $m$ -gon. Some graph invariants of  $m$ -generalized fullerene are presented in this lecture .we determine the diameter and some distances property of  $F(m,k)$  and proof the hamiltonicity of this graph .Then we found both upper bound and lower bound for the number of perfect matching in  $m$ -generalized fullerene and enumerate the exact number of perfect matching in  $F(m,k)$  for some  $m$ .

**Keywords:** Fullerene, perfect matching, graph invariant.

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INVOLUTIONS OF DUAL SPLIT-QUATERNIONS

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**MSC 2000:** 11R52; 53A25; 53A35; 53B30; 70B10; 70E15

**Abstract**

Involutions and anti-involutions, which are self-inverse linear mappings, are useful tools to determine rigid-body (screw) motions. In 3-dimensional Euclidean space  $\mathbb{R}^3$ , a reflection of a vector in a plane can be represented by an involution or anti-involution mapping obtained by using real-quaternions. Also, a reflection of a line about a line in  $\mathbb{R}^3$  can be represented by an involution or anti-involution mapping obtained by using dual-quaternions. In this study, we will represent involution and anti-involution mappings obtained by using dual split-quaternions, and a geometric interpretation of each as rigid-body motions in 3-dimensional Minkowski space  $\mathbb{R}_1^3$ .

**Keywords:** Dual split-quaternions, involutions, rigid-body (screw) motions.

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ASYMPTOTIC EXPANSION OF DOUBLE OSCILLATORY INTEGRALS:  
CONTRIBUTION OF NON STATIONARY CRITICAL POINTS OF THE  
SECOND KIND

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**MSC 2000:** 41A60

**Abstract**

In in this paper, we show that the contribution of a non-stationary critical point of the second kind to the asymptotic expansion of a double oscillatory integral is governed by “the order of contact” between the boundary of the domain of integration and the level curve of the phase through the critical point. Complete asymptotic expansions are derived and the coefficient of the leading term is computed in terms of the original data of the problem. This problem was previously studied by several authors, but only in the special case when the order of contact is minimal.

**Keywords:** Asymptotic expansion, oscillatory integral, critical point of the second kind.

**References**

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DEVELOPMENT OF AN EDUCATIONAL SOFTWARE FOR BASIC  
STATISTICS

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**MSC 2000:** 97U50,62-07

**Abstract**

In this study we aim to develop a dynamic educational software. For this purpose we have developed a software that capable of both describing and analyzing issues of statistics. Programme can both produce random sample and solve statistics. The developed software has been created with the *c#* language with 2900 lines of code. General algorithm of the program, the calculated values compared with the actual values of these values will be presented in this study.

**Keywords:** Educationonal software, statistical software, *c#* programming language.

**References**

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## REGRESSION ANALYSIS ALGORITHM FOR CIRCULAR DATA

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**MSC 2000:** 62J99,68N01

### Abstract

In this study, we aimed to develop a software algorithm that can provide the opportunity to create the foundations of a regression of circular data without any additional information. For this purpose, we have developed special algorithms for circular regression and some basic circular statistics. Some of these are mode, mean, standard deviation and correlation. Algorithms have been developed with c# programming language. It was created for this purpose and approximately 3760 lines of code. We will give obtained results and error rates.

**Keywords:** circular regression, software algorithm, c# programming language.

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CHARACTERIZATION OF  $\mathcal{U}_1(\mathbb{Z}[C_n \times C_4])$ **Tevfik BİLGİN<sup>1</sup>, İsmail Gökhan KELEBEK<sup>2</sup>**<sup>1,2</sup>*Fatih University, İstanbul , Turkey***MSC 2000:** 16U60, 16S34**Abstract**

Let us denote  $C_n^* = C_n \times C_4$  where  $C_n = \langle a : a^n = 1 \rangle$  and  $C_4 = \langle x : x^4 = 1 \rangle$ . In this study it was shown that the unit group of the integral group ring  $\mathbb{Z}C_n^*$  can be written as an internal direct product of unit groups as follows:

$$U(\mathbb{Z}C_n^*) = \pm C_n^* \times F \times U(1 + \mathbb{Z}C_n(1 - x)(1 + x^2)) \times \text{Im}\varphi \times [\text{Ker}\varphi / \langle x^2 \rangle]$$

where  $F$  is the torsion free part of  $U(\mathbb{Z}C_n)$ . At the end we gave two concrete examples.

**Keywords:** Integral group ring, unit group, generators of unit group.

**References**

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## VARIOUS NOTIONS OF MODULE AMENABILITY

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Garmsar, Iran*

**MSC 2010:** 43A20, 43A40, 46H25**Abstract**

The concept of module amenability for a class of Banach algebras that are modules over another Banach algebra was introduced by M. Amini in 2004. He showed that for an inverse semigroup  $S$  with the set of idempotents  $E$ , the semigroup algebra  $l^1(S)$  is module amenable, as a Banach module over  $l^1(E)$ , if and only if  $S$  is amenable. In this talk, we present the notions of module amenability, module contractibility,  $n$ -weak module amenability, module character amenability, module (uniform) approximate amenability, module pseudo-amenability and module pseudo-contractibility for Banach algebras. We also show that under which conditions the semigroup algebra  $l^1(S)$  (as  $l^1(E)$ -module) satisfies in the above mentioned concepts, where  $S$  an inverse semigroup with the subsemigroup of idempotents  $E$ .

**Keywords:** Banach module, inverse semigroup, module amenability.

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SOME NEW THEOREMS IN HILBERT QUASILINEAR SPACES

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**MSC 2000:** 34C10

**Abstract**

This study is concerned with the some new theorems and definitions in Hilbert quasilinear spaces. First, we introduce minimizing vector theorem and some results in Hilbert quasilinear spaces. Next, we provide two main examples: First example is a Hilbert quasilinear space, that does not satisfy the orthogonal decomposition and second example is subset of a Hilbert quasilinear space with the orthogonal decomposition properties. Then, we have from first example that any Hilbert quasilinear space may not satisfy the orthogonal decomposition theorem of Hilbert spaces. Finally, we give some results related to above theorems provide an important contributions to the improvement of the quasilinear functional analysis.

**Key words:** Quasilinear space, quasilinear inner prouct space, quasilinear Hilbert Space, orthogonality.

**References**

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ON STATISTICAL MANIFOLD WITH DUAL CONNECTION AND ITS  
APPLICATIONS**Mustafa BUYUKARSLAN<sup>1</sup>, Oguzhan BAHADIR<sup>2</sup>**<sup>1,2</sup> *Kahramanmaraş Sutcu Imam University, Kahramanmaraş, Turkey***Abstract**

Statistical manifolds was introduced by Amari in 1985. He has studied statistical manifolds in terms of information geometry. Since the geometry of such manifolds includes the notion of dual connection, also called conjugate connection in affine geometry, it is closely related to affine differential geometry. A statistical manifold is, in short, a Riemannian manifold  $(M, g)$  with one additional structure given by a torsion-free and symmetric affine connection  $\nabla$  and its dual connection  $\nabla^*$ , which is also assumed to be torsion-free; we say  $\nabla$  and  $\nabla^*$  are mutually dual whenever  $Xg(Y, Z) = g(\nabla_X Y, Z) + g(Y, \nabla_X^* Z)$  holds for all vector fields  $X, Y, Z$  on  $M$ . In this paper, we gave some fundamental definitions and theorems, then we studied statistical manifolds with dual connection and its applications.

**Keywords:** Statistical manifold, statistical structure, dual connection.

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SOME FIXED POINT THEOREMS SATISFYING MEIR-KEELER TYPE  
CONTRACTIONS VIA RATIONAL EXPRESSION IN 2-METRIC SPACES

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**MSC 2000:** 54H25, 47H10

### **Abstract**

In this paper, we establish some fixed point theorems for Meir-Keeler type contractions via rational expressions and also we obtain some results for mappings satisfying integral type contractions in 2- metric spaces.

**Keywords:** Fixed point, Meir-Keeler type contraction, 2-metric spaces.

### **References**

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A NUMERICAL METHOD FOR NONLINEAR SINGULARLY PERTURBED  
MULTI-POINT BOUNDARY VALUE PROBLEM

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**MSC 2000:** 34B10, 65L05, 65L11, 65L12, 65L20

### Abstract

We consider the following nonlinear singular perturbed multi-point problem:

$$\begin{aligned} -\varepsilon^2 u'' + f(x, u) &= 0, \quad 0 < x < 1, \\ u(0) &= 0, \\ k_0 u(1) &= \sum_{i=1}^m k_i u(s_i) + k_{m+1} \int_0^1 u(x) dx + d, \end{aligned}$$

where  $0 < \varepsilon \ll 1$  is small perturbation parameter, the function  $f(x, u)$  is sufficiently smooth on  $[0, 1] \times \mathbb{R}$ ,  $s_i \in (0, 1)$ ,  $i = 1, 2, \dots, m$ ,  $k_0 \geq 0$ , and furthermore  $\frac{\partial f}{\partial u}(x, u) \geq \alpha > 0$ . The solution  $u(x)$  has boundary layers at  $x = 0$  and  $x = 1$ . This study is concerned with  $\varepsilon$ -uniform numerical method for the nonlinear singularly perturbed multi-point boundary value problem. We describe some properties of the solution of this problem. The numerical method is constructed on Shishkin mesh and the method is shown to be convergent of first order in the discrete maximum norm. Consequently, the numerical experiments which demonstrate the sharpness of our theoretical analysis are presented.

**Keywords:** Singular perturbation, fitted finite difference method, Shishkin mesh, nonlocal boundary condition, uniform convergence.

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REDUCED DIFFERENTIAL TRANSFORM METHOD FOR SIXTH-ORDER  
SINGULARLY PERTURBED BOUSSINESQ EQUATIONMusa CAKIR<sup>1</sup>, Derya ARSLAN<sup>2</sup><sup>1, 2</sup> *Yuzuncu Yil University, Van, Turkey*

MSC 2000: 34D15, 35B25, 65N20, 65N12, 65N15

**Abstract**

We consider the following the ill-posed Boussinesq equation and sixth-order singularly perturbed Boussinesq equation, respectively:

$$u_{tt} = u_{xx} + u_{xx}^2 + u_{xxxx}, \quad \text{and} \quad u_{tt} = u_{xx} + u_{xx}^2 + u_{xxxx} + \epsilon u_{xxxxxx}$$

The purpose of this paper is to obtain the approximate solution of sixth-order singularly perturbed Boussinesq equation and the ill-posed Boussinesq equation ( $\epsilon = 0$ ) by the reduced differential transform method (RDTM). This numerical method for solving a wide variety of linear and nonlinear partial differential equations usually gets the solution in a series form. The suggested algorithm is quite efficient and is practically well suited for use in these problems. Several examples are presented to demonstrate the efficiency and reliability of the RDTM (Because this method yield the desired accuracy only in a few terms and in a series form of the exact solution), and numerical results are discussed, compared with exact solution. The numerical results show that this method is a powerful tool for solving nonlinear singular perturbed PDEs and the results show that the method reduces the numerical calculations.

**Keywords:** Singularly perturbed Boussinesq equation, ill-posed Boussinesq equation, reduced differential transform method.

**References**

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INVERTED DISTANCE AND INVERTED WIENER INDEX

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**MSC 2000:** 05C12

**Abstract**

The Wiener index is the sum of distances between all pairs of vertices of a (connected) graph. In this paper we defined two novel graph invariants; the inverted distance and the inverted Wiener index. The inverted distance of between any two different vertices  $u$  and  $v$  of a simple connected graph  $G$  defined as;  $i(u, v) = D - d(u, v) + 1$  where  $D$  denotes the diameter of  $G$  and  $d(u, v)$  denotes the distance of the vertices  $u$  and  $v$ . The inverted Wiener index of a simple connected graph  $G$  defined as;  $IW(G) = \sum_{u \neq v} i(u, v)$  where the sum is taken over unordered pairs of vertices of  $G$ . We characterized maximum trees with respect to the inverted Wiener index.

**Keywords:** inverted distance, inverted Wiener index, Wiener index, average inverted distance.

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MATRIX-VALUED DIFFERENCE OPERATORS WITH POLYNOMIAL TYPE  
JOST SOLUTIONS ON THE WHOLE AXIS

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**MSC 2000:** 39A05, 39A70, 39A10, 47A05.

### Abstract

The main aim of this paper is to obtain the Jost solutions and some spectral properties of a second order matrix self-adjoint difference equation on the whole axis. In this paper, we investigate the analytical properties and asymptotic behaviors of these Jost solutions. Then, we find continuous spectrum of the operator  $L$  generated by matrix-valued difference expression of second order. At last, we get that the operator  $L$  has a finite number of real eigenvalues.

**Keywords:** Difference equations, discrete operator, Jost solution, eigenvalues, continuous spectrum.

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## SOME INCOMPLETE $Q$ -POLYNOMIALS

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**MSC 2000:** 11B39, 05A30

### Abstract

The  $q$ -analogues of Fibonacci polynomials were studied by Carlitz [6] and Cigler [8, 7]. We use  $q$ -analogues of Fibonacci polynomials to define incomplete  $q$ -Fibonacci polynomials. We obtain some properties and relations between these polynomials.

**Keywords:** Incomplete Fibonacci numbers, incomplete  $q$ -Fibonacci polynomials,  $q$ -Fibonacci polynomials.

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SOME FIXED POINT THEOREMS ON SOFT G-METRIC SPACES

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**MSC 2000: 54A05, 06D72, 47H10**

**Abstract**

In this presentation, the notion of soft G-complete space is introduced and some properties of such spaces are investigated. Then, some fixed point theorems for mappings satisfying sufficient conditions are proved on soft G-metric spaces.

**Keywords:** soft set, soft G-metric space, fixed point.

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ON THE  $Q$ -EXTENSION OF THE  $P$ -ADIC BETA FUNCTION

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**MSC 2000:** 11S80, 33D05

### Abstract

Let  $p$  be a fixed prime number. By  $\mathbb{Z}_p, \mathbb{Q}_p$  and  $\mathbb{C}_p$  we denote the ring of  $p$ -adic integers, the field of  $p$ -adic numbers and the completion of the algebraic closure of  $\mathbb{Q}_p$ , respectively. Y. Morita (1975) defined the  $p$ -adic gamma function  $\Gamma_p : \mathbb{Z}_p \rightarrow \mathbb{Q}_p$  by the formula

$$\Gamma_p(x) = \lim_{n \rightarrow x} (-1)^n \prod_{\substack{1 \leq j < n \\ (j,p)=1}} j$$

Let  $q \in \mathbb{C}_p$  with  $|q-1|_p < 1$  and  $q \neq 1$ , the  $q$ -extension of the  $p$ -adic gamma function is defined by

$$\Gamma_{p,q}(x) = \lim_{n \rightarrow x} (-1)^n \prod_{\substack{1 \leq j < n \\ (j,p)=1}} \frac{1-q^j}{1-q} \quad \text{for } x \in \mathbb{Z}_p,$$

where  $n$  runs over positive integers. We recall that  $\lim_{q \rightarrow 1} \Gamma_{p,q} = \Gamma_p$ .

In the present work we consider the  $q$ -extension of the  $p$ -adic beta function which is defined by

$$B_{p,q}(x, y) = \frac{\Gamma_{p,q}(x) \Gamma_{p,q}(y)}{\Gamma_{p,q}(x+y)}.$$

We obtain some properties of the  $q$ -extension of the  $p$ -adic beta function  $B_{p,q}$ .

**Keywords:**  $p$ -adic number,  $q$ -extension of the  $p$ -adic gamma function,  $q$ -extension of the  $p$ -adic beta function.

### References

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DYNAMICAL STABILITY AND MATHEMATICAL MODELING OF HEROIN  
EPIDEMIC IN URMIA

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**MSC 2000:** 37L15, 37N25, 37N30

### Abstract

Experts say Iranians have turned to opiates partly out of despair at rising unemployment and economic trouble, and partly because alcohol, which they might otherwise use, is illegal. The main reason, though, is obvious. Iran shares a long and porous border with Afghanistan, which produces most of the world's opium poppies. Four million of its 70 million people are addicts. HIV and Injecting Drug have consistently highlighted in recent years the ongoing and persistent nature of opiate and particularly heroin use on a global scale. National prevalence studies have indicated the scale of the problem, but the drug-using career, typically consisting of initiation, habitual use, a treatment-relapse cycle and eventual recovery, is not well understood. We will present a ODE model of opiate addiction, based on the principles of mathematical epidemiology. The aim of this model is to identify parameters of interest for further study, with a view to informing and assisting policy-makers in targeting prevention and treatment resources for maximum effectiveness. An epidemic threshold value  $R_0$ , is proposed for the drug-using career. Sensitivity analysis is performed on  $R_0$  and it is then used to examine the stability of the system. The model we use is as follows:

$$\begin{aligned}\frac{dS}{dt} &= \Delta - \frac{\alpha_1 D_1 S}{N} - \mu S \\ \frac{dD_1}{dt} &= \frac{\alpha_1 D_1 S}{N} - p D_1 + \frac{\alpha_3 D_1 D_2}{N} - (\mu + r_1) \mu S \\ \frac{dD_2}{dt} &= p D_1 - \frac{\alpha_3 D_1 D_2}{N} - (\mu + r_2)\end{aligned}$$

**Keywords:** Mathematical modeling, dynamical stability, reproduction ratio  $R_0$ .

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OPERATOR EQUATIONS GENERALIZING THE NOTIONS OF HANKEL  
AND TOEPLITZ OPERATORS

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**Abstract**

Hankel and Toeplitz operators came into existence with the work of H. Hankel in 1861 and O. Toeplitz in 1911 respectively. Although, the initial appearance of these operators was seen in matrix form, but various equivalent forms were obtained subsequently. In terms of matrices, a Hankel operator is an operator on a Hilbert space whose matrix with respect to an orthonormal basis is constant along each diagonal perpendicular to the main one and a Toeplitz operator is one whose matrix is constant along each diagonal parallel to the main one. In terms of operator equations, Hankel and Toeplitz operators on Hardy spaces are nothing but the solutions of operator equations  $U^*X = XU$  and  $U^*XU = X$  respectively, where  $U$  is the forward unilateral shift and  $U^*$  is its adjoint.

Barria and Halmos in 1982 focused the attention of mathematicians towards a new direction by proposing the operator equation  $U^*XU = \lambda X$  for an arbitrary complex number  $\lambda$ . The study of Hankel and Toeplitz operators has gone a long way with the inception of various classes of operators like slant Hankel, slant Toeplitz, essentially slant Hankel, essentially slant Toeplitz,  $k^{\text{th}}$ -order slant Hankel,  $k^{\text{th}}$ -order slant Toeplitz operators,  $\lambda$ -Hankel operators. The present talk is a motivation of the work of Barria and Halmos that leads to some generalizations of the operator equations characterizing Hankel and Toeplitz operators and has come up as a recent development in this direction.

**Keywords:** Hankel operators, Toeplitz operators, operator equations.

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NUMERICAL APPROXIMATIONS FOR SOME FRACTIONAL STOCHASTIC  
PARTIAL DIFFERENTIAL EQUATIONS

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**Abstract**

In this work, we elaborate and calculate the rate of convergence of several numerical schemes to approximate the solution of some fractional stochastic partial differential equations (FSPDEs); fractional stochastic heat and Burgers equations with gaussian multiplicative noise. In particular, we use Galerkin spectral method in space, Euler method in time and we elaborate a complete scheme. We prove strong convergence and we calculate explicitly the rate of convergence and show its dependence on the fractional power of the Laplacian.

**Keywords:** Strong convergence, Galerkin spectral method, implicit Euler scheme, multiplicative noise, fractional laplacian, Burgers equation.

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SOME PROPERTIES OF SOFT PROXIMITY SPACES

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**MSC 2000:** 54A40, 06D72, 54E05.

