

High School Students' Use of Diagrams in Geometry Proofs

Ruveyda Karaman and Dr. Samuel Otten
 Mathematics Education in Learning, Teaching, and Curriculum
 University of Missouri

Introduction

Background

- Geometry is a subject in which high school students are expected to develop their skills on proving geometric theorems such as claims about the congruency and similarity of triangles (Common Core State Standards Initiative, 2010).
- Diagrams have usually played a significant role to either explore a mathematical argument or use as an analytical tool to solve the geometric problems (Shin et al., 2001).
- Understanding the meaning making with geometric diagrams has become prevalent in mathematics education research recently (e.g. Dimmel & Herbst, 2015; Herbst et al., 2016).

Objective

- Diagrams help reason and do proofs in geometry (Fischbein, 1993), but they are also open to different interpretations (Chazan, 1993).
- Many research efforts have focused on how diagrams convey the meaning but understanding what meanings students draw from the diagrams needs further research.

Research Questions

1. What semiotic resources (in diagrams) do high school students use to prove geometric claims? How do the semiotic resources relate to the quality of reasoning students provide?
2. How do high school students interpret and use geometric diagrams to prove diagram-given geometric claims, and what is the semiotic structure of their proving process?
3. How do high school students produce and use (if at all) diagrams to prove diagram-free geometric claims, and what is the semiotic structure of their proving process (whether or not they produced a diagram)?



Materials & Methods

Participants and Design

- Task-based interviews were used to answer the research questions.
- 9 (5 male, 4 female) high school students whose grade level vary 10-12 participated the study.
- Valid and invalid arguments were welcomed.
- 4 (2 male, 2 female) of the participants invited for the second interview based on their responses to the first interview.

Data Collection and Tasks

- Video-records, verbatim transcripts, written work of students.
- Tasks aligned with CCSS and had varying features.

Interview 1 Task Features					
Participants	Task	Geometric Object	Truth Known	Diagrammatic Register	Accompanying Diagram
5 Participants	1	Isosceles Triangle	Yes	Yes	Not Given
	2	Triangle Midpoints	Yes	Yes	Given/Accurate
	3	Right Triangle	No	Yes	Given/Inaccurate
	4	Right Triangle	Yes	Yes	Given/Accurate
4 Participants	1	Triangle Midpoints	Yes	No	Not Given
	2	Isosceles Triangle	Yes	Yes	Given/Accurate
	3	Right Triangle	No	Yes	Given/Inaccurate
	4	Right Triangle	Yes	Yes	Given/Accurate

Interview 2 Task Features					
Participants	Task	Geometric Object	Truth Known	Diagrammatic Register	Accompanying Diagram
4 Participants	1	Triangle Midpoints	Yes	No	Not Given
	2	Right Triangle	Yes	Yes	Given/Inaccurate
	3	Bisector Isosceles Triangle	Yes	No	Given/Accurate
	4	Pentagon Angles	No	No	Not Given

Sample Tasks

Isosceles Task

Diagram-given

Given: $\triangle ABC$ is isosceles with $AB = AC$.
 Prove: $\angle B = \angle C$.

Prover Task

Given a triangle, the midpoint of any side connects two segments by joining with the other two vertices to the other two sides of the triangle and base, we can draw lines to the center of the triangle. Prove that the perimeter of the inner triangle is half the perimeter of the outer triangle.

Claim: The segment joining the midpoint of two sides of a triangle is parallel to the third side and half its length.

Right Triangle Task

If D is the midpoint of \overline{AB} and $\angle C = 90^\circ$,
 It is claimed that $\triangle CDA$ is a right angle.

The claim is:
 Circle one: Always True Sometimes True Never True

Prove:

Data Analysis and Results

- The data analysis includes 3 phases.
- Phase 1: Coding the symbolic, visual, and gestural semiotic resources in the proof activity.

