Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 852796, 6 pages
http://dx.doi.org/10.1155/2013/852796

Research Article

New Convergence Definitions for Sequences of Sets

Ömer Kişi[1] and Fatih Nuray[1,2]

[1] Faculty of Education, Mathematics Education Department, Cumhuriyet University, Sivas, Turkey
[2] Department of Mathematics, Faculty of Science and Literature, Afyon Kocatepe University, Afyonkarahisar, Turkey

Correspondence should be addressed to Fatih Nuray; fnuray@aku.edu.tr

Received 14 May 2013; Accepted 26 September 2013

Academic Editor: Svatoslav Staněk

Several notions of *convergence* for subsets of metric space appear in the literature. In this paper, we define *Wijsman I-convergence* and *Wijsman I^*-convergence* for sequences of sets and establish some basic theorems. Furthermore, we introduce the concepts of *Wijsman I-Cauchy* sequence and *Wijsman I^*-Cauchy* sequence and then study their certain properties.

1. Introduction and Background

The concept of convergence of sequences of points has been extended by several authors (see [1–9]) to the concept of convergence of sequences of sets. The one of these such extensions that we will consider in this paper is Wijsman convergence. We will define I-convergence for sequences of sets and establish some basic results regarding these notions.

Let us start with fundamental definitions from the literature. The natural density of a set K of positive integers is defined by

$$\delta(K) := \lim_{n\to\infty}\frac{1}{n}\left|\{k \le n : k \in K\}\right|, \tag{1}$$

where $|k \le n : k \in K|$ denotes the number of elements of K not exceeding n ([10]).

Statistical convergence of sequences of points was introduced by Fast [11]. In [12], Schoenberg established some basic properties of statistical convergence and also studied the concept as a summability method.

A number sequence $x = (x_k)$ is said to be statistically convergent to the number ξ if, for every $\varepsilon > 0$,

$$\lim_{n\to\infty}\frac{1}{n}\left|\{k \le n : |x_k - \xi| \ge \varepsilon\}\right| = 0. \tag{2}$$

In this case, we write $st-\lim x_k = \xi$. Statistical convergence is a natural generalization of ordinary convergence. If $\lim x_k = \xi$, then $st-\lim x_k = \xi$. The converse does not hold in general.

Definition 1 (see [13]). A family of sets $I \subseteq 2^{\mathbb{N}}$ is called an ideal on $\mathbb{N}$ if and only if

(i) $\emptyset \in I$;

(ii) for each $A, B \in I$ one has $A \cup B \in I$;

(iii) for each $A \in I$ and each $B \subseteq A$ one has $B \in I$.

An ideal is called nontrivial if $\mathbb{N} \notin I$, and nontrivial ideal is called admissible if $\{n\} \in I$ for each $n \in \mathbb{N}$.

Definition 2 (see [14]). A family of sets $F \subseteq 2^{\mathbb{N}}$ is a filter in $\mathbb{N}$ if and only if

(i) $\emptyset \notin F$;

(ii) for each $A, B \in F$ one has $A \cap B \in F$;

(iii) for each $A \in F$ and each $B \supseteq A$ one has $B \in F$.

Proposition 3 (see [14]). *I is a nontrivial ideal in $\mathbb{N}$ if and only if*

$$F = F(I) = \{M = \mathbb{N} \setminus A : A \in I\} \tag{3}$$

is a filter in $\mathbb{N}$.

Definition 4 (see [14]). Let I be a nontrivial ideal of subsets of $\mathbb{N}$. A number sequence $(x_n)_{n\in\mathbb{N}}$ is said to be I-convergent to ξ ($\xi = I - \lim_{n\to\infty} x_n$) if and only if for each $\varepsilon > 0$ the set

$$\{k \in \mathbb{N} : |x_k - \xi| \ge \varepsilon\} \tag{4}$$

belongs to I. The element ξ is called the I limit of the number sequence $x = (x_n)_{n\in\mathbb{N}}$.