ASTROHELICOIDAL HYPERSURFACES IN 4-SPACE

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ABSTRACT. We consider an astrohelicoidal hypersurface which its profile curve has astroid curve in the four dimensional Euclidean space $\mathbb{E}^4$. We also calculate Gaussian curvature and the mean curvature, and Weingarten relation of the hypersurface. Moreover, projecting hypersurface into 3-spaces, we draw some figures.

1. Introduction

About (hyper)surfaces in the literature, we see some following papers: Arslan et al. [1], Ganchev and Milousheva [4], Güler et al. [6, 7], Güler and Turgay [8], and also some books: Eisenhart [2], Hacisalihoglu [9], Nitsche [10], etc..

In this paper, we consider the astrohelicoidal hypersurface in Euclidean 4-space $\mathbb{E}^4$. We give some fundamental notions of three dimensional Euclidean geometry in section 2. In section 3, we define helicoidal hypersurface. We obtain astrohelicoidal hypersurface, and calculate its curvatures in the last section.

2. Preliminaries

In the rest of this work, we shall identify a vector $(a,b,c,d)$ with its transpose $(a,b,c,d)^t$. We will introduce the first and second fundamental forms, matrix of the shape operator $S$, Gaussian curvature $K$, and the mean curvature $H$ of hypersurface $M = M(u,v,w)$ in Euclidean $4$-space $\mathbb{E}^4$. Let $M$ be an isometric immersion of hypersurface $M^3$ in $\mathbb{E}^4$.

The inner product of vectors $\vec{x} = (x_1, x_2, x_3, x_4)$ and $\vec{y} = (y_1, y_2, y_3, y_4)$ on $\mathbb{E}^4$ is defined by as follows:

$$\vec{x} \cdot \vec{y} = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4.$$ 

The vector product of vectors $\vec{x} = (x_1, x_2, x_3, x_4)$, $\vec{y} = (y_1, y_2, y_3, y_4)$ and $\vec{z} = (z_1, z_2, z_3, z_4)$ on $\mathbb{E}^4$ is defined by as follows:

$$\vec{x} \times \vec{y} \times \vec{z} = \begin{pmatrix}
x_2y_3z_4 - x_2y_4z_3 - x_3y_2z_4 + x_3y_4z_2 + x_4y_2z_3 - x_4y_3z_2 \\
-x_1y_3z_4 + x_1y_4z_3 + x_3y_1z_4 - x_3y_4z_1 - y_1x_4z_3 + y_1x_3z_4 \\
x_1y_2z_4 - x_1y_4z_2 - x_2y_1z_4 + x_2y_4z_1 + y_1x_4z_2 - y_1x_2z_4 \\
-x_1y_2z_3 + x_1y_3z_2 + x_2y_1z_3 - x_2y_3z_1 - x_3y_1z_2 + x_3y_2z_1
\end{pmatrix}.$$ 

Date: July 15, 2019.
2000 Mathematics Subject Classification. Primary 53A05; Secondary 53C42.
Key words and phrases. astrohelicoidal hypersurface, Gauss map, Gaussian curvature, mean curvature.

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This paper is in final form and no version of it will be submitted for publication elsewhere.