**Abstract**

In this work, we continue investigating the properties of soft proximity spaces. Also, we give the notion of a soft  $\delta$ -neighborhood in soft proximity spaces and obtain a few results analogous to the ones that hold for  $\delta$ -neighborhood in proximity spaces. Moreover, we show that each soft uniform space on  $X$  induces a soft proximity space on the same set. Finally, we prove the existences of initial soft proximity spaces.

**Keywords:** soft set, soft proximity, soft  $\delta$ -neighborhood, initial soft proximity.

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CONVERGENCE PROPERTIES ON JAIN-DURRMEYER OPERATORS

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**MSC 2000:** 41A25, 41A35, 41A36

### Abstract

In this talk, we extend Jain operators to Durrmeyer type operators. We deal with some approximation properties of the new operators. Firstly, we estimate quantitative asymptotic formula in terms of weighted modulus of smoothness. Also we present weighted uniform convergence using weighted Korovkin type theorem.

**Keywords:** Jain operators, Durrmeyer operators, modulus of smoothness, Korovkin type theorem.

### References

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NAKAYAMA'S LEMMA FOR ARTINIAN MODULES AND GENERALIZED  
MATLIS DUALITY

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**MSC 2010:** 13E10, 13E05

**Abstract**

The purpose of this study is to produce Nakayama's Lemma for Artinian modules. Note that Nakayama's Lemma is applicable for Noetherian modules. To prove the Artinian case, we develop a generalization of Matlis duality which applies to a complete semi-local Noetherian ring; This enables us to pass back and forth between the category of Noetherian modules and Artinian modules. This technique is used in conjunction with the completion of  $R$  (the ring we define modules over) related to  $R$ -module  $A$ , to show how several result about Artinian modules can be deduced from well-known classical Noetherian results. The classical duality of Matlis was originally developed for a complete local Noetherian ring. We use the fact that such a ring is isomorphic to a direct product of finitely many complete local rings and appeal to the standard version of Matlis' duality.

**Keywords:** Artinian rings and modules, finite dimensional algebras.

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PSEUDO-SLANT SUBMANIFOLD IN SASAKIAN SPACE FORMS

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**MSC 2000:** 53C15, 53C25, 53C17, 53D15 and 53D10.

### Abstract

In this paper, we study the geometry of the pseudo-slant submanifolds of a Sasakian space form. Necessary and sufficient conditions are given for a submanifold to be a pseudo-slant submanifold, pseudo-slant product, mixed geodesic and totally geodesic in Sasakian manifolds. Finally, we give some results for totally umbilical pseudo-slant submanifold in a Sasakian manifold and Sasakian space form.

**Keywords:** Sasakian manifold, Sasakian space form, Slant submanifold, Pseudo-slant submanifold.

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A GENERAL RESULT ON ASYMPTOTIC INTEGRATION OF IMPULSIVE  
DIFFERENTIAL EQUATIONS

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**Abstract**

There is hardly any work about asymptotic integration of differential equations under impulse effect. We consider second order nonlinear impulsive differential equations with fixed moments of impulses. By using principal and nonprincipal solutions we find an asymptotic representation of the solutions depending on a parameter.

**Keywords:** fixed point theory, impulsive differential equations, principal and non-principal solutions, asymptotic integration.

**References**

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SOME FIXED POINT THEOREMS FOR A WEAK PARTIAL METRIC SPACE

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**MSC 2010:** 54H25, 47H10

**Abstract**

In this presentation, we present new developments about contractions on a weak partial metric space. Then, considering this contractions, we give some fixed point theorems for singlevalued mappings on a complete weak partial metric space.

**Keywords:** Fixed point, weak partial metric space, contraction mapping.

**References**

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DIFFERENCE SCHEMES ON ADAPTED MESH FOR THE INITIAL  
BOUNDARY VALUE SOBOLEV PROBLEMS WITH BOUNDARY LAYERS

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**MSC 2000:** 65M06, 34K26, 65M12, 34K28

### Abstract

In this paper, a new adaptive mesh strategy has been developed for solving the linear singular initial-boundary value Sobolev type differential equation in the domain  $D = (0, l) \times (0, T]$ , form as follows:

$$Lu \equiv -\varepsilon \frac{\partial^4 u}{\partial t^2 \partial x^2} + a(x) \frac{\partial^2 u}{\partial t^2} - \varepsilon \frac{\partial^2 u}{\partial x^2} + b(x, t) u = f(x, t), \quad (x, t) \in D, \quad (1)$$

with the initial data

$$u(x, 0) = u(x), \quad \frac{\partial u}{\partial t}(x, 0) = \psi(x), \quad (2)$$

and boundary conditions

$$u(0, t) = u(l, t) = 0. \quad (3)$$

Here  $\varepsilon$  is a small positive parameter ( $0 < \varepsilon \ll 1$ ),  $a(x) \geq \alpha > 0$ ,  $|b(x, t)| \leq b$ ,  $u(x)$ ,  $\psi(x)$  and  $f(x, t)$  are sufficiently smooth functions. For the numerical solution of this problem, we use an finite difference schemes on B-mesh on a non-uniform mesh which is accomplished by the method of integral identities with the use of basis functions and interpolating quadrature rules with weight and remainder term in integral form. The error estimates for the numerical solution are obtained.

**Keywords:** Singular perturbation, Sobolev problem, uniform convergence, difference schemes, Bakhvalov mesh.

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FINITE DIFFERENCE SCHEMES ON SHISHKIN MESH FOR SINGULARLY  
PERTURBED INITIAL-BOUNDARY VALUE SOBOLEV TYPE PROBLEMS

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**MSC 2000:** 65M06, 65M12, 34K28

### Abstract

In this paper, we present a finite difference schemes on piece-wise uniform Shishkin mesh for solving singular perturbation Sobolev problem. We show that the constructed difference scheme is stable and first order uniform convergence. We give a numerical example which illustrate the theoretical results on the uniform accuracy of the discrete problem, as well as the robustness of the method.

**Keywords:** Singular perturbation, difference schemes, Shishkin mesh, Sobolev problem.

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ANALYSIS ON STABILITY OF LIMIT CYCLES AND HOPF BIFURCATION  
IN VAN DER POL NONLINEAR DIFFERENTIAL EQUATION

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**Abstract**

The main purpose of this paper is to discuss the following objectives with the famous Van Der Pol Nonlinear Differential Equation. The Van Der Pol oscillator is an oscillator with nonlinear damping governed by the second-order differential equation:

$$\partial^2 x / \partial t^2 - \epsilon (1 - x^2) \partial x / \partial t + x = 0$$

where  $x$  is the dynamical variable and

$$\epsilon > 0$$

a parameter.

The objectives are as follows:

- (i) Development of general theory and formulae for determining Hopf Bifurcations on any non-linear Differential equations
- (ii) Existence of Chaos, Limit Cycles, Supercritical and Subcritical Hopf Bifurcations of Van Der Pols Oscillator and their Statistical analysis.

**Keywords:** Stability, limit cycles, supercritical, subcritical Hopf bifurcations.

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## REVERSE ZAGREB INDICES OF CARTESIAN PRODUCT OF GRAPHS

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**MSC 2000:** 05C07

### Abstract

Recently the reverse vertex degree and the reverse Zagreb indices have been defined [1]. Let  $G$  be a simple connected graph and  $v$  be a vertex of  $G$ . Then, the reverse vertex degree of the vertex  $v$ ,  $c_v$  defined as follows;  $c_v = \Delta - d_v + 1$ . The first reverse Zagreb alpha index of  $G$  defined as;  $CM_1^\alpha(G) = \sum_{v \in V(G)} c_v^2$ . The first reverse Zagreb beta index of  $G$  defined as;  $CM_1^\beta(G) = \sum_{uv \in E(G)} (c_u + c_v)$ . The second reverse Zagreb index of  $G$  defined as;  $CM_2(G) = \sum_{uv \in E(G)} c_u c_v$ . The chemical predictivity of these novel indices have been investigated in [2]. In this paper, some exact expressions for the reverse Zagreb indices of Cartesian product of two simple connected graphs were determined.

**Keywords:** Reverse vertex degree, reverse Zagreb indices, cartesian product of graphs.

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## Q-CONVERGENCE OF GRADED DIFILTERS

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**MSC 2000:** 54A05, 54A20, 06D10

### Abstract

Convergence of graded difilters have been presented and investigated by the authors in [3]. In this work, using graded Q-dinhd systems defined in [2] the authors define a different convergence type of graded difilters called Q-convergence which has some advantages and some disadvantages in comparison with the convergence defined in [3].

**Keywords:** Texture, q-convergence, graded ditopology, graded difilter, fuzzy topology.

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CHAOS THEORY AND LORENZ ATTRACTORS

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**Abstract**

Chaos theory is one of the fundamental theories in our lives. It ended the so-called deterministic era where everything is predictable. It was thought that the behaviour, whether in the future or the past, of all the physical systems is known and that reaching perfect prediction is a matter of precision and accuracy. In this paper, Chaos theory is introduced along with its origin and history. In addition, Lorenz attractors are also introduced with the famous butterfly representation of Lorenz. Moreover, Applications of the chaos theory are included here. These applications include applications in economics, circuits and meteorology (weather prediction). Finally, the paper is summarized in the conclusion section.

**Keywords:** Chaos, Lorenz, butterfly, MatLab.

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RESEARCH ON USING URANIUM-LEAD RADIOACTIVE DECAY IN  
DETECTING THE AGE OF THE EARTH

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**Abstract**

Since long ago, and the question regarding the exact age of the earth has brought scientists interest all over the world. Before the discovery of the radiometric dating, many estimations for the age of the earth were done, but all of them were far away from the accepted one today. These wrong estimation had a negative effect on other field such as biology, where Darwin faced a big challenge to prove the validity of the theory of evolution. Thanks to science, we now know with a good accuracy that the age of earth is 4.4 Gyr old according to the latest research published in 2014.

**Keywords:** Age of earth, uranium, radioactive decay, radiometric dating.

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ON ORDERING OF TREES BY MULTIPLICATIVE VERSION OF ZAGREB  
INDICES**Mehdi ELIASI<sup>1</sup>***Department of Mathematics and Computer Science, Faculty of Khansar,  
University of Isfahan, P.O.Box 87931133111, Khansar, Iran***MSC 2000:** 05C07**Abstract**

A topological index is a real number related to a molecular graph, which is a graph invariant and which has some chemical application. Let  $G = (V, E)$  be a molecular graph representing of a chemical structures. The first and the second Zagreb indices of  $G$  are defined as:

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)], \quad M_2(G) = \sum_{uv \in E(G)} [d_G(u)d_G(v)],$$

respectively, where  $d_u$  denotes the degree of vertex  $u$ . These indices have been used to study molecular complexity, chirality, ZE-isomerism and hetero-systems. Gutman et al. [1, 2] have recently proposed to consider the multiplicative variants of Zagreb indices as:

$$P_1^*(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)], \quad P_2(G) = \prod_{uv \in E(G)} [d_G(u)d_G(v)],$$

In this paper for chemical trees, we introduce some graph transformations, which decrease  $\Pi_1^*$  and  $\Pi_2$ . By using these operations, we identify classes of trees, which have smallest multiplicative version of Zagreb indices among all chemical trees of order  $n \geq 16$ .

**Keywords:** Zagreb indices, graph operation, chemical tree.

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THE EIGENVALUES (ENERGY LEVELS) OF THE RIEMANN ZETA  
FUNCTION

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**Abstract**

This work presents and examined the communications that were exchanged among some renowned Mathematicians and their thoughts concerning the zeros of the Riemann zeta function. Some meromorphic functions which have the same results as the Riemann zeta function are presented. Matrix representations of these functions are also obtained through which the general form of the point spectral and the trace of the Riemann zeta function were generated. The Riemann Zeta function is presented as a function of complex Variables and thus transformed into a bilinear function, and through the use of Sobolev space theorem, an optimization problem with a variable coefficient is derived. Some methods of solution are also presented.

**Keywords:**Riemann zeta function, meromorphic functions, Matrix representations, Bilinear function, Sobolev space, Optimization variable coefficient.

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GENERALIZED BULLEN TYPE INEQUALITIES FOR LOCAL FRACTIONAL  
INTEGRALS AND ITS APPLICATIONS

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**MSC 2000:** 26D10, 26D15, 26A33, 52A41, 41A55.

### Abstract

In this paper, we establish the generalized Bullen type inequalities involving local fractional integrals on fractal sets  $R^\alpha$  ( $0 < \alpha \leq 1$ ) of real line numbers. Some applications of these inequalities in numerical integration and for special means are given.

**Keywords:** Bullen's inequality, local fractional integral, fractal space, generalized convex function, numerical integration, special means.

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A FITTED NUMEROV METHOD FOR SINGULARLY PERTURBED  
REACTION-DIFFUSION EQUATIONS

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### Abstract

In this paper we considered singularly perturbed reaction-diffusion problem whose solution exhibits boundary layers. We have introduced a simple and efficient computational technique based on Numerov's scheme which is composed of an exponentially fitted difference scheme on uniform mesh. A fitting factor is obtained from the theory of singular perturbations. The method is shown to uniformly convergent with respect to the perturbation parameter. A numerical experiment illustrate in practice the result of convergence proved theoretically.

**Keywords:** Singularly perturbation problems, reaction-diffusion problem, boundary layer, fitting factor, Numerov's method.

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AN EXPONENTIAL FITTED METHOD FOR SINGULARLY PERTURBED  
REACTION-DIFFUSION EQUATIONS

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**MSC 2000:** 34D15, 33F05

### **Abstract**

In this study we consider a numerical method for a singularly perturbed one-dimensional reaction-diffusion problem whose solution exhibits boundary layers. A finite difference scheme is constructed in an equidistant mesh, which gives first and second order uniform convergence in the discrete maximum norm. A fitting factor is introduced in finite difference scheme and is obtained from the theory of singular perturbations. Thomas algorithm is used to solve the system. The method is shown to uniformly convergent with respect to the perturbation parameter. A numerical experiment illustrate in practice the result of convergence proved theoretically.

**Keywords:** Reaction-diffusion, singular perturbation, numerov method.

### **References**

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IMPULSIVE NEUTRAL FRACTIONAL DIFFERENTIAL INCLUSIONS AT  
VARIABLE TIMES

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**MSC 2000:** 26A33, 34A08, 34A37, 34A60, 34K37

**Abstract**

In this work, we establish some sufficient conditions for the existence of solutions for a class of initial value problems for impulsive fractional functional differential inclusions with neutral delay at variable moments.

**Keywords:** Caputo fractional derivative, existence and uniqueness, functional differential inclusions, Impulsive differential inclusions, variable times.

**References**

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NEUTRAL FRACTIONAL DIFFERENTIAL EQUATIONS WITH IMPULSES  
AT VARIABLE TIMES

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**MSC 2000:** 26A33, 34A08, 34A37

**Abstract**

As known, impulsive functional differential equations of integer order with fixed and variable moments and the ones of fractional order with fixed moments take place in the related literature many times (see for instance [1, 2]). However, to the best of our knowledge, the ones of fractional order with variable moments have not been considered yet. In this study, we extend the results of Benchohra and Ouahab [3] having an integer-order impulsive neutral-delay differential equations with variable moments to the fractional order ones.

**Keywords:** Fractional differential equation, Caputo fractional derivative, impulses, variable times.

**References**

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ANTI-INVARIANT SEMI-RIEMANNIAN SUBMERSIONS ADMITTING  
VERTICAL FROM LORENTZIAN SASAKIAN AND PARA SASAKIAN  
MANIFOLDS

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**MSC 2000:** 53C25, 53C15, 53B30.

### Abstract

In this paper we study anti-invariant semi-Riemannian submersions from Lorentzian Sasakian and Para Lorentzian Sasakian onto semi-Riemannian manifolds. We give examples of anti-invariant semi-Riemannian submersions. We survey main results of anti-invariant semi-Riemannian submersions defined on (para) Lorentzian Sasakian manifolds. We investigate necessary and sufficient condition for an anti-invariant semi-Riemannian submersion to be totally geodesic and harmonic.

**Keywords:** Semi-Riemannian submersion, (Para) Sasakian manifold, anti-invariant submersion.

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ON GENERALIZATION OF THE STRONGLY IRREDUCIBLE SUBMODULES

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**MSC 2000:** 13A15

**Abstract**

Throughout this article, all rings are considered to be commutative ring with identity and all modules are unitary. Let  $\alpha$  be a cardinal, and  $\Lambda$  be a set with  $|\Lambda| = \alpha$ . A submodule  $N$  of an  $R$ -module  $M$  such that  $N \in Lat(M)$  is said to be  $\alpha$ -irreducible, if for submodules  $\{N_\lambda\}_{\lambda \in \Lambda}$  of  $M$ , the equation  $\bigcap_{\lambda \in \Lambda} N_\lambda = N$  implies that there exists some  $\lambda_0 \in \Lambda$  such that  $N_{\lambda_0} = N$ . In this work we give generalization for the concept  $\alpha$ -irreducible submodule. Also we call a submodule  $N$  of  $M$  is an  $\alpha$ -strongly irreducible submodule if for submodules  $\{N_\lambda\}_{\lambda \in \Lambda}$  of  $M$ , the inclusion  $\bigcap_{\lambda \in \Lambda} N_\lambda \subseteq N$  implies that there exists some  $\lambda_0 \in \Lambda$  such that  $N_{\lambda_0} \subseteq N$ . Furthermore, we generalize some properties of them.

**Keywords:**  $\alpha$ -irreducible,  $\alpha$ -strongly irreducible, faithful module.

**References**

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RINGS OVER WHICH MONOID RINGS ARE SEMICOMMUTATIVE

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**MSC 2000:** 16S36

**Abstract**

For a monoid  $M$ , we introduce strongly  $M$ -semicommutative rings, which are generalization of strongly semicommutative rings, and we investigate their properties. We show that if  $G$  be a finitely generated Abelian group, then  $G$  is torsion free if and only if there exists a ring  $R$  with  $|R| \geq 2$  such that  $R$  is strongly  $M$ -semicommutative. We also show that if  $R$  be a ring and  $\Delta$  be a multiplicatively closed subse of  $R$  consisting of central regular element. Then  $R$  is strongly  $M$ -semicommutative if and only if  $\Delta^{-1}R$  is strongly  $M$ -semicommutative.

**Keywords:** Semicommutative, strongly  $M$ -semicommutative, u.p.-Monoid.

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ARTIFICIAL INTELLIGENCE BASED MODELING FOR WATER  
TREATMENT

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**MSC 2000:** 92B20

**Abstract**

In this work, two computational methods are developed to predict the photocatalytic removal of AY23 in the presence of Ag-TiO<sub>2</sub> nanoparticles prepared under desired conditions. One is artificial neural network (ANN) approach, another is genetic algorithm (GA) modeling approach. To develop the models, a total of 100 data were used, wherein four parameters, such as initial concentration of dye, UV light intensity, initial dosage of nano Ag-TiO<sub>2</sub> and irradiation time were used as the input variables and removal of AY23 as output variable. The predictive and generalization abilities of the models were comprehensively evaluated using several statistical tests. The comparison between the predicted results by designed models and the experimental data prove that modeling of the removal process of AY23 by using ANN and GA are precise methods to predict the extent of AY23 removal under different conditions. ANN model performed relatively better than the GA model.

**Keywords:** Artificial neural network, genetic algorithm, modeling.

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SYMMETRIC BI-MULTIPLIERS ON INCLINE ALGEBRAS

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**MSC 2000:** 06B35, 06B99, 16B70, 16B99

**Abstract**

In this study, we introduce the notion of  $*$  and  $+$ -symmetric bi-multipliers in incline algebras and research some related properties. Also, we define kernel of  $*$  and  $+$ -symmetric bi-multipliers in incline algebras. Additionally, we state some properties of these  $*$  and  $+$ -symmetric bi-multipliers in integral incline algebras.

