Konuralp Journal of Mathematics
Volume 4 No. 1 pp. 261–274 (2016) ©KJM

SOME CHARACTERIZATIONS OF EULER SPIRALS IN E_1^3

SEMRA SARACOGLU CELİK, YUSUF YAYLI, AND ERHAN GULER

Abstract. In this study, some characterizations of Euler spirals in E_1^3 have been presented by using their main property that their curvatures are linear. Moreover, discussing some properties of Bertrand curves and helices, the relationship between these special curves in E_1^3 have been investigated with different theorems and examples. The approach we used in this paper is useful in understanding the role of Euler spirals in E_1^3 in differential geometry.

1. INTRODUCTION

In three dimensional Euchlidean space E^3, Euler spirals are well-known as the curves whose curvatures evolves linearly along the curve. It is also called Clothoid or Cornu spiral whose curvature is equal to its arclength.

The equations of Euler spirals were written by Bernoulli first, in 1694. He didn't compute these curves numerically. In 1744, Euler rediscovered the curve's equations, described their properties, and derived a series expansion to the curve's integrals. Later, in 1781, he also computed the spiral's end points. The curves were re-discovered in 1890 for the third time by Talbot, who used them to design railway tracks [1].

A new type of Euler spirals in E^2 and in E^3 are given in [1] with their properties. They prove that their curve satisfies properties that characterize fair and appealing curves and reduces to the 2D Euler spiral in the planar case. Furthermore, they require that their curve conforms with the definition of a 2D Euler spiral. Smiliarly, these curves are presented in [6] as the ratio of two rational linear functions and have been defined in E^3 as generalized Euler spirals with some various characterizations. On the other hand, linear relation between principal curvatures of spacelike surfaces in Minkowski space is studied in [3].

In this paper, we present the timelike and spacelike Euler spirals in Minkowski space E_1^3. At first, we give the basic concepts and theorems about the study then we deal with these spirals whose curvatures and torsion are linear. Here, we seek

Date: January 1, 2016 and, in revised form, February 2, 2016.
2000 *Mathematics Subject Classification.* 53A04, 53A05, 53B30 .
Key words and phrases. Curvature, Cornu spiral, Bertrand curve pair.
The author is supported by ...