# Ulisse Dini-type Helicoidal Surface in 3-Space

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## ABSTRACT

In this talk, we study Ulisse Dini-type helicoidal surface in Euclidean 3-space $E^3$. We give some basic notions of the three dimensional Euclidean geometry in section 2. In section 3, we consider Ulisse Dini helicoidal surface. We obtain Ulisse Dini-type helicoidal surface, and calculate its curvatures in the last section.

We calculate the first and second fundamental forms, matrix of the shape operator S, Gaussian curvature K, and the mean curvature H of surface M=M(u,v) in Euclidean 3-space $E^3$.

We define the rotational surface and helicoidal surface in $E^3$. For an open interval $I\subset \mathbb{R}$, let $\gamma: I\to\Pi$ be a curve in a plane $\Pi$ in $E^3$, and let $\ell$ be a straight line in $\Pi$. A rotational surface in $E^3$ is defined as a surface rotating a curve $\gamma$ around a line $\ell$ (these are called the profile curve and the axis, respectively). Suppose that when a profile curve $\gamma$ rotates around the axis $\ell$, it simultaneously displaces parallel lines orthogonal to the axis $\ell$, so that the speed of displacement is proportional to the speed of rotation. Then the resulting surface is called the helicoidal surface with axis $\ell$ and pitch $a\in \mathbb{R}\backslash\{0\}$. We may suppose that $\ell$ is the line spanned by the vector $(0,0,1)^t$.

Moreover, we consider Dini-type helicoidal surface:

$$D(u,v)=\begin{pmatrix}\sin u\cos v\\ \sin u\sin v\\ \varphi(u)+av\end{pmatrix},$$

where $\varphi(u): I\subset\mathbb{R}\to\mathbb{R}$ is a differentiable function for all $u\in I\subset\mathbb{R}\backslash\{0\}$, $0\le v\le 2\pi$ and $a\in\mathbb{R}\backslash\{0\}$.

**Key Words:** Dini-type helicoidal surface, Gauss map, Gaussian curvature, mean curvature.

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