**Keywords:** Symmetric bi-derivations, incline algebras, multipliers, fixed set, kernel.

**References**

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DIRICHLET BOUNDARY VALUE PROBLEM FOR A  $N^{\text{th}}$  ORDER COMPLEX  
DIFFERENTIAL EQUATION

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**MSC 2000:** 30E20, 30E25, 32A55

### Abstract

In this work, we investigate the solvability condition of the problem

$$\partial_{\bar{z}}^n w + c \partial_z \partial_{\bar{z}}^{n-1} w = f(z), \quad f \in L_p(D, \mathbb{C}), \quad p > 2, \quad n = 1, 2, \dots, \quad (1)$$

$$\partial_{\bar{z}}^{k-1} w|_{\partial D} = \gamma_k, \quad \gamma_k \in C(\partial D; \mathbb{C}), \quad 0 \leq k \leq n-1 \quad (2)$$

in the unit disc of complex plane, for  $|c| < 1$ . Moreover, under this condition, we get the unique solution of the problem (1)-(2).

**Keywords:** Dirichlet boundary value problem, Beltrami equation, polyanalytic equation.

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## KIRCHOFF INDEX OF WEIGTED GRAPHS

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Let  $G$  be a simple, connected graph. The Kirchoff index of  $G$  defined as

$$Kf(G) = \sum_{i < j} r_{ij}.$$

In this paper, we define Kirchoff index for the simple connected weighted graphs which edge weights are positive real numbers or positive definite matrices. Furthermore we will give some properties of Kirchoff index for weighted graphs.

**Keywords:** Weighted graphs, Laplacian matrices, Kirchoff index.

**References**

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- [2] D. J. Klein, M. Randic, Resistance distance, *J. Math. Chem.* **12** (1993) 81-95.
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- [7] B. Zhou, N. Trinajstic, A note on Kirchoff index, *Chemical Physics Letters* **455** (2008) 120-123.

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WIENER INDEX OF WEIGTED GRAPHS

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**MSC 2000:** 05C50

**Abstract**

The Wiener index of simple connected  $G$  graph defined as

$$W(G) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n d(i, j).$$

In this paper, we will define of Wiener index of edge-weighted and vertex-weighted graphs, which weights are positive definite matrices. Moreover we will give some properties of Wiener index for this graphs.

**Keywords:** Weighted graphs, Laplacian matrices, Wiener index.

**References**

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- [6] B. Zhou, N. Trinajstic, A note on Kirchoff index, *Chemical Physics Letters* **455** (2008) 120-123.

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ANALYSIS OF THE REASONING SKILLS OF STUDENTS IN SOLVING A  
NON-ROUTINE PROBLEM

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**MSC 2000:** 97A90, 97C30

### **Abstract**

Non-routine problems have a great significance in developing students reasoning skills. Because students attempt to overcome the situation they are in by associating their existing knowledge to arrive at a consistent objective while solving non-routine problems. Naturally these processes necessitate reasoning. This study aims to investigate the reasoning approaches students used in the solution process of a non-routine problem in a milieu designed in compliance with the Theory of Didactical Situations supporting the reasoning skills of seventh grade students. The study was conducted using the qualitative method. Participants of the study were 24 students attending the seventh grade in a middle school in Van province, Turkey. The implementation lasted for 45 minutes. The analysis of the findings of the study was based on the reasoning levels determined by Brousseau and Gibel[1]. As a result, students were able to solve the non-routine problem by establishing interaction with the designed milieu. It could be stated that the designed milieu supported the reasoning skills of the students.

**Keywords:** Theory of didactical situations, reasoning, non-routine problems, 7th grade.

### **References**

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TEXTILE IMAGE CLASSIFICATION USING NAIVE BAYES AND  
MULTI-LAYER PERCEPTRON

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**MSC 68U10 68T10 62H30 62H35**

**Abstract**

In this study, we have designed and implemented textile image classification systems using the methods as Multi-layer Perceptron and Naive Bayes. After the models of the systems are constructed for the classes as "Flowery", "Spotted", "Horizontal Striped", "Vertical Striped", "Plaided", "45 Degree Striped" and "135 Degrees Striped" in training phase, we have computed the success of systems in testing phase. The systems consist of four stages as preprocessing, feature extraction, training and testing [1]. In the preprocessing, first, all textile images are converted to the black-and-white images. Second, the thinning process of the images is performed by skeletonization operation. Third, the Sobel filter is applied to detect the edge of images [2]. In the feature extraction stage, the frequencies of 2x2, 3x3 and 4x4 kernel matrices in the images are calculated for each image. Information gain is also used for the dimension reduction of the images' attribute vectors. In the training stage, the models representing each class are composed by training all attribute vectors. In the testing stage, the systems are evaluated by accuracy and f-measure. As a result, Naive Bayes (The best accuracy and F-measure: 0.944) outperformed Multi-layer Perceptron (The best accuracy: 0.938, The best F-measure: 0.937) in classification accuracy and f-measure.

**Keywords:** Image classification, Naive Bayes, multi-layer perceptron.

**References**

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SOLVING SYSTEMS OF LINEAR DIFFERENTIAL EQUATIONS BY USING  
ARTIFICIAL NEURAL NETWORKS

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**MSC 2000:** 68T01, 68T20, 34A30

### Abstract

Artificial neural networks (ANNs) are well known computational methods which can solve different mathematical problems such as approximating the solution of differential equations. In this work, we solve systems of linear differential equations by using ANN called Multi Layer Perceptron (MLP). We present numerical solutions by MLP and compare them with the analytical solutions. The obtained results show that the artificial neural networks are able to solve systems of linear differential equations.

**Keywords:** Systems of linear differential equations, feedforward neural network, multi layer perceptron, backpropagation algorithm.

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SCHATTEN  $p$ -NORM INEQUALITIES FOR ACCRETIVE-DISSIPATIVE  $2 \times 2$   
OPERATOR MATRICES**İbrahim Halil GUMUS<sup>1</sup>, Omar Hirzallah<sup>2</sup>**<sup>1</sup>*Adiyaman University, Adiyaman, Turkiye*<sup>2</sup>*Hashemite University, Zarqa, Jordan***MSC 2000:** 47B10; 47B44; 47A30**Abstract**

Let  $\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$  be a  $2 \times 2$  bounded linear operator on a Hilbert space  $\mathbb{H} = \mathcal{H} \oplus \mathcal{H}$  with positive real and imaginary parts, where  $\mathcal{H}$  is a complex Hilbert space. It is shown that if  $p \geq 2$ , then

$$\|T_{12}\|_p^p + \|T_{21}\|_p^p \leq 2^{p-1} \sqrt{\|T_{11}\|_p^p \|T_{22}\|_p^p}. \quad (1)$$

where  $\|\cdot\|_p$  stands for the Schatten  $p$ -norm. Our results generalize and improve some earlier results.

**Keywords:** Accretive-dissipative operator, Schatten  $p$ -norm, inequality.

**References**

- [1] A. George, Kh. D. Ikramov, On the properties of accretive-dissipative matrices, *Math. Notes* **77** (2005) 767-776.
- [2] M. Lin, D. Zhou, Norm inequalities for accretive-dissipative operator matrices, *J. Math. Anal. Appl.* **407** (2013) 436-442.
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GENERALIZED BOUR'S THEOREM IN MINKOWSKI SPACE FORM

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**MSC 2010:** 53A35; 53C42

### Abstract

We obtain isometric minimal helicoidal and rotational surfaces using generalized Bour's theorem in three dimensional Minkowski space. In addition, we show that the surfaces preserve minimality when their Gauss maps identically equal, choosing any differentiable functions on the profile curve.

**Keywords:** Gauss map, Gaussian curvature, helicoidal surface, mean curvature, rotational surface.

### References

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ALGEBRAIC SURFACES OF HENNEBERG IN MINKOWSKI 3-SPACE

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**MSC 2010:** 53A35; 53C42

### Abstract

We consider the algebraic Henneberg zero mean curvature surfaces in three dimensional Minkowski space and compute their classes, degrees and integral free representations.

**Keywords:** Henneberg surfaces, Gauss map, mean curvature, class, degree.

### References

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ON WEAK CONTINUITY OF SOFT TOPOLOGICAL SPACES

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**MSC 2000: 54A05, 54C08, 06D72**

**Abstract**

In this presentation, we introduce soft weak continuous function and soft almost continuous function on soft topological spaces. We show that the notion of soft weak continuous and soft almost continuous are independent. We also obtained soft weak continuity and soft almost continuity are strictly weaker than soft continuity. Then we give some basic theorems and results about these new notions.

**Keywords:** soft set, soft topological space, soft continuity.

**References**

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SCHATTEN  $p$ -NORM INEQUALITIES FOR ACCRETIVE-DISSIPATIVE  $2 \times 2$   
OPERATOR MATRICES

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**MSC 2000:** 47B10; 47B44; 47A30

**Abstract**

Let  $\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$  be a  $2 \times 2$  bounded linear operator on a Hilbert space  $\mathbb{H} = \mathcal{H} \oplus \mathcal{H}$  with positive real and imaginary parts, where  $\mathcal{H}$  is a complex Hilbert space. It is shown that if  $p \geq 2$ , then

$$\|T_{12}\|_p^p + \|T_{21}\|_p^p \leq 2^{p-1} \sqrt{\|T_{11}\|_p^p \|T_{22}\|_p^p}. \quad (1)$$

where  $\|\cdot\|_p$  stands for the Schatten  $p$ -norm. Our results generalize and improve some earlier results.

**Keywords:** Accretive-dissipative operator, Schatten  $p$ -norm, inequality.

**References**

- [1] A. George, Kh. D. Ikramov, On the properties of accretive-dissipative matrices, *Math. Notes* **77** (2005) 767-776.
- [2] M. Lin, D. Zhou, Norm inequalities for accretive-dissipative operator matrices, *J. Math. Anal. Appl.*, **407** (2013) 436-442.
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ON COMPARISON OF COHERENT SYSTEMS VIA DYNAMIC SYSTEM  
SIGNATURE

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**MSC 2010:** 62N05, 62G30, 60E15

### Abstract

System signature is extremely useful tool for comparing of coherent systems. Let  $X_i$  's be independent and identically distributed  $n$ -components lifetimes of a system with  $T$  lifetime. The signature of the system is  $\mathbf{s} = (s_1, s_2, \dots, s_n)$  where  $s_i$  is equal to probability of event  $(T=X_{i:n})$  and  $X_{i:n}$  is the  $i$ th order statistics of  $X_i$  's. Also, dynamic system signature is the truncated form of system signature when exactly  $i$  components of the system have failed at time  $t$ . In this study, comparison of new better than used (NBU) and uniformly new better than used (UNBU) properties of aging systems with dynamic system signature by stochastic, hazard rate and likelihood ratio orderings are investigated.

**Keywords:** Coherent system, order statistics, signature, aging, NBU, UNBU, stochastic, Hazard rate and Likelihood ratio orderings.

### References

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A COMBINATORIAL APPROACH TO CATALAN NUMBERS

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**Abstract**

In this paper, we form a bijection between two sets which their cardinalities are Catalan numbers. Also we prove a conjecture in [1] with regard to the equality of two generating functions are connected with these two sets whose cardinalities are Catalan numbers.

**Keywords:** Catalan numbers, generating functions, Dyck paths.

**References**

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## MURNAGHAN-NAKAYAMA RULE FOR JACK POLYNOMIALS

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10, Turkey***Abstract**

For  $\lambda$  is a partition and  $n$  is a nonnegative integer, Murnaghan-Nakayama rule for Schur functions calculates the product of a Schur function  $s_\lambda$  and a power symmetric function  $p_n$ :

$$s_\lambda p_n = \sum_{\nu} (-1)^{ht(\nu/\lambda)} s_\nu,$$

where all partitions  $\lambda \subseteq \nu$  for which  $\nu/\lambda$  is a border strip with  $n$  boxes and the height  $ht(\nu/\lambda)$  of the border strip is the number of rows, minus 1. This is the theorem in [3]. In this work, we investigate Murnaghan Nakayama rule for Jack polynomials. We obtain some combinatorial results and interpretations for some conditions.

**Keywords:** Jack polynomials, symmetric functions, young diagram.

**References**

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SOME RESULTS ON PREDATOR-PREY DYNAMIC SYSTEMS WITH  
BEDDINGTON-DEANGELIS TYPE FUNCTIONAL RESPONSE

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### Abstract

We consider two dimensional predator-prey system with Beddington-DeAngelis type functional response on Time Scales. For this special case we try to find under which conditions the system is permanent and globally attractive. This study will also give beneficial results for continuous and discrete case.

**Keywords:** Predator-prey dynamic system, permanence, global attractivity, Beddington DeAngelis type functional response.

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OPTIMAL CONTROL PROBLEM FOR PROCESSES GIVEN BY  
MULTI-PARAMETER LINEAR STOCHASTIC DYNAMIC SYSTEM

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**MSC 2000:** 49N05, 49J15, 93C10, 93E20

### **Abstract**

In this work, processes represented by multi-parameter linear stochastic dynamic system are investigated [1], [2] and by considering optimal control problem, principle of optimality is proven. Also, for existence of optimal control and corresponding optimal trajectory, proof of theorem of necessity and sufficiency condition is attained.

**Keywords:** Optimal control problem, linear stochastic dynamic system, optimal trajectory.

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ASYMPTOTIC ANALYSIS OF A DYNAMICAL PROBLEM OF  
NON-ISOTHERMAL LINEAR ELASTICITY WITH FRICTION

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**MSC 2000:** 35R35, 76F10, 78M35.

### Abstract

In this paper, we are interested in the study of the asymptotic analysis of a dynamical problem in elasticity with non linear friction of Tresca type. The Lamé coefficients of thin layer are assumed to vary with respect to the thin layer parameter  $\varepsilon$  and depend of the temperature. We prove the existence and uniqueness of the weak solution for the limit problem. The proof is carried out by the use of the asymptotic behaviour when the dimension of the domain tends to zero.

**Keywords:** A priori inequalities, free boundary problems, elasticity system, asymptotic approach, Tresca law.

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ADAPTIVE STEP SIZE NUMERICAL SOLUTION TO FIRST ORDER ODES,  
A REFINEMENT OF EULER'S AND RK METHODS

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October City, Giza, Egypt*

### Abstract

Solving differential equations numerically is a subject that employs a multitude of methods each suitable for certain class of equations. Herein we report a new approach for solving first order ODE numerically that can be considered to be a refinement to Euler and RK methods. Instead of using fixed step sizes as employed in these methods, our method estimates the suitable step size based on an evaluation of the second derivative of the solution curve at or in the vicinity of the iteration point.

**Keywords:** Differential equations, numerical, step size.

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IMPLEMENTING QUANTUM SEARCH ALGORITHM IN THE PRESENCE  
OF DISSIPATION

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**Abstract**

Here, we propose a new physical scheme for perfect implementation of quantum search algorithm in presence of qubits dissipation. It is shown that Grover algorithm is obtained as a special case of the suggested quantum search algorithm. We have generated the the quantum gates to realize the required steps for implementing the algorithm.

**Keywords:** Quantum computation, quantum search algorithm and quantum superconducting.

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APPROXIMATION OF THE SET OF TRAJECTORIES OF CONTROL  
SYSTEM DESCRIBED BY AN AFFINE VOLTERRA TYPE INTEGRAL  
EQUATION

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**MSC 2000:** 93C23

### Abstract

The control system described by an affine Volterra type integral equation is considered. It is assumed that the system is nonlinear with respect to the phase vector and is affine with respect to the control vector. Admissible control functions are chosen from the closed ball of the space  $L_p$ ,  $p > 1$ , with radius  $\mu$  and centered at the origin. Approximation of the set of trajectories of the system generated by all admissible control functions is studied. The set of admissible control functions is replaced by the set which consists of a finite number of control functions and generates a finite number of trajectories. An evaluation of the Hausdorff distance between the set of trajectories of the system and the set consisting a finite number of trajectories is given. This paper extends the results obtained in [1] and [2].

**Keywords:** Integral equation, control system, approximation.

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STRUCTURE OF BASIN AND BIFURCATION PHENOMENA IN  
TWO-DIMENSIONAL PIECEWISE MAPS

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**Abstract**

We present in this work the study of a new bifurcation phenomena for discrete time dynamical systems represented by the iteration of piecewise maps, where the bifurcation theory is much less developed. These phenomena are part of a riche new class of bifurcations. We aim to investigate, via numerical examples, some global bifurcations in the two-dimensional phase plane. The dynamical behaviors, multiple basins with fractal boundary, attractors, route to chaos via bifurcations are further investigated. We also show by numerical simulation different types of bifurcations that can occur in such map, including transitions to chaotic attractors. The property of multistability, i.e. the existence of many coexisting attractors, is a characteristic property of such maps. The problem of the delimitation of the attractors and their basins is studied.

**Keywords:** Piecewise smooth systems, bifurcation, chaos.

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ON OBTAINING STABLE SOLUTION FOR A HYPERBOLIC COEFFICIENT  
CONTROL PROBLEM

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**MSC 2000:** 49J20, 35L20, 49J50

### Abstract

This study deals with obtaining a solution for the hyperbolic coefficient control problem. The set of admissible controls has been taken as a subspace of the space whose elements and their first generalized derivatives are square integrable functions. Obtaining the gradient of the cost functional and proving the Lipschitz continuity on this set, the necessary condition for optimal solution has been given.

**Keywords:** Optimal control, second-order hyperbolic equations, Frechet differentiability.

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ELASTO-PLASTIC DEFORMATION OF AN INCOMPRESSIBLE BENDING  
PLATE WITH CLAMPED BOUNDARY**Feda İLHAN**<sup>1</sup>, **Zahir MURADOĞLU**<sup>2</sup><sup>1</sup>*Abant İzzet Baysal University, Bolu, Turkey*<sup>2</sup>*Kocaeli University, Kocaeli, Turkey***MSC 2000:** 34C10 65N06, 65L10, 35Q74, 35Q90**Abstract**

In this study, the bending problem of an elasto-plastic and homogeneously isotropic incompressible plate is studied. It is assumed that the plate with thickness  $h$  is placed to the coordinate system  $Ox_1x_2x_3$  such that the middle surface of the plate is located in  $Ox_1x_2$  plane. The plate is supposed to be in equilibrium under the action of the loads applied on the upper surface of the plate in the  $x_3$  axis direction, while its lower surface is free. It is known from the deformation theory of plasticity that [1] and [2] as  $w = w(x)$  is the deflection of a point  $x \in \Omega$  on the middle surface of the plate, which is placed in the region  $\Omega = \{(x_1, x_2) \in R^2 : 0 \leq x_\alpha \leq l_\alpha, \alpha = 1, 2\}$ , satisfies the following nonlinear biharmonic equation:

$$Aw \equiv \frac{\partial^2}{\partial x^2} \left[ g(\xi^2(w)) \left( \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \frac{\partial^2 w}{\partial y^2} \right) \right] + \frac{\partial^2}{\partial x \partial y} \left[ g(\xi^2(w)) \left( \frac{\partial^2 w}{\partial x \partial y} \right) \right] + \frac{\partial^2}{\partial y^2} \left[ g(\xi^2(w)) \left( \frac{\partial^2 w}{\partial y^2} + \frac{1}{2} \frac{\partial^2 w}{\partial x^2} \right) \right] = F(x, y) \quad (1)$$

A numerical solution for the boundary value problem related to the fourth order nonlinear PDE for a bending plate with clamped boundary condition by using finite difference method is obtained. Test functions are used for verifying the applicability of the computer program. Accuracy of the approximate solutions of numerical examples showed effectiveness of the given approach.