Codes for types of semiotic resources	Visual	Symbolic	Gestural
	VF-Draw a new figure/diagram VLD-Trying an exact drawing	SF-Label Figure SS-Label side	GF-Pointing at figure/diagram GS-Pointing at side/segment
VR-Redraw figure	SP-Label point	GL-Pointing at line	
VS-Draw side/segment	SV-Label vertex	GV-Pointing at ray	
VL-Draw line	SA-Label angle	GP-Pointing at point	
VY-Draw ray	SRV-Relabel vertex	GA-Pointing at angle	
VP-Draw point	SRA-Relabel angle	GH-Pointing at hatch/tick marks	
VA-Draw angle marking	SRS-Relabel side	GV-Pointing at vertex	
VH-Draw hatch/tick marks	SG-Use geometric symbols	GC-Pointing at a calculation in the work	
VSC-Draw hatch/tick marks to show side or segment congruency	SAS-Writing algebraic symbols	GM-Referring to movement	
VAC-Draw angle congruency	SE-Writing an equation	GT-Turning the paper	

- Phase 2: The level of reasoning in proving

Proving Actions			
CLAIM	INVEST	REFINE	ACCEP
Stating the overall claim to be proved or disproved	Investigating or guessing the truth value of a problem	Making a refinement or modification of a claim	Stating an accepted definition or previous result
STRUC	END	JUST	
Identifying the structure of the argument or proof	Stating the end of the proof	Justifying a step in the argument	
SUM	STEP		
Summarizing all or part of the argument	Providing a step in the argument		
ABOUT			
Making a general statement about a proof			
Proving Interactions			
CLAR	CRIT	CONF	
Clarifying claim or arguments	Critiquing claims or arguments	Confirming the validity of arguments	

Preliminary Results

- Sample student work on diagram-given/inaccurate task.

2. Right Triangle Task

$\triangle ABC$ is equilateral and D is the midpoint of BC. Prove that $\triangle ABC$ is a right triangle.

Because D is the midpoint of BC, it creates a 90° angle between $\angle BAC$. Causing $\triangle ABC$ to be a right triangle. Line AD is a line segment splitting angles $\angle DAB$ and $\angle DAC$ into two 45° angles, meaning $\triangle CAB$ is a right triangle.

- Sample student work on a truth-unknown task.

3. Right Triangle Task

If D is the midpoint of \overline{AB} and $\angle C = 90^\circ$,
 It is claimed that $\triangle CDA$ is a right angle.

The claim is:
 Circle one: Always True Sometimes True Never True

Prove:

If $AB \cong BC$ and B is the midpoint of \overline{AC} , then $\triangle ABC$ is isosceles. Since \overline{BB} is a segment that is a side to both triangles, both triangles $\triangle ABD$ and $\triangle BDC$ are isosceles triangles because they have two congruent sides. Thus, will always be isosceles triangles with the congruent sides being the same length as the other triangles congruent sides. The isosceles however will not always line up to create a right angle.

- Sample student work on a non-diagrammatic register task.

3. Bisector Ray Task

Given a triangle, if an angle bisector ray emanating from one vertex meets the opposite side at a right angle, then the overall triangle is isosceles.

References

- Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*.
- Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics*, 24(4), 359-387.
- Dimmel, J. K., & Herbst, P. G. (2015). The Semiotic Structure of Geometry Diagrams: How Textbook Diagrams Convey Meaning. *Journal for Research in Mathematics Education*, 46(2), 147-195.
- Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics*, 24(2), 139-162.
- Herbst, P., Dimmel, J., & Erickson, A. (2016, April). *High school mathematics teachers' recognition of the diagrammatic register in proof problems*. Paper presented at the American Educational Research Association, Washington, DC.
- Shin, S. J., Lemon, O., & Mumma, J. (2001). Diagrams. Retrieved from <http://olito.stanford.edu/entries/diagrams/>.

Contact Information

Samuel Otten, otens@missouri.edu
 Ruveyda Karaman, rk577@mail.missouri.edu

