Computers and Structures 161 (2015) 55–63

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Quasi-harmonic Bézier approximation of minimal surfaces for finding forms of structural membranes

Gang Xu [a,*], Timon Rabczuk [b], Erhan Güler [c], Qing Wu [a], Kin-chuen Hui [d], Guozhao Wang [e]

[a] Hangzhou Dianzi University, 310018 Hangzhou, PR China
[b] Institute of Structural Mechanics, Bauhaus-University Weimar, Marienstr. 15, D-99423 Weimar, Germany
[c] Department of Mathematics, Bartin University, 74100 Bartin, Turkey
[d] Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong
[e] Department of Mathematics, Zhejiang University, 310027 Hangzhou, PR China

ARTICLE INFO

Article history:
Received 3 February 2015
Accepted 11 September 2015
Available online 2 October 2015

Keywords:
Form finding
Minimal surfaces
Bézier approximation
Plateau–Bézier problem
Quasi-harmonic method
Multi-patch structure

ABSTRACT

Numerical approximation of minimal surface is an important problem in form-finding of structural membranes. In this paper, we present a novel approach to construct minimal surface from a given boundary by quasi-harmonic Bézier approximation. A new energy functional called *quasi-harmonic energy functional* is proposed as the objective function to obtain the *quasi-harmonic Bézier surface* from given boundaries. The quasi-harmonic mask is also proposed to generate approximate minimal surfaces by solving a sparse linear system. We propose a framework to construct multi-patch quasi-harmonic Bézier approximation from *N*-sided boundary curves. The efficiency of the proposed methods is illustrated by several modeling examples.

1. Introduction

Minimal surfaces are important in many engineering and physics applications, such as object segmentation [2,7] and mechanics of cellular materials [8]. Of particular interest is the similarity between the minimal surface problem and form-finding problems which are used for the design of membrane structures [4–6,13,29]. Determining the minimal surface for a pre-defined boundary leads to surfaces that are in a self-equilibrium. They are like soap films in a uniform state of isotropic stresses and do not play only a major role in fluid mechanics but are also of importance for the design of structural membranes, which are commonly used to provide covers for outdoor areas.

A minimal surface is a surface with vanishing mean curvature. As the mean curvature is the variation of the area functional, minimal surfaces include the surfaces minimizing the area with a fixed boundary. The Plateau problem is to find a surface of minimum area that spans a given curve. This problem is named after J. Plateau, who performed an extensive series of experiments using metal wire to represent curves and soap films to model minimal spanning surfaces. As we know, only a few minimal surfaces have been found in closed form. Hence, numerical methods have been devised to construct approximated minimal surface. Brakke proposed an approach to compute a parametric minimal surface with the finite element method [3]. Direct simulation of surface tension forces on a grid of marker particles is used for the minimal surface approximation in [17,10]. Chopp proposed a level set method to handle topological changes with linear convergence [9]. Jung et al. proposed a variational level set approach for the surface area minimization of triply-periodic surfaces [18]. They showed that the Schwartz P, Schwartz D, and Schoen G minimal surfaces are local minima of the total surface area under a volume fraction constraint. Tråsdahl and Rønquist presented an algorithm for finding high order numerical approximations of minimal surfaces with a fixed boundary [28]. Their algorithm employs parametrization with high-order polynomials and an iterative approach is used to achieve the final minimal surface from a given initial surface. Koohestani proposed an approach for the form-finding of the minimal surface membranes using discrete models and nonlinear force density method [20].

If the given boundary is a polynomial curve, in most cases, the solution of the Plateau problem is not a polynomial surface. In order to find an approximation Bézier solution of the Plateau problem, J. Monterde proposed the Plateau–Bézier problem, which is to find the surface of minimal area from among all Bézier

* Corresponding author.
E-mail addresses: xugangzju@gmail.com (G. Xu), timon.rabczuk@uni-weimar.de (T. Rabczuk), eguler@bartin.edu.tr (E. Güler), kchui@mae.cuhk.edu.hk (K.-c. Hui), wanggz@zju.edu.cn (G. Wang).

http://dx.doi.org/10.1016/j.compstruc.2015.09.002
0045-7949/