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Cheng–Yau Operator and Gauss Map of Rotational Hypersurfaces in 4-Space

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Abstract. We consider rotational hypersurface in the four-dimensional Euclidean space $\mathbb{E}^4$. We study the Gauss map $\mathbf{G}$ of rotational hypersurface in $\mathbb{E}^4$ with respect to the so-called Cheng–Yau operator L_1 acting on the functions defined on the hypersurfaces. We obtain the classification theorem that the only rotational hypersurface with Gauss map $\mathbf{G}$ satisfying $L_1\mathbf{G} = \mathbf{AG}$ for some 4×4 matrix $\mathbf{A}$ are the hyperplanes, right circular hypercones, circular hypercylinders, and hyperspheres.

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Keywords. Euclidean spaces, Cheng–Yau operator, finite type mappings, rotational hypersurfaces, L_k-operators.

1. Introduction

The notion of finite type immersion of submanifolds of a Euclidean space has been used in classifying and characterizing well-known Riemannian submanifolds [5]. Chen posed the problem of classifying the finite type surfaces in the three-dimensional Euclidean space $\mathbb{E}^3$. A Euclidean submanifold is said to be of Chen finite type if its coordinate functions are a finite sum of eigenfunctions of its Laplacian Δ [5]. Further, the notion of finite type can be extended to any smooth functions on a submanifold of a Euclidean space or a pseudo-Euclidean space. Then the theory of submanifolds of finite type has been studied by many geometers.

Takahashi [27] stated that minimal surfaces and spheres are the only surfaces in $\mathbb{E}^3$ satisfying the condition $\Delta r = \lambda r$, $\lambda \in \mathbb{R}$. Ferrandez et al. [12] proved that the surfaces of $\mathbb{E}^3$ satisfying $\Delta H = AH$, $A \in Mat(3, 3)$ were either minimal, or an open piece of sphere or of a right circular cylinder. Choi and Kim [8] characterized the minimal helicoid in terms of pointwise 1-type Gauss map of the first kind.

Dillen et al. [9] proved that the only surfaces in $\mathbb{E}^3$ satisfying $\Delta r = Ar + B$, $A \in Mat(3, 3)$, $B \in Mat(3, 1)$ are the minimal surfaces, the spheres

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