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HENNEBERG'S ALGEBRAIC SURFACES IN MINKOWSKI 3-SPACE

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Abstract. We consider the Henneberg's algebraic zero mean curvature surfaces in three dimensional Minkowski space, and compute their classes, degrees and integral free representations. We also draw some figures of the algebraic surfaces.

1. Introduction

Classical and first non-orientable minimal surface in $\mathbb{E}^3$ were introduced by Henneberg [4, 5].

After Weierstrass' representation in [13] in three dimensional Euclidean space $\mathbb{E}^3$, Kobayashi [8] gave an analogous Weierstrass-type representation for conformal spacelike surfaces with mean curvature identically 0, called maximal surfaces, in 3-dimensional Minkowski space $\mathbb{E}^{2,1}$. Unlike the case of minimal surfaces in $\mathbb{E}^3$, maximal surfaces have singularities, generally. Details about singularities of maximal surfaces can be found in [2, 12]. We remark that Magid [9] gave a Weierstrass-type representation for timelike surfaces with mean curvature identically 0, called timelike minimal surfaces, in $\mathbb{E}^{2,1}$, see also [7].

In Section 2, we give the Henneberg minimal surface in (u, v) coordinates via Weierstrass representation in $\mathbb{E}^3$. In Section 3, we compute algebraic equation defining Henneberg surface $\mathcal{H}(u, v)$ in terms of running coordinates, and obtain class and degree of it. In Section 4, using integral free form of Weierstrass, we give an algebraic function of Henneberg surface. We review Weierstrass-type representations for spacelike maximal and timelike minimal surfaces in $\mathbb{E}^{2,1}$ in Section 5 of this paper, and give explicit parametrizations for spacelike maximal and timelike minimal Henneberg surfaces. We calculate algebraic surfaces of the spacelike maximal and timelike minimal Henneberg surfaces in terms of their coordinates in Section 6. Then give degrees and classes of the surfaces. Finally, we find some algebraic

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