**Keywords:** Biharmonic equation, finite difference method, elasto-plastic plate, deflection, plasticity.

**References**

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REAL HYPERSURFACES IN COMPLEX PROJECTIVE SPACE WITH  
WEAKLY CONSTANT HOLOMORPHIC CURVATURE

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**MSC 2000:** 53C25, 53C40

**Abstract**

The homogeneous real hypersurfaces in  $\mathbb{C}P^{n+1}$  were classified by Ryoichi Takagi [3] in 1973.

For a homogeneous real hypersurfaces in  $\mathbb{C}P^n$  we have  $g \in \{2, 3, 5\}$ , where  $g$  is number of distinct principal curvatures. Zhen Qi Li [2] prove that  $g \in \{2, 3, 5\}$  for all isoparametric real hypersurfaces in  $\mathbb{C}P^n$  with constant principal curvature. Also, Kimura [1] completed this results.

In this paper, we study isoparametric Hopf hypersurfaces in complex projective space  $\mathbb{C}P^n$  such that structural vectors field  $\xi$  is a principle vector field and with weakly constant holomorphic sectional curvature.

**Keywords:** Hopf hypersurfaces, complex projective space, weakly constant holomorphic curvature.

**References**

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MOORE-PENROSE INVERSE OF WEIGHTED COMPOSITION OPERATORS  
ON  $L^p$ -SPACES

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**MSC 2000:** Primary 47B33, Secondary 47B38.

**Abstract**

For a weighted composition operator  $W : f \mapsto u.(f \circ \varphi)$ , we give an explicit formula for the Moore-Penrose inverse of  $W$  on  $L^2(\Sigma)$  and then determine the Hyers-Ulam stability constant for  $W$  on  $L^p(\Sigma)$ , in terms of conditional expectation operator.

**Keywords:** Weighted composition operator, Hyers Ulam stability, Moore Penrose inverse, conditional expectation.

**References**

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ARTIFICIAL NEURAL NETWORK METHOD FOR SOLVING FRACTIONAL  
FREDHOLM INTEGRAL EQUATIONS

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**MSC 2000:** 45B05

**Abstract**

For the last decade, several authors demonstrated the performance of artificial neural network models over other traditional testing methods[1, 2]. The current research, aimed to present a global optimization technique based on combination of neural networks approach and power series method for the numerical solution of a fractional Fredholm type integro-differential equation involving the Caputo derivative. The mentioned problem to be solved approximately for the unknown series coefficient via a three-layer feed-forward neural architecture. In other words, an accurate truncated power series representation of the solution function is achieved when a suitable learning algorithm is used for the suggested neural architecture. As applications of the present iterative approach, some kinds of integral equations are investigated. The achieved simulations are compared with the results obtained by some existing algorithms.

**Keywords:** Fractional Fredholm equation, generalized power series expansion, ANNs approach, Caputo fractional derivative, approximate solution.

**References**

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NUMERICAL SOLUTION FOR SOME WEAKLY SINGULAR NONLINEAR  
VOLTERRA INTEGRAL EQUATIONS

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Shahid Chamran University, Ahvaz, Iran*

**Abstract**

We consider a second kind weakly singular nonlinear Volterra integral equation defined by a compact operator and derive a Nyström type interpolant of the solution based on zeroes of orthogonal Chebyshev polynomials on  $[0, 1]$  with the weight function 1, (see,[1]). We prove the convergence of the interpolant and derive convergence estimates. For equations with nonlinearity of algebraic kind, we improve the rate of convergence by using a smoothing transformation. Some numerical examples are given.

**Keywords:** Volterra integral equations, weakly singular kernels, Chebyshev polynomials.

**References**

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STABILITY OF CAUCHY FUNCTIONAL EQUATION AND QUADRATIC  
EQUATION BY GENERALIZED OPERATIONS

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**MSC 2000:** 65M06, 65M12

### Abstract

In 1940 S. M. Ulam proposed the famous Ulam stability problem. In 1941 D. H. Hyers solved this problem for additive mappings subject to the Hyers condition on approximately additive mappings. In this paper we generalize the Hyers result for the Ulam stability problem for pseudo-analysis. Core of the construction presented here consists of generalized pseudo-operations which are generating of classical operations. We consider generated pseudo-operations of the following forms:

$$x \oplus y = g^{-1}(g(x) + g(y)), \quad x \otimes y = g^{-1}(g(x)g(y))$$

where  $g$  is a strictly monotone generating function. Using this type of pseudo-operations, an extension the stability of Hyers-Ulam-Rassias and quadratic equations is investigated and proved.

**Keywords:** Pseudo addition, pseudo multiplication, pseudo-analysis, stability, cauchy functional equation, quadratic functional equation.

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A COMPUTATIONAL MODEL FOR THE SIMULATION OF  
ATHEROSCLEROTIC PLAQUES

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**Abstract**

Atherosclerosis is a chronic inflammatory process in which the arterial wall develops a plaque as a result of the build up of cholesterol and other fatty materials in the interior surface of the wall, and is the most common disease of the arterial system. This study focuses on the implementation of a coupled reaction diffusion model in two dimensions with a cross-sectional geometry of the artery which reveals the interaction between various factors that affect the growth of the plaque. The Darcy equations are implemented to model the intramural flow through the arterial wall. The interaction between the macrophages and the oxidized LDLs are modeled by a system of coupled reaction diffusion equations. A pseudo-Stokes equation is used to compute the longterm growth velocity field of the wall, which is then used for the evolution of the geometry of the plaque.

**Keywords:** Atherosclerosis, modeling and simulation, reaction diffusion advection.

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COLLOCATION FINITE ELEMENT SOLUTIONS FOR STEFAN PROBLEM  
WITH NEUMANN BOUNDARY CONDITION

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**MSC 2000:** 65D07, 65L60, 41A15, 35R37

**Abstract**

In this paper, we are going to consider one-dimensional Stefan problem with time-dependent Neumann boundary condition. For the problem, collocation finite element schemes constructed with variable space grid method and boundary immobilisation method are used. The newly obtained numerical results are represented for temperature distribution, the position of moving boundary and the velocity of moving boundary. The numerical results reached in this study have been compared with exact solutions and other numerical results obtained by finite difference method based on isotherm migration method. And they are found to be in good agreement with each other.

**Keywords:** Stefan problems, variable space grid method, boundary immobilisation method, collocation finite element method.

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ON THE CONVERGENCE OF NEWTON-LIKE METHOD FOR SOLVING  
NONLINEAR EQUATIONS IN BANACH SPACES

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**MSC 2000:** 47H10, 49M15

### Abstract

In this paper, we introduce a Newton-like method for solving nonlinear equations in a Banach space. We present a local and semi-local convergence analysis of our method. Also, we show that our method is better than the Newton method and the S-iteration processes of Newton-like [5] and we give numerical examples to support of our results.

**Keywords:** Newton's method, fixed point, nonlinear operator equations, Newton-like method.

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## ON ALMOST PRIME IDEALS

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This work consists only of a survey [1]. In this talk, we study almost prime ideals. Throughout this study,  $R$  denotes commutative ring with identity. We give some theorems about characterization of almost prime ideals.

Theorem : For a proper ideal  $I$  of  $R$  the following are equivalent:

1.  $I$  is almost prime.
2. For  $x \in R - I$ ,  $(I : x) = I \cup (I^2 : x)$ .
3. For  $x \in R - I$ ,  $(I : x) = I$  or  $(I : x) = (I^2 : x)$ .
4. For ideals  $A$  and  $B$  of  $R$  with  $AB \subseteq I$ , but  $AB \not\subseteq I^2$ , then  $A \subseteq I$  or  $B \subseteq I$ .

Theorem : For a proper ideal  $I$  of  $R$  the following are equivalent:

1.  $I$  is  $n$ -almost prime.
2. For  $x \in R - I$ ,  $(I : x) = I \cup (I^n : x)$ .
3. For  $x \in R - I$ ,  $(I : x) = I$  or  $(I : x) = (I^n : x)$ .
4. For ideals  $A$  and  $B$  of  $R$  with  $AB \subseteq I$ , but  $AB \not\subseteq I^n$ , then  $A \subseteq I$  or  $B \subseteq I$ .

Theorem : Let  $R$  and  $S$  be any two commutative rings. Then an ideal of  $R \times S$  is almost prime if and only if it has one of the following three forms,

1.  $I \times S$ , where  $I$  is an almost prime ideal of  $R$ .
2.  $R \times J$ , where  $J$  is an almost prime ideal of  $S$ .
3.  $I \times J$ , where  $I$  is an idempotent ideal of  $R$  and  $J$  is an idempotent ideal of  $S$ .

**Keywords:** Almost prime ideals,  $n$ -almost prime, idempotent ideal.

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GROUP STRUCTURE OF MARKOV POLYGONS

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**MSC 2000:** 20F65

**Abstract**

This paper presents defining Markov n-polygons and shows group structures on these sets.

**Keywords:** Markov, group.

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A NEW REGULAR MATRIX DEFINED BY FIBONACCI NUMBERS AND ITS  
APPLICATIONS

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**MSC 2000:** 11B39, 46B45

### Abstract

The main goal of this study is to introduce the sequence spaces  $l_p(F)$ ,  $l_\infty(F)$ ,  $c(F)$  and  $c_0(F)$  by means of a new infinite Toeplitz matrix  $F$  of Fibonacci numbers and examine some algebraic and topological properties of these spaces where  $1 \leq p < \infty$ .

**Keywords:** Regular matrix, fibonacci number, sequence space.

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CONE METRIC SPACES AND CONE TWO METRIC SPACES

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**Abstract**

There are many generalization forms of metric spaces. Some of them are; fuzzy metric space, cone metric space, K-metric space and K-normed space, etc.. In 2007, Chinese mathematicians Zang and Huang described the cone metric spaces as unaware of the existence of the K-metric and K-normed which was defined and used in the 20th century. At both of them, E Banach space was handled instead of reel number. Then, by making further, Huang and Zang gave the definition of convergence of series at cone metric spaces [1]. In this presentation, cone metric spaces which are a generalisation of metric spaces will be handled. Namely, cone two metric spaces will be examined by using two metric spaces instead of known d-metric space by looking at cone metric spaces. The knowledge about convergence of series will be given by using two metric at cone metric spaces. Some theorems and definitions related to cone two metric spaces will be examined and concluded with examples.

**Keywords:** Metric spaces, cone metric spaces, cone two metric spaces.

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ON  $\Lambda$ - SEMICONSERVATIVE FK SPACES

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MSC 2000 : 46A35, 46A45, 40C05

### Abstract

An FK space  $X \supset \phi$  is  $\Lambda$ - semiconservative FK space if  $X^f \subset \lambda(S)$ , where  $\lambda(S)$  is the space of  $\lambda$ - convergent series and  $X^f$  is  $f$ - (or sequential) dual of  $X$ , that is

$$X^f = \{(f(\delta^k)) : f \in X'\}.$$

In this work, we give some definitions and theorems related with  $\Lambda$ - semiconservative FK spaces.

**Keywords:** FK spaces,  $\beta$ - dual,  $f$ - dual, semiconservative FK spaces.

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HOPF BIFURCATIONS OF A RATIO-DEPENDENT PREDATOR-PREY MODEL  
INVOLVING TWO DISCRETE MATURATION TIME DELAYS

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**MSC 2010:** 34K13, 34K18, 34K20

## Abstract

In this talk, we give a detailed Hopf bifurcation analysis of a ratio-dependent predator-prey system involving two different discrete delays. By analyzing the characteristic equation associated with the model, its linear stability is investigated. Choosing delay terms as bifurcation parameters the existence of Hopf bifurcations is demonstrated. Furthermore, some of the bifurcation properties including direction, stability and period are mentioned. Finally, theoretical results are supported by some numerical simulations.

**Keywords:** Hopf bifurcation, delay differential equation, time delay, stability, periodic solutions, population dynamics.

## References

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FRACTIONAL BOUNDARY VALUE PROBLEMS (BVPS) AND LYAPUNOV TYPE  
INEQUALITY

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**MSC 2000:** 34B05, 34A08

## Abstract

In this talk we prove a sufficient condition for the existence and uniqueness of solutions of linear fractional differential equations involving sequential derivative with Riemann Liouville fractional derivative by using Lyapunov type inequality. As far as we know, this approach is quite new and the connection between BVPs and Lyapunov type inequality obtained for these kind of fractional differential equations is given for the first time.

**Keywords:** Boundary value problems, sequential fractional derivative, Lyapunov type inequalities.

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ON  $S_1$ ,  $B_1$  NEAR RINGS

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**MSC 2000:** 16Y30

### Abstract

A near-rings  $N$  is  $S_1$  near ring if for every  $a \in N$ , there exists  $x \in N^*$  where  $N^* = N - \{0\}$ , such that  $aNa = xa$ . A right near-ring  $N$  is a  $B_1$  near-ring if for every  $a \in N$ , there exists  $x \in N^*$ , where  $N^* = N - \{0\}$ , such that  $Nax = Nxa$ . We discuss some of their properties, obtain a characterisation and also a structure theorem.

**Keywords:**  $S_1$  near ring,  $B_1$  near-ring.

### References

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A GENERALIZED STATIC MEAN-VARIANCE PORTFOLIO OPTIMIZATION

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**MSC 2000:** 91G10, 90C20

## Abstract

The foundations of Modern Portfolio Theory (MPT) was established in the 1950's by Markowitz [1, 2]. The aim of Markowitz's portfolio selection problem is to maximize the expected return of a portfolio and minimize its variance as the measure of risk. The only sources of uncertainty in the standard Mean-Variance portfolio optimization problem are the future price of assets. Martellini and Urošević [3] extend the standard model to a single period model with an uncertain exit time. In this paper we generalize the the Markowitz's Mean-Variance model which covers the model of Martellini and Urošević [3]. It is shown that under some general circumstances the set of optimal portfolios in the generalized model coincides with the standard Markowitz's model.

**Keywords:** Mean-Variance portfolio optimization, Optimal portfolio, Uncertain exit time

## References

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FRactal Calculus and Applications

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**MSC 2000:** 26A33, 28A80, 28A25, 37Fxx

### Abstract

Fractals are the shape of many objects in the nature. Fractal geometry was studied by Benoit Mandelbrot. Fractals are often sets with fractional dimension. Mathematician and researchers have tried to establish analysis on fractals using measure theory but it is not algorithmic. Riemann method as algorithmic method has been generalized to define derivative and integral on fractal sets and curves. Recently, the researcher try to apply these equation in physics and engineering as a new mathematical models to have better approximation for the real problems.

**Keywords:** Fractal calculus, fractional local calculus, fractional dimension, local fractional derivatives.

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RECURRENCE RELATION FOR THE MOMENTS OF ORDER STATISTICS FROM A  
BETA-PARETO DISTRIBUTION

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**MSC 2000:** 62G30

### Abstract

In this paper, a novel cumulative distribution function (*c.d.f.*) for beta-pareto (*BP*) distribution, through two distinct practical frames, is developed. However, the presented models are obviously more pragmatic than the ones being demonstrated in previous works, in the case of extending the further relations. Then, using the exhibited *c.d.f.s*, certain recurrence relations for the single and product moments of the order statistics of a random sample of size  $n$  arising from beta-Pareto distribution are derived.

**Keywords:** Order statistics, single and product moments, recurrence relations, beta-Pareto.

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THE CONCEPT OF WEAK  $(\psi, \alpha, \beta)$  CONTRACTIONS IN PARTIALLY ORDERED  
METRIC SPACES

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**MSC 2000:** 41A65, 41A15, 47H09, 47H10, 54H25

## Abstract

In this paper, we investigate generalized weak  $(\psi, \alpha, \beta)$  contractions in partially ordered sets in order to establish extensions of Banach, Kannan and Chatterjea's fixed point theorems in this setting.

**Keywords:** Fixed point, weak contractions, Kannan fixed point theorem, Chatterjea fixed point theorem, partially ordered set.

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ON GENERALIZED SOME INEQUALITIES FOR  $S$ - CONVEX FUNCTIONS

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**MSC 2000:** 26D07, 26D15

## Abstract

In this paper, a general integral identity for differentiable mapping is derived. Then, we extend some estimates of the right hand and left hand side of a Hermite- Hadamard-Fejér type inequality for functions whose first derivatives absolute values are  $s$  convex. Some applications for special means of real numbers are also provided. The results presented here would provide extensions of those given in earlier works.

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EXPONENTIAL CUBIC B-SPLINE BASED SOLUTIONS OF  
ADVECTION-DIFFUSION EQUATION

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## Abstract

Differential quadrature method (DQM), which was first proposed by Bellman et al. [1], based on exponential cubic B-spline functions [2] has been set up to simulate the solutions of the Advection-Diffusion equation numerically. The reduction of the equation to an ordinary differential equation system (ODS) has been performed by the use of differential quadrature method. Then, the resultant ODS has been integrated in time by using Fehlberg fourth-fifth order Runge-Kutta method with degree four interpolant.

Two initial boundary value problems modeling the transport of the initial concentration along a channel and fade out of an initial pulse have been studied. Existence of analytical solutions for both problems provides to measure the error between exact and numerical solutions. In order to check the efficiency and validity of the method, the discrete maximum error norm has been computed for various space step sizes and time step sizes.

**Keywords:** Differential quadrature method, Exponential cubic B-spline, Advection-Diffusion equation.

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INEQUALITIES AND EXPONENTIAL DECAY OF CERTAIN DIFFERENTIAL  
EQUATIONS OF FIRST ORDER IN TIME VARYING DELAY

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**MSC 2000:** 34D20, 34C11

## Abstract

In this paper, we give sufficient conditions to guarantee exponential decay of solutions to zero of the time varying delay differential equation of first order. By using the Lyapunov-Krasovskii functional approach, we establish new results on the exponential decay of solutions, which include and improve some related results in the literature.

**Keywords:** Exponential stability, instability, Lyapunov functional, time delay.

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## GENERALIZED FRACTIONAL MAXIMAL OPERATOR ON GENERALIZED LOCAL MORREY SPACES

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In this talk, we study the continuity properties of the generalized fractional maximal operator  $M_\rho$  on the generalized local Morrey spaces  $LM_{p,\varphi}^{\{x_0\}}$  and generalized Morrey spaces  $M_{p,\varphi}$ . We find conditions on the triple  $(\varphi_1, \varphi_2, \rho)$  which ensure the Spanne type boundedness of  $M_\rho$  from one generalized local Morrey space  $LM_{p,\varphi_1}^{\{x_0\}}$  to another  $LM_{q,\varphi_2}^{\{x_0\}}$ ,  $1 < p < q < \infty$  and from  $LM_{1,\varphi_1}^{\{x_0\}}$  to the weak space  $WLM_{q,\varphi_2}^{\{x_0\}}$ ,  $1 < q < \infty$ . We also find conditions on the pair  $(\varphi, \rho)$  which ensure the Adams type boundedness of  $M_\rho$  from one generalized Morrey space  $M_{p,\varphi^{\frac{1}{p}}}$  to another  $M_{q,\varphi^{\frac{1}{q}}}$  for  $1 < p < q < \infty$  and from  $M_{1,\varphi}$  to  $WM_{q,\varphi^{\frac{1}{q}}}$  for  $1 < q < \infty$ . In all cases the conditions for the boundedness of  $M_\rho$  are given in terms of supremal-type integral inequalities on  $(\varphi_1, \varphi_2, \rho)$  and  $(\varphi, \rho)$ , which do not assume any assumption on monotonicity of  $\varphi_1(x, r)$ ,  $\varphi_2(x, r)$  and  $\varphi(x, r)$  in  $r$ .

