J. Math. Kyoto Univ. (JMKYAZ)
46-1 (2006), 47–63

Bour's theorem in Minkowski 3-space

By

Erhan GÜLER and Aysel Turgut VANLI

Abstract

In this study, we show that a generalized helicoid with null axis is isometric to a rotation surface with null axis so that helices on the helicoid correspond to parallel circles on the rotation surface in three dimensional Minkowski space. Moreover, we obtained that these surfaces are minimal. An addition, if these surfaces have the same Gauss map, we can determine them.

1. Introduction

In classical surface geometry in Euclidean space, it is well known that the right helicoid (resp. catenoid) is the only ruled (resp. rotation) surface which is minimal. Moreover, a pair of these two surfaces has interesting properties. That is, they are both members of a one-parameter family of isometric minimal surfaces and have the same Gauss map. This pair is a typical example for minimal surfaces. On the other hand, the pair of the right helicoid and the catenoid has following generalization.

Bour's Theorem. *A generalized helicoid is isometric to a rotation surface so that helices on the helicoid correspond to parallel circles on the rotation surface* [1], [6].

In this generalization, original properties that they are minimal and preserve the Gauss map are not generally kept.

In [4], T. Ikawa showed that a generalized helicoid and a rotation surface have isometric relation by Bour's theorem in Euclidean 3-space. He determined pairs of surfaces with an additional conditional that they have the same Gauss map on Bour's theorem. About helicoidal surfaces in Euclidean 3-space, M. P. do Carmo and M. Dajczer [2] proved that, by using a result of E. Bour [1], there exists a two-parameter family of helicoidal surfaces isometric to a given helicoidal surface. By making use of this parametrization, they found a representation formula for helicoidal surfaces with constant mean curvature. Furthermore they proved that the associated family of Delaunay surfaces is made up by helicoidal surfaces of constant mean curvature.

Received February 4, 2005

References

[1] E. Bour, *Memoire sur le deformation de surfaces*, Journal de l'Êcole Polytechnique, XXXIX Cahier (1862), 1–148.

[2] M. P. Do Carmo and M. Dajczer, *Helicoidal surfaces with constant mean curvature*, Tohoku Math. J. **34** (1982), 425–435.

[3] J. Hano and K. Nomizu, *Surfaces of revolution with constant mean curvature*, Tohoku Math. J. **36** (1984), 427–437.

[4] T. Ikawa, *Bour's theorem and Gauss map*, Yokohama Math. J. **48** (2000), 173–180.

[5] ———, *Bour's theorem in Minkowski geometry*, Tokyo J. Math. **24** (2001), 377–394.

[6] T. Otsuki, *Differential geometry*, Asakura Publishing, 1961.

[7] N. Sasahara, *Spacelike helicoidal surfaces with constant mean curvature in Minkowski* 3*-space*, Tokyo J. Math. **23** (2000), 477–502.