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A NEW LIFETIME DISTRIBUTION

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The well-known statistical distributions such as Exponential, Weibull and Gamma distributions have been commonly used for analysing the different types of lifetime data. In this paper, following the idea of the extension of T-X family of distributions, we propose a new type of exponential distribution. We define the survival function, the hazard function and the mean time to failure related to this new distribution. Type II censoring procedure is also considered for this distribution. Additionally, stress-strength reliability and the maximum likelihood (ML) estimators of the unknown parameters are obtained. As an application, a real data set is used to show that the proposed distribution gives best fit than the alternative ones.

***Keywords**: T-X family of distributions, Reliability Function, Hazard Rate, Type II Censoring, Stress-Strength Probability.*

Powszechnie znane rozkłady statystyczne, takie jak rozkład wykładniczy, Weibulla czy rozkład Gamma znajdują szerokie zastosowanie w analizie różnych typów danych dotyczących cyklu życia. W niniejszym artykule, zaproponowano nowy typ rozkładu wykładniczego, w oparciu o rozszerzenie rodziny rozkładów TX. Zdefiniowano funkcję przeżycia, funkcję ryzyka (hazardu) oraz średni czas do uszkodzenia w odniesieniu do proponowanego rozkładu. Rozpatrywano także procedurę ucinania typu II dla nowego rozkładu. Dodatkowo określono niezawodność wytrzymałościową oraz estymatory największej wiarygodności nieznanych parametrów. Sposób wykorzystania proponowanego rozkładu zilustrowano wykorzystując rzeczywisty zbiór danych. Wykazano, że daje on lepsze dopasowanie niż modele alternatywne.

***Słowa kluczowe**: rodzina rozkładów TX, funkcja niezawodności, współczynnik ryzyka, ucinanie typu II, prawdopodobieństwo wytrzymałości.*

1. Introduction

Exponential distribution has been broadly applied in the context of reliability. One reason for the popularity of the exponential distribution in reliability modeling is that it is the limiting lifetime distribution of a series system of substantially similar components [18]. The exponential distribution is also important for its "memoryless" property.

The probability density function (pdf) and the cumulative distribution function (cdf) of the exponential distribution are given by:

$$f(x) = \lambda e^{-\lambda x},\ x > 0 \tag{1}$$

$$F(x) = 1 - e^{-\lambda x},\ x > 0 \tag{2}$$

respectively, where λ is the shape parameter, which is also known as the rate parameter (failure, death, arrival or transition). The reliability function and the hazard function of the exponential distribution are given by:

$$R(x) = 1 - F(x) = e^{-\lambda x},\ \ x > 0 \tag{3}$$

$$h(x) = \frac{f(x)}{R(x)} = \lambda,\ x>0 \tag{4}$$

However, the exponential distribution does not provide a significant fitting for some real life applications, where the failure rates are not constant. In recent years, many authors have proposed different types of generalization of the exponential distribution to overcome this problem, see [1, 4, 5, 8 and 11]. Some of these proposed life-time distributions have decreasing failure rate, which are common tools in biology and engineering, while the others have increasing failure rate which have been used in risk analysis. In this paper, we propose a new generalization of exponential distribution, which includes both increasing and decreasing failure rate. This property yields a great flexibility to fit the life time data obtained by any field of subject.

In literature, there are various types of extensions of the exponential distribution. [8-9] proposed an extension of the exponential distribution, which is called the generalized exponential (GE) distribution. Following the idea of the GE distribution, lots of extension procedures for exponential distribution has been introduced, [3], [7], [14] and [15].

Recently, [2] proposed a new technique to generate continuous probability distributions. The methodology proceeds as follows; Let X be arandom variable whose pdf is $f(x)$ and cdf is $F(x)$ and let T be continuous random variable with pdf $h(x)$ defined on the interval $[a, b]$. The cdf of new family of distribution can be obtained by:

$$G(x) = \int_a^{W[F(x)]} h(y)dy \tag{5}$$

where $W[F(x)]$ is differentiable and monotonically non-decreasing on the interval $[a,b]$. It should be also noted that $W[F(x)] \to a$ as $x \to -\infty$ and $W[F(x)] \to b$ as $x \to \infty$. The corresponding pdf of X can be written as:

$$g(x)=\left\{\frac{d}{dx}W\left[F(x)\right]\right\}h\left\{W\left[F(x)\right]\right\}. \tag{6}$$

The random variable T is called "transformed" into a new cdf $G(x)$ through the function $W\left[F(x)\right]$ known as "transformer". So, $G(x)$ is called "Transformed-Transformer" or "$T-X$" distribution.

We propose a new $W\left[F(x)\right]$ described in (5) with additional parameter θ. We define $W\left[F(x)\right]$ as follows:

$$W\left[F(x)\right]=\frac{e^{-\theta F(x)}-1}{e^{-\theta}-1} \tag{7}$$

where $\theta\in\mathcal{R}$. It can be seen that $W\left[F(x)\right]\to 0$ as $x\to-\infty$ and $W\left[F(x)\right]\to 1$ as $x\to-\infty$. Therefore, in order to use such a function defined in (6), it must be used a random variable, whose pdf is defined on the interval $[0,1]$. We use uniform distribution whose pdf is $h(y)=1, 0<y<1$.