**Keywords:** Generalized fractional maximal operator, generalized local Morrey spaces.**References**

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FOURTH ORDER APPROXIMATE METHOD FOR A TIME-DELAYED  
PSEUDO-PARABOLIC EQUATION

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**MSC 2000:** 65M15, 65M20, 65L05, 65L70

### **Abstract**

In this study we consider the one dimensional initial-boundary Sobolev problem with delay. For solving this problem numerically, we construct fourth order difference-differential scheme and obtain the error estimate for its solution. Further, for the time variable we use the appropriate Runge-Kutta method for the realization of our difference-differential problem. Numerical results supporting the theory are presented.

**Keywords:** Sobolev problem, delay difference scheme, error estimate.

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A NEW APPROACH TO ONE PARAMETER MOTION

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**MSC 2000:** 51J15, 51P05

## Abstract

In our study, we study a different approach to one parameter motion. We think that while one of the planes is fixed, the other is deformation on the plane with shear motion. By this way, we will calculate the velocity connection and pole curve that occurred by the movement.

**Keywords:** Planar motion, pole curve, shear mapping.

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THE LEVEL CURVES AND SURFACES ON TIME SCALES

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**MSC 2000:** 51P05, 26E70;

### **Abstract**

The general idea of this paper is to study level curves and surfaces by considering delta gradient functions on time scales. Aided by the definition of the delta gradient function, some geometric structures of level curves and surfaces are investigated.

**Keywords:** Time scales, level curves and surface, delta calculus.

### **References**

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THE DUAL-VARIABLE BERNSTEIN POLYNOMIALS

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**MSC 2000:** 33E99, 26B12

**Abstract**

In this paper we introduce the concept of dual Bernstein polynomials and give its some analysis properties. In particular, we investigate some the limit and derivation of dual-variable Bernstein polynomials.

**Keywords:** Dual-variable Bernstein polynomials, limit, derivations.

**References**

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DISTANCE MEASURES FOR TEMPORAL INTUITIONISTIC FUZZY SETS

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**MSC 2000:** 03E72, 46S40.

## Abstract

In this study, we proposed distance measures for temporal intuitionistic fuzzy sets and investigated some properties of these distance measures. Also, we gave numerical examples for TIFS and its distance measures. We compared these distance measures defined with two and three parameters in terms of reliability and applicability.

**Keywords:** distance measure, temporal intuitionistic fuzzy set, intuitionistic fuzzy set.

## References

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ANOTHER DESCRIPTION ON THE UNITS OF INTEGRAL GROUP RING OF  
DICYCLIC GROUP OF ORDER 12

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**MSC 2000:** 16S34, 16U60

### Abstract

There are many kind of open points with varying difficulty on characterization of unit group of the integral group ring of a given group. In this work, we explicitly give a description of unit group of integral group ring of the group  $T_{12} = \langle a, b : a^6 = 1, b^2 = a^3, bab^3 = a^5 \rangle$ .

**Keywords:** Unit problem, group ring, integral group ring, dicyclic group.

### References

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A POSSIBLE KEY EXCHANGE PROTOCOL OVER GROUP RINGS

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**MSC 2000:** 94A60, 11T71, 14G50

### Abstract

Key exchange protocols are such methods for parties who want to generate shared cryptographic keys that they can send secret messages to each other securely through an insecure channel. In this paper, we first construct a possible key exchange protocol over group rings by giving a concrete example and discuss the security of the system.

**Keywords:** group rings, units, cryptographic keys, security.

### References

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OSCILLATION OF MIXED TYPE THIRD ORDER NONLINEAR DIFFERENTIAL  
EQUATION WITH PIECEWISE CONSTANT ARGUMENTS

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**MSC 2000:** 34K11

### Abstract

In this paper, we study the oscillatory and asymptotic behavior of all solutions of the mixed type third order nonlinear differential equation with piecewise constant arguments.

**Keywords:** Third order differential equation, piecewise constant arguments, oscillation

### References

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PULSED CHEMOTHERAPY MODEL

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**Abstract**

A pulsed chemotherapeutic treatment model is considered in this work. We prove existence of nontrivial periodic solutions by the mean of Lyapunov-Schmidt bifurcation method. In this model we consider the case of application of two drugs, the first one P with continuous effect, it appears in the differential equations, and the second one T with instantaneous effects expressed by impulse equations. The study of existence of bifurcated nontrivial periodic solutions is based on the competition parameter values.

**Keywords:** Chemotherapy, impulsive differential equations, stability.

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GENERAL NON-MARKOVIAN DYNAMICS OF OPEN QUANTUM SYSTEMS AND  
SPECTRAL DENSITY OF COMPLEX SYSTEMS USING EXCEPTIONAL  
ORTHOGONAL POLYNOMIALS

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### **Abstract**

According to Ref [1], we study a simple structure of mapping the environment of an open quantum system onto infinite chain representations with nearest neighbour interactions where the system only couples to the first element in the chain. In this paper we explore various properties of exceptional orthogonal polynomials and then use exceptional jacobi polynomials that it can be applied to three types of the bath spectral density sub-Ohmic, Ohmic, and super-Ohmic in open quantum systems.

**Keywords:** Exceptional orthogonal polynomials, spectral density, open quantum systems.

### **References**

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PHONON SPECTRAL DENSITY OF THE FMO LIGHT-HARVESTING COMPLEX  
WITH ASSOCIATED AND GENERALIZED JACOBI POLYNOMIALS

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**Abstract**

Energy transfer systems like Fenna-Matthews-Olson (FMO) complex shows quantum coherence between sites of Bacteriophylla molecules in protein environment. In this paper we consider phonon spectral density(PSD) of protein environment in FMO complex and provide a assessment of PSD using associated and generalized jacobi polynomials.

**Keywords:** associated and generalized jacobi polynomials, Phonon spectral density, FMO light-harvesting.

**References**

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BOUNDARY VALUE PROBLEM FOR A STURM-LIOUVILLE OPERATOR WITH  
PIECEWISE CONTINUOUS COEFFICIENT

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**MSC 2000:** 34L10, 34L40

### Abstract

In this paper, a self adjoint boundary value problem with a piecewise continuous coefficient on the positive half line  $[0, \infty)$  is considered. The resolvent operator is constructed and the expansion formula with respect to eigenfunctions is obtained. The spectrum of the operator is discussed.

**Keywords:** Sturm-Liouville operator, resolvent operator, expansion formula.

### References

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ON THE INVERSE PROBLEM FOR A CLASS OF DIRAC OPERATORS

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**MSC 2000:** 34A55, 34L40.

### Abstract

In this paper, we consider the direct and inverse problems for a class of Dirac operators with spectral parameter dependent in boundary condition. The asymptotic formulas of eigenvalues, eigenfunctions and normalizing numbers of this problem are investigated, spectral data is defined by the sets of eigenvalues and normalizing numbers. The expansion formula with respect to eigenfunctions is obtained. The main equation is derived. The uniqueness of inverse problem according to spectral data is proved.

**Keywords:** Dirac operator, main equation, inverse problem.

### References

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HEAT SOURCE USING THE CONJUGATE GRADIENT METHOD WITH ADJOINT  
PROBLEM

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**MSC 2000:** 17B56.

## Abstract

In this paper, we illustrate the solution of the inverse problem of estimating the temperature-dependent heat Source. Inverse problems of estimating temperature-dependent properties and have been generally solved by the conjugate gradient method with adjoint problem. One-dimensional formulation of heat conduction problem in a slab was used. An unknown heat source are estimated using conjugate gradient method. Finally, the results obtained from inverse method cover the exact values properly.

**Keywords:** The inverse heat conduction, heat source, conjugate gradient method, function estimation.

## References

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A CLASSIFICATION OF RAMANUJAN COMPLEMENTS OF UNITARY CAYLEY  
GRAPHS

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**MSC 2000:** 05C50

### Abstract

The unitary Cayley graph on  $n$  vertices,  $X_n$ , has vertex set  $\mathbb{Z}_n$ , and two vertices  $a$  and  $b$  are connected by an edge if and only if they differ by a multiplicative unit modulo  $n$ , i.e.  $\gcd(ab, n) = 1$ . A  $k$ -regular graph  $X$  is Ramanujan if and only if  $\lambda(X) \leq 2\sqrt{k-1}$  where  $\lambda(X)$  is the second largest absolute value of the eigenvalues of the adjacency matrix of  $X$ . We obtain a complete characterization of the cases in which the complements of unitary Cayley graph  $\bar{X}_n$  is a Ramanujan graph.

**Keywords:** Graph, Cayley graph, Ramanujan graph.

### References

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- [2] W. Klotz, T. Sander, Some properties of unitary Cayley graphs, *The Electronic Journal of Combinatorics* **14** (2007) .

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THE PROBLEM OF CLOTHSELLER'S SON

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**MSC 2000:** 05C50

**Abstract**

When wrapped cloth area is intended to calculate without unfolding, how can it be done? This question which, I call, is the problem of clothseller's son is discussed in details in this paper.

**Keywords:** Wrapped, curvature length, clothseller.

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NILPOTENT LIE ALGEBRAS AND BETTI NUMBER BEHAVIOR

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**MSC 2000:** 17B56.

## Abstract

Three general problems concerning the cohomology of a (real or complex) nilpotent Lie algebra: first of all, determining the Betti numbers exactly; second, determining the distribution these Betti numbers follow; and finally, estimating the size of the individual cohomology spaces or the total cohomology space. We show how spectral sequence arguments can contribute to a solution in a concrete setting. For one-dimensional extensions of a Heisenberg algebra, we determine the Betti numbers exactly. We then show that some families in this class have a M-shaped Betti number distribution, and construct the first examples with an even more exotic Betti number distribution. Finally, we discuss the construction of (co)homology classes for split metabelian Lie algebras, thus proving the Toral Rank Conjecture for this class of algebras.

**Keywords:** Lie algebra, cohomology, Betti numbers.

## References

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MIRRORS ON HURWITZ SURFACES

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**MSC 2000:** 05C10, 30F10

## Abstract

A compact Riemann surface  $X$  is called symmetric if it admits an anti-conformal involution  $\sigma: X \rightarrow X$ , which is called a symmetry of  $X$ . The fixed-point set of  $\sigma$  consists of disjoint simple closed geodesics on  $X$ , which are called the mirrors of  $\sigma$ . Let  $g > 1$  be a positive integer and  $\mu(g)$  be the maximum number of conformal automorphisms of all Riemann surfaces of genus  $g$ . Then it is known that  $\mu(g) \leq 84(g - 1)$ . A Riemann surface of genus  $g$  admitting  $84(g - 1)$  conformal automorphisms is called a Hurwitz surface. In this study we find an upper bound for the number of mirrors on Hurwitz surfaces.

**Keywords:** Riemann surface, Hurwitz surface, symmetry, mirror.

## References

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MSC 2000: 11S80, 33D05

**Abstract**

Let  $p$  be a fixed prime number. By  $\mathbb{Z}_p$ ,  $\mathbb{Q}_p$  and  $\mathbb{C}_p$  we denote the ring of  $p$ -adic integers, the field of  $p$ -adic numbers and the completion of the algebraic closure of  $\mathbb{Q}_p$ , respectively.

J. Diamond (1977) gave a definition for the  $p$ -adic log gamma function  $G_p : \mathbb{C}_p \setminus \mathbb{Z}_p \rightarrow \mathbb{C}_p$  by the Volkenborn integral

$$G_p(x) := \int_{\mathbb{Z}_p} ((x+u) \log_p(x+u) - (x+u)) du$$

where  $\log_p$  is the  $p$ -adic logarithm function.

In the present work we consider the  $p$ -adic log beta function and we obtain some its properties.

**Keywords:**  $p$ -adic number,  $p$ -adic logarithm function,  $p$ -adic log gamma function,  $p$ -adic log beta function.

**References**

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A MODIFIED GALERKIN METHOD FOR SOLVING INTEGRAL EQUATIONS OF  
THE SECOND KIND

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**MSC 2000:** 45E05. 45J05

### **Abstract**

A Some important classes of integral equations of the second kind is considered. The problem is investigated by using a modified Galerkin method. We prove the existence of the solution for the approximate equation, and we perform the error analysis.

**Keywords:** Galerkin method, projection methods, singular integral equations.

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OPTIMAL PORTFOLIO STRATEGIES UNDER VARIOUS RISK MEASURES

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## Abstract

In this research, we search for optimal portfolio strategies in the presence of various risk measures that are common in financial applications. Particularly, we deal with the static optimization problem with respect to Value at Risk, Expected Loss and Expected Utility Loss measures. To do so, under the Black-Scholes model for the financial market, Martingale method is applied to give closed-form solutions for the optimal terminal wealths, then via representation problem the optimal portfolio strategies are achieved. We compare the performances of these measures on the terminal wealths and optimal strategies of such constrained investors. Finally, we present some numerical results to compare them in several respects to give light to further studies.

**Keywords:** Portfolio optimization, value at risk, expected loss, expected utility loss, Black-Scholes model, Martingale method, risk constraints.

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GLOBAL STABILITY ANALYSIS OF A GENERAL SCALAR DIFFERENCE  
EQUATION

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**MSC 2000:** 39A10,39A30

## Abstract

We consider a general first order scalar difference equation with and without Allee effect. The model without Allee effect represents asexual reproduction of a species while the model including Allee effect represents sexual reproduction. We analyze global stabilities of both models analytically and compare the results obtained. Numerical simulations are included to support the analytical results. We conclude that Allee effect decreases global stability of a nonnegative fixed point of the model. This result is different from the local stability behavior of the same fixed point of the model.

**Keywords:** Allee effect, discrete-time models, global stability.

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THE  $R$ -WHITNEY NUMBERS LIKED TO GENERALIZED BERNOULLI  
POLYNOMIALS

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### Abstract

The main object of this paper is to give an application of the  $r$ -Whitney numbers on the values at rational arguments of the high order Bernoulli and Euler polynomials. The obtained formulas generalize the known expressions of the Bernoulli numbers of both kinds.

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REVISIT SCHEME OF ADOMIAN DECOMPOSITION METHOD FOR NON  
HOMOGENEOUS HEAT EQUATION

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**Abstract**

In this paper, a new revisit scheme of the Adomian decomposition method is proposed to solve the homogenous and non-homogenous initial and boundary value problem of heat equation, leading to the same solution as the one obtained by the separation of variables method. A numerical example is thus given to prove that the presented method is reliable, efficace and can be employed to derive successfullt analytical approximate solutions of heat equation. **Keywords:** Heat equation, adomian decomposition method, separation of variables method.

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*F*-CONTRACTIONS ON METRIC SPACES AND SOME RELATED FIXED POINT  
RESULTS

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**MSC 2010:** 54H25, 47H10

## Abstract

In this work, we present recent developments about *F*-contractions on a metric space. Then, considering *F*-contractions, we give some fixed point theorem for singlevalued and multivalued mappings on complete metric spaces.

**Keywords:** Fixed point, single-valued mapping, multivalued mappings, *F*-contraction.

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## THE SKEW INVERSE SEMIGROUP RING

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MSC 2000: 20M18, 16W22

**Abstract**

For a given partial action  $\pi$  of an inverse semigroup  $S$  on an associative algebra  $\mathcal{A}$ , we introduce the notation of *skew inverse semigroup ring*  $\mathcal{A} \rtimes_{\pi} S$ , and prove that this construction is associative algebra under some conditions on a partial action  $\pi$ . At the end we define the concept of *strongly associative algebra* and we show that a semiprime algebra  $A$  is strongly associative. We refer to the treatises [1, 2, 3] for a thorough treatment of the concepts of partial actions, actions, and crossed products. Let  $\pi = (\{\pi_s\}_{s \in S}, \{X_s\}_{s \in S})$  be a partial action of  $S$  on  $A$ , and let  $L = \{\sum_{s \in S} a_s \delta_s : a_s \in X_s\}$  the set of all formal finite sums, with the following multiplication:

$$(a_s \delta_s) \cdot (b_t \delta_t) = \pi_s(\pi_{s^*}(a_s) b_t) \delta_{st}.$$

With the aid of multiplier algebra, instead of using approximate identity of  $C^*$ -algebra as in [3], we will prove that if for each  $s \in S$  the ideal  $X_s$  is  $(L, R)$ -associative then  $L$  is associative, so, it is an algebra. Let  $I$  be the ideal generated by the set  $\{a \delta_r - a \delta_t : \text{where } r \leq t \text{ and } a \in X_r\}$ , then  $A \rtimes_{\pi} S$  is the quotient algebra  $\frac{L}{I}$ , hence, it is an associative algebra.

**Keywords:** Partial action, inverse semigroup, multiplier algebra.

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WHEN THE UNIVERSAL INVERSE SEMIGROUP  $Pr(S)$  OF INVERSE SEMIGROUP  
 $S$  IS  $E^*$ -UNITARY

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MSC 2000: 20M18, 16W22

## Abstract

In this work, we will consider the notion of *partial actions* of groups and *partial actions* of inverse semigroups on sets, for more details about these concepts the reader is referred to [1, 3]. At first, for a finite group  $G$  we will prove that if the order of  $G$  is greater than one then  $G$  admits a partial action which is not a homomorphism. We will prove our claim by using the *universal inverse semigroup*  $S(G)$  associated to a group  $G$ , more information can be found in [3]. Also, we will consider the *universal inverse semigroup*  $Pr(S)$  that A. Buss and R. Exel in [3] associated to an inverse semigroup  $S$ . Recall that an inverse semigroup  $S$  is  $E^*$ -unitary if for  $s \in S$  and  $e \in E(S)$ ,  $e \leq s$  implies that  $s \in E(S)$ . We will show that an inverse semigroup  $S$  is  $E^*$ -unitary if and only if  $E(S)$ , the set of all idempotents, is a filter. Our main Theorem for an inverse semigroup  $S$  is that:

**Theorem.** An inverse semigroup  $S$  is  $E^*$ -unitary if and only if  $Pr(S)$  is  $E^*$ -unitary inverse semigroup.

**Keywords:** Partial action, universal inverse semigroup,  $E^*$ -unitary inverse semigroup.

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GENERALIZED NEWTON TRANSFORMATION AND ITS APPLICATION TO  
TRANSVERSAL SUBMANIFOLDS

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### Abstract

In this paper, we study some properties of generalized Newton transformation  $T_U$  of a family of endomorphisms, this quantities is a generalisation of the natural Neton transformation. As application we establish a relation between the transversality of two submanifolds and ellipticity of  $T_U$ . The second part of our work is to determine a flux formula wich can help us to minimise the higher order mean curvature of a submanifold by the geometry of its boundary.

**Keywords:** Generalized Newton transformation, transversal submanifolds, Neton transformation.

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GENERALIZED MARKOV PROCESSES

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**Abstract**

Generalized Markov processes are long memory Markov processes , they can be , among others, solution of stochastic differential equation delay. Statistical techniques of these processes must be developed to describe these processes in order to apply forecasting techniques.

**Keywords:** Markov processes, stochastic differential equation, statistical techniques.

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EULER-LAGRANGE AND HAMILTON-JACOBI EQUATIONS ON A RIEMANN  
ALMOST CONTACT MODEL OF A CARTAN SPACE OF  
ORDER K

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**MSC 2000:** 70H03, 70H05, 70G45

## Abstract

Lagrangians and Hamiltonians have many applications in various fields, as: Mathematics, Physics, Optimal Control Theory, Dynamic Systems, Economy, Biology, etc.[1]. Since one can construct geometries of higher-order Lagrange space and higher-order Hamilton space over the manifolds  $T^k M$  and  $T^{*k} M$  of a manifold  $M$  respectively, manifold theory has an important role to describe "Euler-Lagrange and Hamilton (-Jacobi) equations" and also "Lagrangian and Hamiltonian mechanics" of a given manifold [2],[3].

Therefore, in this paper, we obtain Euler-Lagrange and Hamilton-Jacobi equations on a Riemann Almost Contact Model of a Cartan Space of order k. In the conclusion we discuss some results about related mechanical system.

