On $\mathcal{J}_G$-convergence of folner sequence on amenable semigroups

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Abstract: In this paper, the concepts of $\sigma$-uniform density of subsets $A$ of the set $\mathbb{N}$ of positive integers and corresponding $\mathcal{J}_\sigma$-convergence of functions defined on discrete countable amenable semigroups were introduced. Furthermore, for any Folner sequence inclusion relations between $\mathcal{J}_\sigma$-convergence and invariant convergence also $\mathcal{J}_\sigma$-convergence and $\mathcal{V}_\lambda$-convergence were given. We introduce the concept of $\mathcal{J}_\sigma$-statistical convergence and $\mathcal{J}_\sigma$-lacunary statistical convergence of functions defined on discrete countable amenable semigroups. In addition to these definitions, we give some inclusion theorems. Also, we make a new approach to the notions of $[\mathcal{V}, \lambda]$-summability, $\sigma$-convergence and $\lambda$-statistical convergence of Folner sequences by using ideals and introduce new notions, namely, $\mathcal{J}_\sigma-[\mathcal{V}, \lambda]$-summability, $\mathcal{J}_\sigma-[\lambda]$-statistical convergence of Folner sequences. We mainly examine the relation between these two methods as also the relation between $\mathcal{J}_\sigma$-statistical convergence and $\mathcal{J}_\sigma-[\lambda]$-statistical convergence of Folner sequences introduced by the author recently.

Keywords: Folner sequence, amenable group, inferior, superior, $\mathcal{J}$-convergence.

1 Introduction

Statistical convergence of sequences of points was introduced by Fast [5]. Schoenberg [27] established some basic properties of statistical convergence and also studied the concept as a summability method.

The natural density of a set $K$ of positive integers is defined by

$$\delta(K) := \lim_{n \to \infty} \frac{1}{n} \{k \leq n : k \in K \},$$

where $|k \leq n : k \in K|$ denotes the number of elements of $K$ not exceeding $n$.

A number sequence $x = (x_k)$ is said to be statistically convergent to the number $L$ if for every $\varepsilon > 0$,

$$\lim_{n \to \infty} \frac{1}{n} \{k \leq n : |x_k - L| \geq \varepsilon \} = 0.$$ 

In this case we write $st \lim x_k = L$. Statistical convergence is a natural generalization of ordinary convergence. If $\lim x_k = L$, then $st \lim x_k = L$. The converse does not hold in general.

By a lacunary sequence we mean an increasing integer sequence $\theta = \{k_r\}$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. Throughout this paper the intervals determined by $\theta$ will be denoted by $I_r = (k_{r-1}, k_r)$.

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