High School Students' Use of Diagrams in Geometry Proofs

Ruveyda Karaman and Dr. Samuel Otten
Mathematics Education in Learning, Teaching, and Curriculum
University of Missouri

Introduction

Background

- Geometry is a subject in which high school students are expected to develop their skills on proving geometric theorems such as claims about the congruency and similarity of triangles (Common Core State Standards Initiative, 2010).
- Diagrams have usually played a significant role to either explore a mathematical argument or use as an analytical tool to solve the geometric problems (Shin et al., 2001).
- Understanding the meaning making with geometric diagrams has become prevalent in mathematics education research recently (e.g. Dimmel &Herbst, 2015; Herbst et al., 2016).

Objective

- Diagrams help reason and do proofs in geometry (Fischbein, 1993), but they are also open to different interpretations (Chazan, 1993).
- Many research efforts have focused on how diagrams convey the meaning but understanding what meanings students draw from the diagrams needs further research.

Research Questions

1. What semiotic resources (in diagrams) do high school students use to prove geometric claims? How do the semiotic resources relate to the quality of reasoning students provide?
2. How do high school students interpret and use geometric diagrams to prove diagram-given geometric claims, and what is the semiotic structure of their proving process?
3. How do high school students produce and use (if at all) diagrams to prove diagram-free geometric claims, and what is the semiotic structure of their proving process (whether or not they produced a diagram)?

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Materials & Methods

Participants and Design

- Task-based interviews were used to answer the research questions.
- 9 (5 male, 4 female) high school students whose grade level vary 10-12 participated the study.
- Valid and invalid arguments were welcomed.
- 4 (2 male, 2 female) of the participants invited for the second interview based on their responses to the first interview.

Data Collection and Tasks

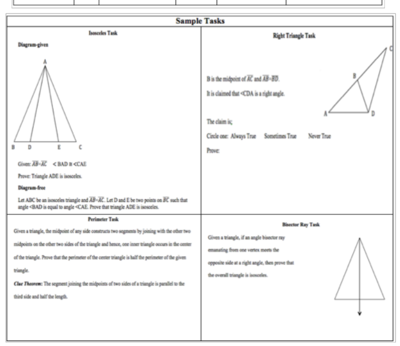
- Video-records, verbatim transcripts, written work of students.
- Tasks aligned with CCSSI and had varying features.

Interview 1 Task Features

	Task	Geometric Object	Truth Known	Diagrammatic Register	Accompanying Diagram
5 Participants	1	Isosceles Triangle	Yes	Yes	Not Given
	2	Triangle Midpoints	Yes	Yes	Given/Accurate
	3	Right Triangle	No	Yes	Given/Inaccurate
	4	Right Triangle	Yes	Yes	Given/Accurate
4 Participants	1	Triangle Midpoints	Yes	No	Not Given
	2	Isosceles Triangle	Yes	Yes	Given/Accurate
	3	Right Triangle	No	Yes	Given/Inaccurate
	4	Right Triangle	Yes	Yes	Given/ Accurate

Interview 2 Task Features

4 Participants	Task	Geometric Object	Truth Known	Diagrammatic Register	Accompanying Diagram
	1	Triangle Midpoints	Yes	No	Not Given
	2	Right Triangle	Yes	Yes	Given/Inaccurate
	3	Bisector Isosceles Triangle	Yes	No	Given/Accurate
	4	Pentagon Angles	No	No	Not Given



Data Analysis and Results

- The data analysis includes 3 phases.
- Phase 1: Coding the symbolic, visual, and gestural semiotic resources in the proof activity.

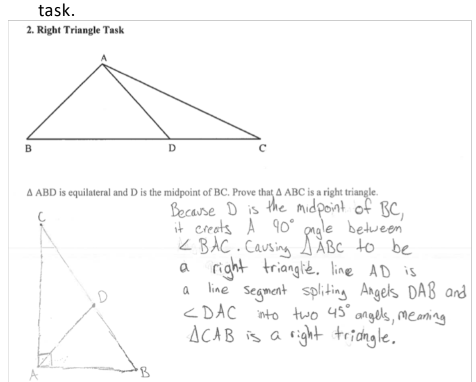
	Visual	Symbolic	Gesture
Codes for types of semiotic resources	VF–Draw a new figure/diagram	SF–Label Figure	GF–Pointing at figure/diagram
	VED–Trying an exact drawing	SS–Label side	GS–Pointing at side/segment
	VR–Redraw figure	SP–Label point	GL–Pointing at line
	VS–Draw side/segment	SV–Label vertex	GY-Pointing at ray
	VL–Draw line	SA–Label angle	GP–Pointing at point
	VY–Draw ray	SRV–Relabel vertex	GA–Pointing at angle
	VP–Draw point	SRA–Relabel angle	GH–Pointing at hatch/tick marks
	VA–Draw angle marking	SRS–Relabel side	GV–Pointing at vertex
	VH–Draw hatch/tick marks	SG–Use geometric symbols	GC–Pointing at a calculation in the work
	VSC–Draw hatch/tick marks to show side or segment congruency	SAS–Writing algebraic symbols	GM–Referring to movement
	VAC–Draw angle congruency	SE–Writing an equation	GT–Turning the paper

- Phase 2: The level of reasoning in proving

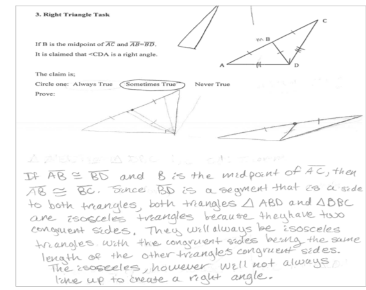
Proving Actions					
CLAIM	Stating the overall claim to be proved or disproved	INVEST	Investigating or guessing the truth value of a problem	REFINE	Making a refinement or modification of a claim
STRUC	Identifying the structure of the argument or proof	END	Stating the end of the proof	ACCEP	Stating an accepted definition or previous result
SUM	Summarizing all or part of the argument	STEP	Providing a step in the argument	JUST	Justifying a step in the argument
ABOUT	Making a general statement about a proof				
Proving Interactions					
CLAR	Clarifying claim or arguments	CRIT	Critiquing claims or arguments	CONF	Confirming the validity of arguments

Preliminary Results

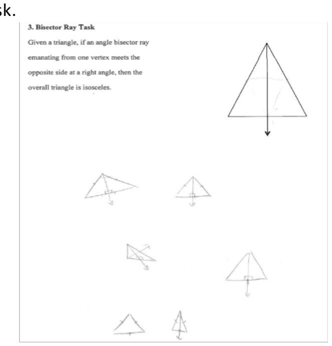
- Sample student work on diagram-given/inaccurate task.



- Sample student work on a truth-unknown task.



- Sample student work on a non-diagrammatic register task.



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Contact Information

Samuel Otten, ottensa@missouri.edu
Ruveyda Karaman, rk5f7@mail.missouri.edu