**Keywords:** Cartan manifold, mechanical systems, Lagrange and Hamilton equations.

## References

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EXISTENCE AND REGULARITY OF THE SOLUTION FOR NONLINEAR AND  
OBLIQUE PROBLEMS WITH FRICTION

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**MSC 2000:** 35B40, 35C20

### **Abstract**

In this paper we consider the nonlinear boundary value problem governed by a stationary perturbed elasticity system with mixed boundary conditions (Tresca-Dirichlet- maximal monotone graph), in a smooth domain. We first establish the existence result and some estimates for weak solutions of its approached problem. A specific regularity of the displacement field is obtained. The proof is based on the approach of maximal monotone graph by its Yosida regularization and the contraction method.

**Keywords:** Regularity, elasticity, maximal monotone graph.

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ON NILPOTENT ELEMENTS IN ORE EXTENSIONS

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**MSC 2000:** 16S36

## Abstract

We introduce the notion of nil- $(\alpha, \delta)$ -compatible rings which is a generalization of reduced rings and  $(\alpha, \delta)$ -compatible rings. In this paper we are concerned with the set of nilpotent elements in the ring of skew polynomial in the more general situation that, when  $R$  is semicommutative and has  $(\alpha, \delta)$ -condition, then we show that the following generalization of equality for polynomial rings holds  $nil(R)[x; \alpha, \delta] = nil(R[x; \alpha, \delta])$ .

**Keywords:** Compatible rings, skew polynomial rings.

## References

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## MIXING TYPE THEOREM FOR POWER BOUNDED MEASURES

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MSC 2000: 47B07; 30H05

## Abstract

Let  $G$  be a locally compact abelian group with dual group  $\Gamma$ . By  $M(G)$  and  $L^1(G)$  respectively, we denote the convolution measure algebra and the group algebra of  $G$ . For  $n \in \mathbb{N}$ , by  $\mu^n$  we denote  $n$ -times convolution power of  $\mu \in M(G)$ . A measure  $\mu \in M(G)$  which satisfies  $\sup_{n \in \mathbb{N}} \|\mu^n\| < \infty$  is called *power bounded*. For a power bounded measure  $\mu \in M(G)$ , we have  $|\widehat{\mu}(\gamma)| \leq 1$  for all  $\gamma \in \Gamma$ , where  $\widehat{\mu}$  is the Fourier-Stieltjes transform of  $\mu$ . We put

$$\mathcal{E}_\mu := \{\mu \in \Gamma : |\widehat{\mu}(\gamma)| = 1\}.$$

The main result is as follows.

**Theorem.** *If  $\mu \in M(G)$  is power bounded, then*

$$\lim_{n \rightarrow \infty} \|\mu^{n+1} * f - \mu^n * f\| = 0, \quad \forall f \in L^1(G),$$

*if and only if  $\widehat{\mu}(\mathcal{E}_\mu) = \{1\}$ .*

**Keywords:** Group algebra, measure algebra, weak mixing.

## References

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HEAT TRANSFER IN HYDRO MAGNETIC OSCILLATORY FLOW PAST AN  
IMPULSIVELY STARTED POROUS LIMITING SURFACE WITH HEAT  
GENERATION/ABSORPTION

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**MSC 2000:** 76W05

## Abstract

This work report the analytical solution of two dimensional hydromagnetic oscillatory flow of a viscous, incompressible and electrically conducting fluid, past a porous, infinite limiting surface with variable suction. The fluid limiting surface is moved impulsively, with a constant velocity, either in the direction of the flow or in the opposite direction, in the presence of a transverse magnetic field. Solutions were obtained for velocity, induced magnetic and temperature fields, and expression were obtained for skin friction, electric current density and rate of heat transfer at the limiting surface. Variations of the emerging flow condition were presented graphically and discussed.

**Keywords:** Hydromagnetic, oscillations, limiting surface, magnetic field, suction, current density.

## References

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## GRAPHICAL METHOD FOR INTERVAL VALUED BIMATRIX GAMES

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We consider a two player non zero strategic (bimatrix) game, in which the players payoffs are given by two interval matrices of the same dimension for player *I* and player *II*. Interval valued bimatrix whose entries are closed intervals as follows;

$$(\tilde{A}, \tilde{B}) = \begin{bmatrix} ([a_{11L}, a_{11R}], [b_{11L}, b_{11R}]) & \cdots & ([a_{1nL}, a_{1nR}], [b_{1nL}, b_{1nR}]) \\ \vdots & \vdots & \vdots \\ ([a_{m1L}, a_{m1R}], [b_{m1L}, b_{m1R}]) & \cdots & ([a_{mnL}, a_{mnR}], [b_{mnL}, b_{mnR}]) \end{bmatrix}$$

In this work, we have adapted graphical method for interval valued bi matrix games. In addition the interval valued bi matrix games 2x2 Nash equilibrium is attained by graphical method.

**Keywords:** Bimatrix games, Nash equilibria, interval matrix.

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FORECASTING BY ADAPTIVE DOUBLE EXPONENTIAL SMOOTHING

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**Abstract**

In this paper we discuss about  $\alpha$  and  $\beta$  where are estimated through some process. We estimating the constant of exponential smoothing using adaptive double exponential smoothing method allow a smoothing parameter to change over time, in order to adapt to changes in the characterizes of the time series ,we finding a new exponential smoothing by fixing value to  $\alpha$  and  $\beta$  to check a goodness of fit we use mean square error ,AIC ,  $x^2$  to test the best model and we compare the method with appropriate ARMA model and forecasting it.

**Keywords:** Forecast, constant, adaptive, normality, mean square error.

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LAPLACE EQUATION WITH TRIPLE-INVERSE SQUARE POTENTIALS ON  
EUCLIDEAN SPACE AND APPLICATIONS**Mohamed Vall OULD MOUSTAPHA**<sup>1</sup><sup>1</sup> *Univerité des Sciences, de Technologie et de Médecine, Nouakchott-Mauritanie*

MSC 2000: 35J05, 35J08, 35K08

**Abstract**

The Dirichlet problem for the Laplace equation attached to the Schrödinger operator with triple-inverse square potential on the Euclidian space  $(R^+)^3$ :

$$\Delta_{\nu,\mu,\eta} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + V(x, y, z) \quad (1)$$

where

$$V(x, y, z) = \frac{1/4 - \nu^2}{x^2} + \frac{1/4 - \mu^2}{y^2} + \frac{1/4 - \eta^2}{z^2} \quad (2)$$

and  $\nu, \mu, \eta$  are real parameters,is solved in terms of the three variables Lauricella hypergeometric functions  $F_A^{(3)}$  and  $F_C^{(3)}$ .

Our principal tools are the Hankel transforms and the special functions of mathematical physics. In the paper [3] another type of problems associated to the operator (1) is considered.

**Keywords:** Laplace equation, inverse square potential, Lauricella hypergeometric functions.

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(G,(H-M))-CONVEX DOMINATED FUNCTIONS

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**MSC 2000:** Primary 26D15, Secondary 26D10, 05C38

### Abstract

In this paper, a new definition of  $(g, (h - m))$ -convex dominated functions is introduced and then several integral inequality for this type functions are presented.

**Keywords:** Convex dominated function, Hermite-Hadamard inequality,  $(h - m)$ -convex function.

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THE RELATIONSHIP BETWEEN  $n^{\text{th}}$  LUCAS NUMBER AND A SEQUENCE  
DEFINED BY M-SEQUENCES

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## Abstract

In this work, we consider the sequence whose  $n$ th term is the number of M-sequences of length  $n^{\text{th}}$  [6]. We define the set of integer vectors  $E(n)$  on the sequence. We show that the cardinality of  $E(n)$  is the  $n^{\text{th}}$  Lucas number  $L_n$ . We also give some theorem related to  $L_n$  and  $E(n)$ .

**Keywords:**  $n^{\text{th}}$  Lucas Number, M-Sequences, cardinality.

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AN ALTERNATIVE PROOF FOR A LEMMA USED IN THE TRACE FORMULA FOR  
GL(2) OVER A NUMBER FIELD

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**MSC 2000:** 11F72

## Abstract

In this note, we give an alternative proof for a particular part of the lemma given in [3] (p. 201, Lemma 9.9) and which is used in [4] (p. 234, Lemma 6.5).

**Keywords:** GL(2), symmetric square, trace formula, number field.

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ON ESTIMATION OF UNKNOWN PARAMETERS OF  
EXPONENTIAL-LOGARITHMIC DISTRIBUTION BY CENSORED DATA

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**MSC 2000:** 62N02

### **Abstract**

The problem of estimation of parameters of Exponential-Logarithmic distribution in the case of censored data is considered. We used pseudo maximum likelihood method and construct a procedure to solve this problem. Theorem of consistency is proved. Also, simulation is used to study the properties of estimators derived.

**Keywords:** Exponential-Logarithmic distribution, pseudo maximum likelihood estimators, consistent estimators, partly censored data.

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ON EXISTENCE OF WEYL DERIVATIVE OF FUNCTIONS IN LORENTZ SPACE  
WITH QUASI-MONOTONE FOURIER COEFFICIENTS

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## Abstract

General notations, definitions and results relating to Lorentz spaces, quasi-monotone sequences, best approximation, class of  $2\pi$ -periodic functions which belong to Lorentz space having quasi-monotone Fourier coefficients and Weyl derivative. Finally the sufficient and necessary condition of existence of Weyl derivative of functions belonging to Lorentz space according to their quasi-monotone Fourier coefficients and properties of best approximation.

**Keywords:** Lorentz spaces, quasi-monotone sequences, best approximation, Weyl derivative.

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BEAUTIFUL NUMBER 6174

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**Abstract**

In this article we try to solve "Mysterious number 6174" or "Kaprekar's constant". We solve this problem in two step or two theorem. This problem is this: Consider a 4-digit number (which is not a multiple for 1111). Sort the digits in bigger-smaller, and reverse order. Subtract the smaller number from the bigger one. Perform the same operation with the remainder (it is called Kaprekar operation). After a number of steps we reach 6174. For example:

$$3452 \Rightarrow 5432 - 2345 = 3087$$

$$3087 \Rightarrow 8730 - 378 = 8352$$

$$8352 \Rightarrow 8532 - 2358 = 6174$$

The question is why these numbers reach 6174?

This question was told by an Indian scientist Kaprekar who had lived from 1905 till 1986. A question that after about 50 years no one is able to solve it with mathematical rules and reasons.

As what was talked about, lots of people have tried to solve this abstruse question but they couldn't yet.

However in all of their deoecton, s.th is ctlective and that is:

If aet all toe numbers, after a levea ne have a particular irouu of digit npmbers that the magnitude of them is 91. And again gf set them from the smsllest till the biggest, the mlgnitude will change do 30. Awt they are shhwn in series named "S".

$$S = \{9810, 9771, 9621, 9531, 9441, 8820, 8721, 8622, 8532, 8442, 8730, 7731, 7632, 7533, 7443, 8640, 7641, 6642, 6543, 6444, 8550, 7551, 6552, 5553, 5544, 9990, 9981, 9972, 9963, 9954\}$$

In firss step or throrem, we peoof that why these numbert arrive to the series "S" with parameter and rules of mathematic. After that we proof that why the numbers of series "S" arrive to 6174 with graph.

**Keywords:** Mysterious number, 6174, Kaprekar.

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## ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF GENERALIZED LIÉNARD SYSTEM

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In this paper autonomous Liénard system

$$\dot{x} = \phi(z - F(x)), \quad \dot{z} = -g(x), \quad (1)$$

is considered. Some sufficient and necessary conditions are presented for the system to have a positive and a negative semiorbit which starts at a point on the curve  $z = F(x)$  and approaches the origin without intersecting the  $x$ -axes which are very important in the theory of oscillation and global asymptotic stability of the solutions of this system. The following conditions on the system are considered.

**(C<sub>1</sub>)**  $F(x)$  and  $g(x)$  are continuous on  $\mathbb{R}$  with  $F(0) = 0$  and  $xg(x) > 0$  for  $x \neq 0$  and  $\phi(u)$  is continuous differentiable and strictly increasing with  $\phi(0) = 0$  and  $\phi(\pm\infty) = \pm\infty$ .

**(C<sub>2</sub>)** For any fixed number  $k > 0$ , there exist  $M(k) > 0$  with  $M(k) \equiv k$  for  $0 < k \leq 1$  such that

$$|\phi(ku)| \leq M(k)\phi(|u|) \quad \text{for all } u.$$

**Keywords:** Global asymptotic stability, Liénard system.

**References**

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MULTIPLE SOLUTIONS TO NONHOMOGENEOUS ELLIPTIC KIRCHHOFF  
EQUATIONS IN  $\mathbb{R}^N$

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**Abstract**

In this work, we use variational methods to study the existence and multiplicity of solutions for a nonhomogeneous Kirchoff equation involving the critical Sobolev exponent.

**Keywords:** Nonhomogeneous Kirchoff equation, the critical Sobolev exponent, multiple solutions.

**References**

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A NEW NUMERICAL APPROACH FOR SOLVING TIME-FRACTIONAL PARTIAL  
DIFFERENTIAL EQUATIONS

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**MSC 2000:** 26A33

### Abstract

In this research, we introduced a new method based on variational iteration method with an auxiliary parameter for solving time-fractional partial differential equations. The convergence of method is showed by using Banach fixed point theorem. Maximum error bound is given. The fractional derivatives are taken in the Caputo sense. Some nonlinear time-fractional partial differential equations are solved by proposed method. The numerical results show that a new method is very effective and convenient.

**Keywords:** Time-fractional partial differential equation, Caputo derivative, variational iteration method, auxiliary parameter.

### References

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A COMBINATION OF VIM AND ASYMPTOTIC EXPANSION FOR SINGULARLY  
PERTURBED CONVECTION-DIFFUSION PROBLEM

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**MSC 2000:** 65L10, 34D15, 41A60

## **Abstract**

In this study, the alternative approach of variational iteration method (VIM) with an auxiliary parameter is introduced for solving singularly perturbed convection-diffusion problems. Asymptotic expansion performed on boundary layer region. The regular region is solved by the alternative approach of variational iteration method with an auxiliary parameter. Linear and nonlinear problems are solved by using the presented method. The numerical results show that the presented method is very effective for this type problems.

**Keywords:** Singularly perturbed, convection-diffusion, variational iteration method, asymptotic expansion.

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EFFECTS OF TRANSPIRATION ON G-JITTER FULLY DEVELOPED MIXED  
CONVECTION FLOW IN A VERTICAL CHANNEL

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**Abstract**

This paper presents an exact solution for fully developed mixed convection flow in a vertical channel induced due to g-jitter by taking into account the effect of suction/injection on the channel walls. The channel walls are maintained at different constant temperatures. The closed form expressions for velocity field, temperature field, skin-friction, and pressure gradient are obtained. The results are presented for various values of controlling parameters. It is found that the presence of transpiration breaks the symmetry of velocity and temperature fields. The results indicate that transpiration can be used as an effective tool to control the flow reversal. **Keyword** g-jitter, mixed convection, porous plates, suction/injection.

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EULER SPIRALS IN SPACE FORMS

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**MSC 2000:** 53A04, 53A05.

## Abstract

We consider Euler spirals in Euclidean 3-space and in three dimensional Minkowski space using some different characterizations. Additionally, we show that all Euler spirals are generalized Euler spirals and also all logarithmic spirals are generalized Euler spirals.

**Keywords:** Curvature, Cornu spiral, Bertrand curve pair.

## References

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CHARACTERIZATIONS OF QUATERNIONIC SOME SURFACES IN MINKOWSKI  
3-SPACE

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**MSC 2000:** 53B30

### **Abstract**

In this paper, we study the bisector of split quaternionic curves in Minkowski 3-space. Moreover, given two rational split quaternionic curves, we show that the bisector surface is rational.

**Keywords:** Bisector surface, Split quaternion, Minkowski space.

### **References**

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NEW PARAMETRIC REPRESENTATION OF A SURFACE FAMILY WITH COMMON  
SMARANDACHE ASYMPTOTIC BY USING BISHOP FRAME

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**MSC 2000:** 53A35

## Abstract

In this paper, we study common smarandache asymptotic curves on a surface in the Euclidean 3-space. By utilizing the Bishop frame, the surface family can be expressed as a linear combination of the components of the local frame in Euclidean 3-space. With this parametric representation, we derive the necessary and sufficient condition for the given smarandache curve to be the common asymptotic curve on the surface.

**Keywords:** Asymptotic curve, Bishop frame, Smarandache curve.

## References

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SOME GENERALIZED OSTROWSKI TYPE INEQUALITIES INVOLVING LOCAL  
FRACTIONAL INTEGRALS AND APPLICATIONS

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**MSC 2000:** 26D07, 26D10, 26D15, 26A33.

## Abstract

In this study, we establish the generalized Ostrowski type inequality involving local fractional integrals on fractal sets  $R^\alpha$  ( $0 < \alpha \leq 1$ ) of real line numbers. Some applications for special means of fractal sets  $R^\alpha$  are also given. The results presented here would provide extensions of those given in earlier works.

**Keywords:** Generalized Ostrowski inequality, generalized Hölder's inequality, generalized convex functions.

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GENERALIZED STEFFENSEN INEQUALITIES FOR LOCAL FRACTIONAL  
INTEGRALS

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**MSC 2000:** 26D15, 26A33.

## Abstract

Firstly we give a important integral inequality which is generalized Steffensen's inequality. Then, we establish weighted version of generalized Steffensen's inequality for local fractional integrals. Finally, we obtain several inequalities related these inequalities using the local fractional integral.

**Keywords:** Steffensen's inequality, local fractional integral, fractal space, generalized convex function.

## References

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LEMKE-HOWSON ALGORITHM FOR TWO-PERSON NON-ZERO SUM GAMES

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**MSC 2000:** 91A05, 91A10

## Abstract

Lemke-Howson Algorithm is effective method to find at least one Nash Equilibrium (N.E) in the pure strategies and mixed strategies for bimatrix (or two-person non-zero sum) games. The algorithm was first introduced in [1] and algebraic method specifying N.E points was generated for bimatrix games. In this work, bimatrix games are studied and Lemke-Howson Algorithm is investigated to find N.E.

**Keywords:** Bimatrix games, Nash equilibrium, Lemke-Howson algorithm.

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THE INFLUENCE OF THERMAL RADIATION, MASS DIFFUSION AND  
FRACTIONAL PARAMETERS ON MHD FLOW OVER A VERTICAL PLATE THAT  
APPLIES TIME DEPENDENT SHEAR TO THE FLUID

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**MSC 2010:** 35Q35, 62P30, 62P35, 65L10

## Abstract

Exact expressions for velocity field, temperature and mass concentration corresponding to the radiative flow of an MHD viscous fluid over an infinite plate that applies time dependent shear to the fluid have been calculated. These expressions are obtained by using Laplace transform of corresponding fractional differential equations. The expression of temperature and mass concentration of fluid have been presented in series form. However, velocity field is presented in the form of integral solutions. All exact expressions satisfy initial and boundary conditions. Some significant limiting cases of fluid parameters and of fractional parameters have been discussed. The influence of fluid and fractional parameters on fluid motion have been analyzed through graphical illustrations. Two special cases of shear stress; shear stress in the form of Heaviside function and oscillating shear stress have also been taken into account to compare the behavior of fluid motion graphically.

**Keywords:** MHD viscous fluid, fractional derivatives, thermal radiation, exact solutions.

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STABILITY AND ACCURACY OF RBF DIRECT METHOD FOR SOLVING A  
DYNAMIC INVESTMENT MODEL

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**MSC 2000:**49Mxx; Secondary 37Mxx.

