Rough $I_2$-lacunary statistical convergence of double sequences

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Abstract
In this paper, we introduce and study the notion of rough $I_2$-lacunary statistical convergence of double sequences in normed linear spaces. We also introduce the notion of rough $I_2$-lacunary statistical limit set of a double sequence and discuss some properties of this set.

Keywords: Statistical convergence; $I$-convergence; Rough convergence; Lacunary sequences; Double sequences

1 Introduction
Throughout the paper, $\mathbb{N}$ and $\mathbb{R}$ denote the set of all positive integers and the set of all real numbers, respectively. The concept of convergence of a sequence of real numbers has been extended to statistical convergence independently by Fast [1] and Schoenberg [2]. This concept was extended to the double sequences by Mursaleen and Edely [3]. Lacunary statistical convergence was defined by Fridy and Orhan [4]. Çakan and Altay [5] presented multidimensional analogues of the results presented by Fridy and Orhan [4].

The idea of $I$-convergence was introduced by Kostyrko et al. [6] as a generalization of statistical convergence which is based on the structure of the ideal $I$ of subset of the set of natural numbers. Kostyrko et al. [7] studied the idea of $I$-convergence and extremal $I$-limit points. Das et al. [8, 9] introduced the concept of $I$-convergence of double sequences in a metric space and studied some properties of this convergence. A lot of development have been made in area about statistical convergence, $I$-convergence and double sequences after the work of [1, 2, 10–28].

The notion of lacunary ideal convergence of real sequences was introduced in [29]. Das et al. [30, 31] introduced new notions, namely $I$-statistical convergence and $I$-lacunary statistical convergence by using ideal. Belen et al. [32] introduced the notion of ideal statistical convergence of double sequences, which is a new generalization of the notions of statistical convergence and usual convergence. Kumar et al. [33] introduced $I$-lacunary statistical convergence of double sequences. Further investigation and applications on this notion can be found in [34].

The idea of rough convergence was first introduced by Phu [35] in finite-dimensional normed spaces. In another paper [36] related to this subject, Phu defined the rough continuity of linear operators and showed that every linear operator $f : X \to Y$ is $r$-continuous at every point $x \in X$ under the assumption $\dim Y < \infty$ and $r > 0$, where $X$ and $Y$ are normed