### **Abstract**

In this paper we consider a Dynamic investment model. In the model, firm's objective is maximizaing discounted sum of profits over an interval of time. The model assumes that firm's capital in time  $t$  increases with investment and decreases with depreciation rate that can be expressed by means of differential equation.

We propose a direct method for solving the problem based on Radial Basis Functions(RBFs). The authors describe operational matrices of RBFs and use them to reduce the variational problem to a static optimization problem which can be solved via some optimization techniques. Next, we describe some economic interpretation of the solution. Finally, the accuracy and stability of the Multiquadric (MQ), Inverse Multiquadric (IMQ) RBFs are illustrated by conducting some numerical experiments.

**Keywords:** RBFs, accuracy, stability, variational problems, dynamic investment problem.

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APPROXIMATING THE RIEMANN-STIELTJES INTEGRAL IN TERMS OF  
SIMPSON'S RULE

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**Abstract**

Error bounds in approximating the Riemann-Stieltjes integral in terms of Simpson's rule are given. Applications for approximating the Riemann integral of a two-function product are provided as well.

**Keywords:** The Riemann-Stieltjes integral, Simpson's rule, error bounds.

**References**

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MATRIX REPRESENTATION OF SOFT POINTS AND ITS APPLICATION

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**MSC 2000:** 03G25, 20D05

### Abstract

The innovation about soft point in this study is, we define soft point's soft matrix form which was not described before for each set of parameters. The matrix representation of soft points is useful for storing all soft points that can be obtained in all different parameters. We then apply it to some important inequalities in classical set theory and observe them as soft equalities in soft set theory. Finally, we prove that our proposed soft matrix provides every soft point that changes with each parameter that takes place in a soft set and enables detailed examination in application of soft set theory.

**Keywords:** Soft set, soft point, soft matrix, soft matrix form of soft point.

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ON WIJSMAN  $I$ -LACUNARY STATISTICAL CONVERGENCE OF ORDER  $\alpha$  OF  
SEQUENCES

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**MSC 2000:** 40A05

## Abstract

In this talk, we introduce the concepts of Wijsman  $I$ -lacunary statistical convergence of order  $\alpha$ , Wijsman  $I$ -statistical convergence of order  $\alpha$  and Wijsman strongly  $I$ -lacunary statistical convergence of order  $\alpha$  of sequences of sets and investigated between their relationship.

**Keywords:**  $I$ -convergence, Wijsman convergence, lacunary sequence.

## References

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ULAM STABILITY OF SOME VOLTERRA EQUATIONS

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**MSC 2000:** 45J05, 47H10, 45M10

## Abstract

In 1940, Ulam posed the following problem: “Give conditions in order for a linear mapping near an approximately linear mapping to exist” [1]. In 1941, this problem was solved by Hyers [2] in the case of Banach space. In 1978, the result of Hyers was generalized by Rassias [3] for approximately linear mappings. The results obtained by Hyers and Rassias can be applied to the case various differential and integral equations [4, 5, 6].

In this talk, we analyse the Hyers-Ulam stability and Hyers-Ulam-Rassias stability of some Volterra equations.

**Keywords:** Hyers-Ulam stability, Hyers-Ulam-Rassias stability, Volterra equations.

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BLOW UP OF SOLUTIONS FOR A NONLINEAR TIMOSHENKO EQUATION WITH  
POSITIVE INITIAL ENERGY

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**MSC 2000:** 74K10,35B99

### **Abstract**

In this work, initial-boundary value problem of a nonlinear Timoshenko equation [1] is investigated. Finite time blow up of solutions with arbitrary positive initial energy is proved under some conditions on the initial data by using potential well method [2] and the method of Levine [3].

**Keywords:** Timoshenko equation, blow-up, high initial energy.

### **References**

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COMPARAISON OF DIFFERENTIAL OPERATORS IN GEVREY SPACE

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**MSC 2000:** 34C10

## Abstract

The aim of this work is to find links between the comparaison of differential operators and their areas of Gevrey vectors. We show that if two hypoelliptic differential operators have equally strength, their spaces of Gevrey vectors coincide.

**Keywords:** Operator of constant strength, hypoelliptic operators, Gevrey spaces vectors.

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ALGORITHMS IN MINIMAL FERRER GRAPH CONSTRUCTIONS

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**MSC 2000:** 05C85, 05A18, 68R05

**Abstract**

In this paper, we give some algorithms and technical properties to construct minimal Ferrer graphs [3] coming from Ferrer relation [2] on  $P_n$  and  $C_n$  by using set cover method [1].

**Keywords:** Graph algorithms, partitions of sets, combinatorics

**References**

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A COMPUTATIONAL APPROACH TO SYLLOGISTIC ENGLISH SENTENCES WITH  
DITRANSITIVE VERBS IN FORMAL SEMANTICS

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**MSC 2000:** 03B65, 68W01, 03C80

## **Abstract**

In this paper, we give some computational aspects of entailment tools in natural English sentences with ditransitive verbs which taking a subject, a direct object and an indirect object in formal semantics of natural language as in [2]. We also present some algorithms and an implementation of reasoners for the sentences and generators for quantifiers [1].

**Keywords:** Logic of natural languages, algorithms (General), logic with extra quantifiers and operators

## **References**

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HYPERBOLIC SMOOTHING METHOD FOR SUM-MAX PROBLEMS

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**Abstract**

In this study, an approach for solving nonsmooth optimization problem, which includes sum of finite maximums of smooth functions is proposed. Minimum  $l_1$ -norm approximations is a particular case of this problem. In this approach, the problem is reformulated in order to use the hyperbolic smoothing function and the relationship between the original problem and reformulated problem are proved. This approach allows us to use conventional smooth optimization methods.

**Keywords:** Hyperbolic smoothing method, sum-max problem, nonsmooth optimization.

**References**

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TAUBERIAN CONDITIONS FOR THE  $(C, \alpha)$  INTEGRABILITY OF FUNCTIONSÜmit TOTUR<sup>1</sup>, İbrahim ÇANAK<sup>2</sup><sup>1</sup>*Adnan Menderes University, Department of Mathematics, Aydın, Turkey*<sup>2</sup>*Ege University, Department of Mathematics, Izmir Turkey*

MSC 2000: 40A10, 40C10, 40D05, 40G05

**Abstract**

For a real-valued continuous function  $f(x)$  on  $[0, \infty)$ , we define

$$s(x) = \int_0^x f(u)du \text{ and } \sigma_\alpha(x) = \int_0^x \left(1 - \frac{u}{x}\right)^\alpha f(u)du$$

for  $x > 0$ . We say that  $\int_0^\infty f(u)du$  is  $(C, \alpha)$  integrable to  $L$  for some  $\alpha > -1$  if the limit  $\lim_{x \rightarrow \infty} \sigma_\alpha(x) = L$  exists.

It is known that  $\lim_{x \rightarrow \infty} s(x) = L$  implies  $\lim_{x \rightarrow \infty} \sigma_\alpha(x) = L$  for all  $\alpha > -1$ . The aim of this paper is twofold. First, we introduce some new Tauberian conditions for  $(C, \alpha)$  integrability method under which the converse implication is satisfied and improve classical Tauberian theorems for the  $(C, \alpha)$  integrability method. Next, we give short proofs of some classical Tauberian theorems as special cases of some of our results.

**Keywords:** Divergent integrals, Cesàro integrability,  $(C, \alpha)$  integrability, Tauberian theorems.

**References**

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THE  $(C, \alpha, \beta)$  INTEGRABILITY OF FUNCTIONS AND A TAUBERIAN THEOREM

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MSC 2000: 40A10, 40C10, 40D05, 40E05

### Abstract

For a continuous function  $f(T, S)$  on  $\mathbb{R}_+^2 = [0, \infty) \times [0, \infty)$ , we define its integral on  $\mathbb{R}_+^2$  by

$$F(T, S) = \int_0^T \int_0^S f(t, s) dt ds,$$

and its  $(C, \alpha, \beta)$  mean by

$$\sigma_{\alpha, \beta}(T, S) = \int_0^T \int_0^S \left(1 - \frac{t}{T}\right)^\alpha \left(1 - \frac{s}{S}\right)^\beta f(t, s) dt ds,$$

where  $\alpha > -1$ , and  $\beta > -1$ . We say that  $\int_0^\infty \int_0^\infty f(t, s) dt ds$  is  $(C, \alpha, \beta)$  integrable to  $L$  if  $\lim_{T, S \rightarrow \infty} \sigma_{\alpha, \beta}(T, S) = L$  exists.

We prove that if  $\lim_{T, S \rightarrow \infty} \sigma_{\alpha, \beta}(T, S) = L$  exists for some  $\alpha > -1$  and  $\beta > -1$ , then  $\lim_{T, S \rightarrow \infty} \sigma_{\alpha+h, \beta+k}(T, S) = L$  exists for all  $h > 0$  and  $k > 0$ .

Next, we prove that if  $\int_0^\infty \int_0^\infty f(t, s) dt ds$  is  $(C, 1, 1)$  integrable to  $L$  and

$$T \int_0^S f(T, s) ds = O(1)$$

and

$$S \int_0^T f(t, S) dt = O(1)$$

then  $\lim_{T, S \rightarrow \infty} F(T, S) = L$  exists.

**Keywords:** The  $(C, \alpha, \beta)$  integrability, improper double integral, convergence in Pringsheim's sense, Tauberian conditions and theorems.

### References

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ON TAUBERIAN REMAINDER THEOREMS FOR CESÀRO SUMMABILITY  
METHOD OF NONINTEGER ORDER

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**MSC 2000:** 40E05,40G05

### Abstract

Let  $A_n^\alpha$  be defined by the generating function  $(1-x)^{-\alpha-1} = \sum_{n=0}^{\infty} A_n^\alpha x^n$ , ( $|x| < 1$ ), where  $\alpha > -1$ . For a real sequence  $u = (u_n)$ , Cesàro means of the sequence  $(u_n)$  of noninteger order  $\alpha$  are defined by

$$\sigma_n^{(\alpha)}(u) = \frac{1}{A_n^\alpha} \sum_{j=0}^n A_{n-j}^{\alpha-1} s_j.$$

We say that a sequence  $(u_n)$  is  $(C, \alpha)$  summable to a finite number  $s$ , where  $\alpha > -1$  if

$$\lim_{n \rightarrow \infty} \sigma_n^{(\alpha)}(u) = s. \quad (1)$$

A sequence  $(u_n)$  is called  $\lambda$ -bounded by  $(C, \alpha)$  method of summability if

$$\lambda_n(\sigma_n^{(\alpha)}(u) - s) = O(1), \quad (2)$$

with  $\lim_{n \rightarrow \infty} \sigma_n^{(\alpha)}(u) = s$ .

In this paper, we prove some Tauberian remainder theorems for Cesàro summability method of noninteger order  $\alpha > -1$ .

**Keywords:** Tauberian remainder theorem,  $\lambda$ -bounded series,  $(C, \alpha)$  summability.

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A FINITE DIFFERENCE METHOD FOR SMOOTH SOLUTION OF SYSTEM OF  
LINEAR WEAKLY SINGULAR VOLTERRA INTEGRAL EQUATIONS

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**Abstract**

In this paper we propose a new numerical method for the smooth solution of a system of linear Volterra integral equations. This method is a generalization of the finite difference method proposed in [3] for scalar linear Volterra integral equations. Error analysis of this method are presented via asymptotic expansion of the absolute error and verification of the accuracy are examined by two illustrative test problems.

**Keywords:** System of integral equation, linear integral equation, weakly singular Volterra.

**References**

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STABILITY AND BOUNDEDNESS OF SOLUTIONS OF VOLTERRA  
INTEGRO-DIFFERENTIAL EQUATIONS

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### Abstract

In 2003, Vanualailai and Nakagiri [2] considered the nonlinear Volterra integro-differential equation without delay

$$\frac{d}{dt}[x(t)] = A(t)f(x(t)) + \int_0^t B(t,s)g(x(s))ds, \quad (1)$$

where  $t \geq 0$ ,  $x \in \mathfrak{R}$ ,  $A(t) : [0, \infty) \rightarrow (-\infty, 0)$ ,  $f, g : \mathfrak{R} \rightarrow \mathfrak{R}$  are continuous functions, and  $B(t, s)$  is a continuous function for  $0 \leq s \leq t < \infty$ . Vanualailai and Nakagiri [2] studied the stability of solutions of equation (1) by defining a suitable Lyapunov functional.

In this paper, we consider the nonlinear Volterra integro-differential equation with delay

$$x'(t) = -a(t)f(x(t)) + \int_{t-\tau}^t B(t,s)g(x(s))ds + p(t), \quad (2)$$

where  $t \geq 0$ ,  $\tau$  is a positive constant, fixed delay,  $x \in \mathfrak{R}$ ,  $a(t) : [0, \infty) \rightarrow (0, \infty)$ ,  $p : [0, \infty) \rightarrow \mathfrak{R}$ ,  $f, g : \mathfrak{R} \rightarrow \mathfrak{R}$  are continuous functions with  $f(0) = g(0) = 0$ ,  $B(t, s)$  is a continuous function for  $0 \leq s \leq t < \infty$ . We investigate the stability of zero solution and boundedness of solutions of equation (2) by defining suitable Lyapunov functionals, when  $p(t) \equiv 0$  and  $p(t) \neq 0$ , respectively.

**Keywords:** Stability and boundedness, Volterra integro-differential equations, Lyapunov functionals.

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ON THE ASYMPTOTIC BEHAVIORS OF SOLUTIONS CERTAIN NON-LINEAR  
NEUTRAL EQUATIONS WITH MULTIPLE DEVIATING ARGUMENTSCemil TUNÇ<sup>1</sup>, Yener ALTUN<sup>2</sup><sup>1,2</sup>Yüzüncü Yıl University, Van, Turkey**Abstract**

In this paper, we obtain sufficient conditions for all solutions of neutral equation of the form

$$\frac{d}{dt} \left[ x(t) + \sum_{i=1}^2 c_i(t)x(t - \tau_i(t)) \right] + \sum_{i=1}^2 p_i(t)x(t) + \sum_{i=1}^2 q_i(t)h_i(x(t))x(t - \sigma_i(t)) \\ + r(t) \int_{t-\delta(t)}^t x(s) ds = 0$$

to approach zero as  $t \rightarrow \infty$ . The technique of proof involves defining an appropriate Lyapunov functional. The obtained result includes and improves some results in the literature.

**Keywords:** Non-linear neutral equations, deviating arguments, Lyapunov functional.

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ON EXPONENTIAL STABILITY OF SOLUTIONS OF NEUTRAL DIFFERENTIAL  
SYSTEM WITH MULTIPLE VARIABLE DELAYSCemil TUNÇ<sup>1</sup>, Melek GÖZEN<sup>2</sup><sup>1,2</sup> *Yüzüncü Yıl University, Van, Turkey***Abstract**

In this work, we establish sufficient conditions which guarantee the solutions of neutral delay differential system of the form

$$\begin{cases} \dot{x}(t) = A(t)x(t) + \sum_{i=1}^n B_i(t)x(t - h_i(t)) + \sum_{i=1}^n C_i(t)\dot{x}(t - h_i(t)) + f_1(t, x(t)) \\ \quad + f_2(t, x(t - h_1(t)), \dots, x(t - h_n(t))) + f_3(t, \dot{x}(t - h_1(t)), \dots, \dot{x}(t - h_n(t))) \\ x(s) = \phi(s), \dot{x}(s) = \varphi(s), s \in [-h_i, 0], (i = 1, 2, \dots, n) \end{cases}$$

are globally exponentially stable. The obtained result includes and improves some results in the literature.

**Keywords:** Neutral delay differential system, Lyapunov functional, globally exponentially stable.

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ON THE STABILITY AND BOUNDEDNESS OF DIFFERENTIAL EQUATIONS OF  
THIRD ORDER WITH RETARDED ARGUMENT

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### Abstract

In this paper, we study the stability, boundedness and ultimately boundedness of solutions of the following  $t$  vector functional differential equations of third order with retarded argument,  $\tau_1 > 0$  :

$$X''' + \Psi(X')X'' + G(X') + cX(t - \tau_1) = P(t, X, X', X'').$$

By using the Liapunov-Krasovskii functional approach, new stability, boundedness and ultimately boundedness criteria are obtained for the considered functional differential equation. The obtained results extend and improve some recent results in the literature.

**Keywords:** Stability and boundedness, retarded argument, Liapunov-Krasovskii functional approach.

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ON THE EXISTENCE OF PSEUDO ALMOST PERIODIC SOLUTIONS TO A CLASS  
OF LASOTA-WAZEWSKA MODEL DIFFERENTIAL EQUATION

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### Abstract

In this work, we discussed a class of Lasota-Ważewska model differential equation. Using exponential dichotomy method, under suitable conditions, we give certain sufficient conditions which guarantee the existence of positive pseudo almost periodic solutions of this model. By this study we extend and improve some earlier result in the literature.

**Keywords:** Pseudo almost periodic solution, Lasota-Ważewska model differential equation, exponential dichotomy.

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OSCILLATION THEOREMS FOR SECOND-ORDER NONLINEAR DIFFERENTIAL  
EQUATIONS WITH NONLINEAR DAMPING

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**Abstract**

We present new oscillation criteria for certain classes of second-order nonlinear differential equations with nonlinear damping term. The obtained results essentially generalize some existing results and are not covered those of Huang and Meng [Y. Huang, F. Meng, Oscillation of second-order nonlinear ODE with damping, Appl. Math. Comput. 199(2008), 644-652]. Examples are also provided to show the importance of our results.

**Keywords:** Nonlinear differential equations, second order, oscillation, damping term.

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## ON THE OSCILLATION OF A CLASS OF DAMPED FRACTIONAL DIFFERENTIAL EQUATIONS

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MSC 2000: 34A08, 34C10

**Abstract**

Using Riccati type transformations, the authors establish some new oscillation criteria for the fractional differential equation

$$(D_{0+}^{1+\alpha}y)(t) + p(t)(D_{0+}^{\alpha}y)(t) + q(t)f(G(t)) = 0, \quad t > 0, \quad (1)$$

where  $D_{0+}^{\alpha}y$  is the Riemann-Liouville fractional derivative of order  $\alpha$  of  $y$ ,  $G(t) = \int_0^t (t-s)^{-\alpha} y(s) ds$  and  $\alpha \in (0, 1)$ . Examples are provided to illustrate the relevance of the results.

**Keywords:** Oscillatory solutions, fractional differential equation, integral averaging technique, Riccati transformation.

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ON GENERALIZED NULL BERTRAND CURVES IN  $\mathbb{E}_2^4$

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**MSC 2000:** 53C50, 53C40

## Abstract

In the present paper, generalized Cartan null Bertrand curves in semi-Euclidean 4-space with index 2 is considered. Because the  $(1, 3)$ -normal planes of the Cartan null curves is timelike, the  $(1, 3)$ -Bertrand mate curves of the curves can be pseudo null curves, non-null curves or Cartan null curves, respectively. Thus, we give the necessary and sufficient conditions for these three cases to be  $(1, 3)$ -Bertrand curves and we also give the related examples.

**Keywords:** Generalized Bertrand curve, Semi-Euclidean Space, Cartan null curve.

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## A NOTE ON LATTICE MODULE

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Let  $L$  be a multiplicative lattice and  $M$  be a lattice  $L$ -module. In this presentation, we acquaint a topology said to be the Zariski topology over  $\sigma(M)$ , the collection of all prime elements of an  $L$ -module  $M$ . We investigate some results on the Zariski topology over  $\sigma(M)$ . Our aim is to characterize the lattice modules whose prime spectrum satisfy some of the separation axioms between  $T_0$  and  $T_1$  which are  $T(\beta)$ ,  $T(\beta')$ ,  $T(\varepsilon)$ ,  $T_{ES}$ ,  $T_D$ ,  $T_{DD}$ ,  $T_Y$  and  $T_{YS}$ .

**Keywords:** Prime spectrum of lattice module, prime element, separation axiom.

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SYMMETRY GROUPS OF PETRIE POLYGONS

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**MSC 2000:** 30F10, 05C10, 05C25

## Abstract

A regular map  $\mathcal{M}$  on a Riemann surface  $X$  is an embedding of a finite connected graph  $\mathcal{G}$  into  $X$  such that the components of  $X - \mathcal{G}$  are identical regular, which are called the faces of  $\mathcal{M}$ . A Petrie polygon of  $\mathcal{M}$  is a polygon such that every two consecutive sides, but no three, belong to a face of  $\mathcal{M}$ . In this study we determine the symmetry group of a Petrie polygon of a regular map.

**Keywords:** Regular map, Petrie polygon, symmetry group.

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GENERALIZED SZÁSZ-MIRAKYAN-DURRMEYER OPERATORS AND THEIR  
APPROXIMATION PROPERTIES

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**MSC 2000:** 41A25, 41A35, 41A36

## Abstract

In this talk, we consider Durrmeyer modifications of the generalized Szász-Mirakyan operators based on a function  $\rho$  which is continuously differentiable  $\infty$  times on  $[0, \infty)$ , such that  $\rho(0) = 0$  and  $\inf_{x \in [0, \infty)} \rho'(x) \geq 1$ . We investigate the weighted approximation properties of the operators and uniform convergence of the operators over unbounded intervals is presented. We obtain direct approximation properties of the operators in terms of the moduli of smoothness and a quantitative Voronovskaya theorem is given.

**Keywords:** Szász-Durrmeyer operators, weighted modulus of continuity, quantitative Voronovskaya theorem.

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ON GENERALIZED DURRMEYER OPERATORS

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**MSC 2000:** 41A25, 41A35, 41A36

## Abstract

In this talk, we extend the studies recently introduced sequence of Ibragimov Gadjiev Durrmeyer operators. These operators include well known Durrmeyer operators as Szasz Durrmeyer, Baskakov Durrmeyer and Generalized Baskakov Durrmeyer operators.

We establish a Voronovskaya type theorem in simultaneous approximation. We present rate of convergence of the derivatives of operators. Some special cases of new operators are presented as examples.

**Keywords:** Ibragimov-Gadjiev-Durrmeyer operators, weighted modulus of continuity, simultaneous approximation.

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$\mathcal{I}$ -CESARO SUMMABILITY OF SEQUENCES OF SETS

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**MSC 2010:** 40A05; 40A35

## Abstract

In this paper, we defined concept of Wijsman  $\mathcal{I}$ -Cesàro summability for sequences of sets and investigate the relationship between the concepts of Wijsman strongly  $\mathcal{I}$ -Cesàro summability, Wijsman strongly  $\mathcal{I}$ -lacunary summability, Wijsman  $p$ -strongly  $\mathcal{I}$ -Cesàro summability and Wijsman  $\mathcal{I}$ -statistical convergence.

**Keywords:** Cesàro summability, statistical convergence, lacunary sequence,  $\mathcal{I}$ -convergence, sequence of sets, Wijsman convergence.

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ASYMPTOTIC NORMALITY OF PARAMETRIC PART IN PARTIALLY LINEAR  
MODELS IN THE PRESENCE OF MEASUREMENT ERROR

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## Abstract

The interest in study measurement error model is growing with the publication of series of papers on various topics. In literature semiparametric partially linear model relating a response  $Y$  to predictors  $(X, X^*)$  with function  $X^T\beta + g(X^*)$  when the  $X^*$ s are unobserved and with additive error is mainly considered with the assumption that the measurement error has a known distribution. Our study gives more detailed answer to the question of "in situation of if independent variable has an unknown distribution in a semiparametric regression model how regression functions and densities predictions could be obtained?". We derived an estimator of  $\beta$  and demonstrated the resulting estimator  $\hat{\beta}$  is asymptotically normal. In the application, the performances of  $\hat{\beta}$  and  $\hat{g}_n(x^*)$  are investigated through Monte Carlo experiments.

**Keywords:** Errors in variables, measurement error, partially linear model, semiparametric regression, unknown error density.

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COEFFICIENT ESTIMATES FOR A NEW SUBCLASS OF CLOSE-TO-CONVEX  
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MSC 2000: Primary 30C45, Secondary 33C45

**Abstract**

Let  $\mathbb{D}$  be the unit disk  $\{z : |z| < 1\}$ ,  $A$  be the class of functions analytic in  $D$ , satisfying the conditions

$$f(0) = 0 \quad \text{and} \quad f'(0) = 1. \quad (1)$$

In this presentation, we will introduce a new subclass of close-to-convex functions denoted by  $KQ_h(\gamma, \lambda)$  with  $\gamma \in \mathbb{C} \setminus \{0\}$ ,  $\lambda \in [0, 1]$ . Here  $h(z)$  is a convex function which satisfies the conditions  $h(0) = 1$  and  $Re(h(z)) > 0$  ( $z \in \mathbb{D}$ ). We will give upper bounds for Taylor coefficients of functions which belong to  $KQ_h(\gamma, \lambda)$ . We will obtain some earlier results as special cases of our parameters.

**Keywords:** Analytic functions, close-to-convex functions, coefficients estimates.

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SOME TOPOLOGICAL PROPERTIES OF THE SPACES OF ALMOST NULL AND  
ALMOST CONVERGENT DOUBLE SEQUENCES

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**MSC 2010:** 40C05, 40G10

### Abstract

Let  $\mathcal{C}_{f_0}$  and  $\mathcal{C}_f$  denote the spaces of almost null and almost convergent double sequences, respectively. We show that  $\mathcal{C}_{f_0}$  and  $\mathcal{C}_f$  are BDK-spaces, barrelled and bornological but they are not monotone and so not solid. Additionally, we establish that both of the spaces  $\mathcal{C}_{f_0}$  and  $\mathcal{C}_f$  include the space  $\mathcal{BS}$  of bounded double series.

**Keywords:** Double sequence, Pringsheim convergence, almost convergence.

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ON ALMOST  $C(\alpha)$ -MANIFOLD SATISFYING SOME CONDITIONS ON THE WEYL  
PROJECTIVE CURVATURE TENSOR

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**MSC 2000:** 53C15, 53C44, 53D10.

## Abstract

In the present paper, we have studied the curvature tensors of almost  $C(\alpha)$ -manifolds satisfying the conditions  $P(\xi, X)R = 0$ ,  $P(\xi, X)\tilde{Z} = 0$ ,  $P(\xi, X)P = 0$ ,  $P(\xi, X)S = 0$  and  $P(\xi, X)\tilde{C} = 0$ . According these cases, we classified almost  $C(\alpha)$ -manifolds, where  $P$  is the Weyl projective curvature tensor,  $\tilde{Z}$  is the concircular curvature tensor,  $S$  is the Ricci tensor and  $\tilde{C}$  is quasi-conformal curvature tensor.

**Keywords:** Almost  $C(\alpha)$ -manifold, Weyl projective curvature tensor, concircular curvature tensor.

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JOST SOLUTION AND SPECTRUM OF THE DISCRETE STURM-LIOUVILLE  
EQUATIONS WITH HYPERBOLIC EIGENPARAMETER**Nihal YOKUS<sup>1</sup>, Nimet COSKUN<sup>2</sup>**<sup>1,2</sup>*Karamanoğlu Mehmetbey University, Karaman, Turkey***MSC:** 39A70 47A10 47A75**Abstract**

In this paper, we consider the boundary value problem (BVP) for the discrete Sturm-Liouville equation

$$a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1} = \lambda y_n, n \in \mathbb{N} \quad (1)$$

$$(\gamma_0 + \gamma_1 \lambda)y_1 + (\beta_0 + \beta_1 \lambda)y_0 = 0 \quad (2)$$

where  $(a_n)$  and  $(b_n)$ ,  $n \in \mathbb{N}$  complex sequences,  $\gamma_i, \beta_i \in \mathbb{C}$ ,  $i = 0, 1$ . By taking  $\lambda$  as a hyperbolic eigenparameter, we obtain exponential type Jost solution of this BVP (1)-(2). Discussing the analytical properties and asymptotic behaviour of Jost solution, we prove that this boundary value problem has a continuous spectrum filling the segment  $[-2, 2]$ . We also prove that BVP (1)-(2) has finite number of eigenvalues and spectral singularities.

**Keywords:** Difference equations, eigenparameter, spectral analysis, Jost solution, discrete equations

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ACHIEVING THE LARGEST PRIMES: ALGORITHM AND RELATIONS IN ORDER

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**MSC 2000:** 11N05, 11N32, 11N80

## Abstract

Prime numbers are currently attained through an exceptional situation in the area of numbers theory and cryptography. The trend for accessing to the largest prime numbers due to using Mersenne theorem, although resulted in vast development of related numbers, it has reduced the speed of accessing to prime numbers from one to four years. Mersenne primes are prime numbers of the form  $2^n - 1$ , where  $n$  is necessarily a prime number. This paper focuses on attaining theorems that are more extended than Mersenne theorem with accelerating the speed of accessing to prime numbers. Since the reason for frequently using Mersenne theorem has found an efficient formula for accessing to the largest prime numbers, this paper provides some relations for prime numbers defined in several formulas for attaining prime numbers in any interval. Therefore, according to flexibility of these relations, it could be found a new branch in the field of accessing to great prime numbers through providing an algorithm at the end of this paper for finding the largest prime numbers.

**Keywords:** Generalization the Mersenne's theorem, relations of prime numbers, algorithm.

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## **Abstracts of Posters**

LOCAL STABILITY ANALYSIS AND ALLEE EFFECTS IN A NONLINEAR  
DISCRETE-TIME POPULATION MODEL INVOLVING DELAY

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**MSC 2000:** 39A10; 39A30

### Abstract

In this study, I will present the effect of Allee factor on the local stability of equilibrium point of the discrete-time population model involving delay generated for  $k = 1$  in [1] with a different approach. The results demonstrate that the Allee effect either reduces the local stability of equilibrium point of the population dynamic model or increases.

**Keywords:** Allee effect, discrete-time models, local stability

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AB INITIO INVESTIGATION OF THE PHYSICAL PROPERTIES OF PEROVSKITE  
RbCdBr<sub>3</sub> STRUCTURE

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## Abstract

We investigated the physical properties such as energy band structure, density of states for electrons and optical properties for RbCdBr<sub>3</sub> compound using the density functional theory under local density approximation and the generalized gradient approximation in Abinit and Wien2k package programs. The calculated results show that the energy band structure of this crystal has an indirect band gap of value 2.693 eV from high symmetry point G to a point between T-Z high symmetry points. Besides, the real and imaginary parts of complex dielectric function, refractive index, energy-loss functions for volume and surface, coefficients of extinction, reflectivity and absorption along the crystallographic axes are investigated. The obtained results are in agreement with the experimental ones.

**Keywords:** DFT, electronic properties, structural properties.

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ON THE NOTION OF  $\theta$ -OPENNESS

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**MSC 2000:** 54D15, 54C08, 54C10

## Abstract

$\theta$ - $g\delta p$ -continuity which are related to  $\delta p$ -normal spaces are presented. More properties of  $\delta p$ -normal spaces and almost  $\delta p$ -normal spaces are researched.

**Keywords:**  $\theta$ - $g\delta p$ -continuity,  $\delta p$ -normal space,  $g\delta p$ -closed set.

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AB INITIO CALCULATION OF PHYSICAL PROPERTIES OF RbGeCl<sub>3</sub>

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## Abstract

We have calculated the structural and electronic properties of RbGeCl<sub>3</sub> compound using the density functional theory within the generalized gradient approximation and the local density approximation. The calculated results for each physical property are presented and then compared with available experimental and previous theoretical data. To our knowledge, structural properties and energy band structure of RbGeCl<sub>3</sub> compound has not been investigated using DFT. Therefore, we investigated the physically properties such as structural and volume optimizations, electronic energy band structure and density of states of valance electrons of this crystal.

**Keywords:** DFT, electronic properties, structural properties.

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CONNECTEDNESS IN MONOTONE ORDERED SPACE

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**MSC 2000:** 34C10

## Abstract

In this paper, we introduce and study the notion of connectedness in monotone ordered space which is a generalization of monotone spaces.

We study ordered semi-separated sets and ordered connected sets in monotone ordered space.

**Keywords:** Monotone ordered space, ordered semi-separated set, ordered connected sets.

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GLOBAL OPTIMIZATION PROBLEM OF LIPSCHITZ FUNCTIONS USING  
A-DENSE CURVES

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MSC 2000: 49M30

## Abstract

In this paper, we study a coupling of the Alienor method with the algorithm of Piyavskii-Shubert. The classical multidimensional global optimization methods involves great difficulties for their implementation to high dimensions. The Alienor method allows to transform a multivariable function into a function of a single variable for which it is possible to use efficient and rapid method for calculating the the global optimum. This simplification is based on the using of a reducing transformation called Alienor.

**Keywords:**The Alienor method, algorithm of Piyavskii-Shubert, global optimization method,  $\alpha$ -dense curves.

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STRUCTURAL AND ELECTRONIC PROPERTIES OF  $A_xD_{1-x}B_yC_{1-y}$  QUATERNARY  
ALLOYS VIA FIRST PRINCIPLES

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## Abstract

We have investigated the structural and electronic properties of cubic AB, AC, DB and DC compounds and their new  $A_xD_{1-x}B_yC_{1-y}$  quaternary alloys, using the density functional theory. Structural properties of these quaternary alloys are calculated with the Perdew and Wang local-density approximation. The lattice constants of  $A_xD_{1-x}B_yC_{1-y}$  quaternary alloys were computed by Vegards law. The band gap of  $A_xD_{1-x}B_yC_{1-y}$  is related by the compositions  $x$  and  $y$ . There is no theoretical examining on  $A_xD_{1-x}B_yC_{1-y}$  quaternary alloys and requires experimental confirmation.

**Keywords:** Structural properties, electronic properties, quaternary alloys.

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STRUCTURAL, DYNAMIC AND THERMODYNAMIC PROPERTIES OF  $CsPbBr_3$   
COMPOUND IN CUBIC PEROVSKITES VIA FIRST PRINCIPLES

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## Abstract

The electronic, dynamic and thermodynamic properties of  $CsPbBr_3$  compound in the cubic perovskite phase are systematically investigated using the first principles calculations. The generalized gradient approximation was used for exchanged and correlation interaction. The theoretically calculated lattice constant and band gap are found to be good in agreement with the other theoretical and experimental results. We studied dynamic and thermodynamic properties of  $CsPbBr_3$  in cubic perovskite phase and have not encountered any available data for dynamic and thermodynamic properties of  $CsPbBr_3$  in cubic perovskite phase. We calculated the entropy, constant-volume specific heat, Helmholtz free energy for  $CsPbBr_3$ .

**Keywords:** Electronic properties, dynamic properties, thermodynamic properties.

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FIRST PRINCIPLE CALCULATION OF ELECTROINIC BAND STRUCTURE OF  
LuRhO<sub>3</sub> COMPOUND

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## Abstract

The structural and volume optimization, electronic band structure density of states of valance electrons of crystal LuRhO<sub>3</sub> have been investigated using the density functional methods, local density approximation and generalized gradient approximation, in Abinit package program. The obtained band structure and calculated density of states of valance electrons for LuRhO<sub>3</sub> compound shows that the electronic structure of crystal has a semiconductor state. Besides, the structural and volume optimization has been performed. To our knowledge in literature, the structural and volume optimization, energy band structure and density of states of valance electrons LuRhO<sub>3</sub> crystal have not been investigated as either experimentally or theoretically. Therefore, in present work, using density functional theory, we investigated the physical properties such as the structural and volume optimization, energy band structure and density of states of valance of this compound.

**Keywords:** Structural properties, electronic band structure, density of states.

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ON THE OPTIMAL CONTROL PROBLEM IN A PARABOLIC SYSTEM

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**MSC 2000:** 35K20, 35D30, 49J20

## Abstract

Optimal control problems in the parabolic problems for different types of cost functionals have been examined by several authors [1, 2, 3, 4]. In this study, we study the optimal control problem governed by on linear parabolic differential equation with Dirichlet boundary conditions. The control function is at the right hand side of the equation. The cost functional consist of deviation in the  $L_2$ -norm of the solution of the system at the final time from a given target, plus  $L_2$ -norm of the control. It is proved that the Frechet derivation of the cost functional can be found via the solution of the adjoint parabolic problem. The results are illustrated by an numerical example.

**Keywords:** Second-order parabolic equations, weak solutions, optimal control.

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STRUCTURAL, DYNAMIC AND OPTICAL PROPERTIES OF DOUBLE-LAYER  
HEXAGONAL BiTeCl CRYSTAL

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## Abstract

The geometric structural optimization, density of states for phonons, phonon band structure and optical features of double-layer hexagonal BiTeCl crystal have been investigated by linearized augmented plane wave method using the density functional theory under the generalized gradient and the local density approximations in this study. Ground state properties of new-type ferroelectric BiTeCl structure were studied. Calculated lattice parameters, ground state properties and experimental results are consistent. Linear dielectric functions and related properties were calculated. The frequency dependent phonon spectrum of hexagonal BiTeCl crystal was calculated using the harmonic approximation model.

**Keywords:** Structural properties, dynamic properties, optic properties.

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STRUCTURAL, ELASTIC AND THERMODYNAMIC PROPERTIES OF HEXAGONAL  
BiTeBr CRYSTAL

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## Abstract

The geometric structural optimization, elastic properties and thermodynamic functions of total energy (E) Helmholtz free energy (F), constant volume heat capacity (Cv) and entropy (S) have been investigated by linearized augmented plane wave method using the density functional theory under the generalized gradient and local density approximations for hexagonal BiTeBr crystal in this study. Ground state properties of topographic ferroelectric BiTeBr structure were studied. Calculated lattice parameters, ground state properties and experimental results are consistent with literature. We could not find any experimental data to compare our calculated results for thermodynamic and elastic properties.

**Keywords:** Structural properties, thermodynamic properties, elastic properties.

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STRUCTURAL, ELASTIC AND ELECTRONIC PROPERTIES OF BiTeI CRYSTAL

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## Abstract

The geometric structural optimization, elastic constants and related properties, electronic density of states and energy band structure of hexagonal BiTeI crystal have been investigated by linearized augmented plane wave method using the density functional theory under the generalized gradient and local density approximations in this study. Calculated lattice parameters, ground state properties and experimental results are consistent. Elastic constants and related properties were calculated. No experimental data we could find in literature to be able to compare elastic properties.

**Keywords:** Structural properties, elastic properties, electronic properties.

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CONDITIONS FOR CONVERGENCE AND SUBSEQUENTIAL CONVERGENCE OF  
REGULARLY GENERATED SEQUENCES**Sefa Anıl SEZER**<sup>1</sup>, **İbrahim ÇANAK**<sup>2</sup><sup>1</sup> *İstanbul Medeniyet University, İstanbul, Turkey*<sup>2</sup> *Ege University, İzmir, Turkey***MSC 2000:** 40A05, 40E05.**Abstract**

Regularly generated sequences were first introduced in [1]. It is said that  $(u_n)$  is regularly generated by a sequence  $\alpha = (\alpha_n)$  in some sequence space  $\mathcal{A}$  and  $\alpha$  is called a generator of  $(u_n)$  if

$$u_n = \alpha_n + \sum_{k=1}^n \frac{\alpha_k}{k}.$$

In this work we obtain necessary conditions under which the regularly generated sequence  $(u_n)$  converges or converges subsequentially.

**Keywords:** Subsequential convergence, regularly generated sequences, slowly oscillating sequences, summability by weighted means.

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