Definition: Let $X(a<X<b)$ be a random variable, whose pdf is $f(x)$ and cdf $F(x)$. Let Y be a uniform random variable defined on the interval $[0,1]$. Then:

$$G(x)=\int_0^{\frac{e^{-\theta F(x)}-1}{e^{-\theta}-1}}dy=\frac{e^{-\theta F(x)}-1}{e^{-\theta}-1} \tag{8}$$

is a cdf of new family of "$T-X$" distribution. The corresponding pdfof this new family can be defined as:

$$g(x)=\frac{\theta f(x)e^{-\theta F(x)}}{1-e^{-\theta}}, a<x<b. \tag{9}$$

In this paper, we propose a new life time distribution by using the pdf and the cdf of the exponential distribution.

2. Uniform-Exponential Distribution

Consider the exponential distribution with parameter λ and let $f(x)$ and $F(x)$ be the pdf and the cdf of exponential distribution, corresponding to the definition,

$$g(x)=\frac{\theta\lambda e^{-\lambda x}e^{-\theta\left(1-e^{-\lambda x}\right)}}{\left(1-e^{-\theta}\right)},\ x>0 \tag{10}$$

is defined as uniform-exponential (UE) distribution. The cdf of X can be written as:

$$G(x)=\frac{e^{-\theta\left(1-e^{-\lambda x}\right)}-1}{e^{-\theta}-1}, x>0. \tag{11}$$

Figure 1 shows the pdfs of the UE distributions for different θ values. It can be seen that if θ tends to 0, the pdf becomes the original distribution. Additionally, if θ>0, then the pdf becomes more positively skewed and by the same way if θ<0, then the pdf becomes more negatively skewed. From now on, we call θ as the skewness parameter, since it determines the shape (skewness) of the distribution. It should be also noted that, the location parameter μ may be added to the distribution.

The moment generating function (mgf) of the UE distribution can be found as:

$$M_X(t)=\int_0^{\infty}e^{tx}\frac{\theta\lambda e^{-\lambda x}e^{-\theta\left(1-e^{-\lambda x}\right)}}{\left(1-e^{-\theta}\right)}dx \tag{12}$$

To solve (12) firstly, take $u=-e^{-\lambda x}$, then the integral becomes

$$\frac{\theta}{\left(1-e^{-\theta}\right)}\int_{-1}^{0}e^{-\theta(1+u)}(-u)^{-\frac{t}{\lambda}}du \tag{13}$$

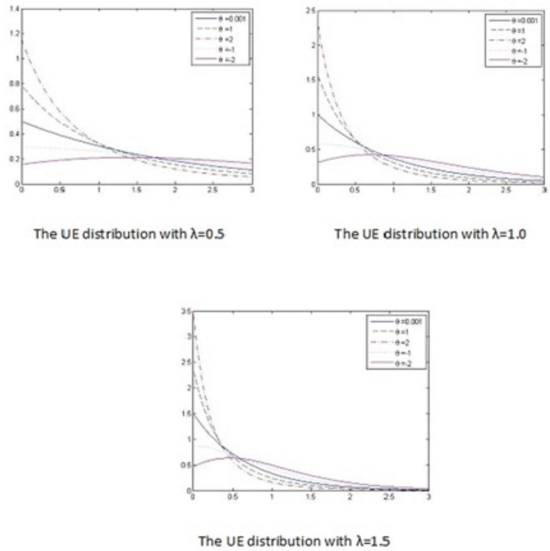

Fig. 1. The pdfs of UE distribution

By transforming $1+u=w$, (13) returns:

$$\frac{\theta}{\left(1-e^{-\theta}\right)}\int_0^1 e^{-\theta w}(1-w)^{-\frac{t}{\lambda}}dw \tag{14}$$

Using the below expansion:

$$e^{-t}=\sum_{i=0}^{\infty}\frac{(-1)^i t^i}{i!} \tag{15}$$

we get:

$$\frac{\theta}{\left(1-e^{-\theta}\right)}\sum_{i=0}^{\infty}\frac{(-1)^i\theta^i}{i!}\int_0^1 w^i(1-w)^{-\frac{t}{\lambda}}dw \tag{16}$$

The integral in (16) is a typical beta function. Therefore, the mgf of UE distribution can be obtained as:

$$M_X(t)=\frac{\theta}{\left(1-e^{-\theta}\right)}\sum_{i=0}^{\infty}\frac{(-1)^i\theta^i}{i!}B\left(i+1,-\frac{t}{\lambda}+1\right) \tag{17}$$

The expected value of UE distribution can be obtained by differentiating the mgf and taking $t=0$.

$$\frac{\theta}{\left(1-e^{-\theta}\right)}\left[\frac{1}{\lambda}-0.75\frac{\theta}{\lambda}+0.31\frac{\theta^2}{\lambda}-0.09\frac{\theta^3}{\lambda}+0.02\frac{\theta^4}{\lambda}+o(\theta,\lambda)\right]=\frac{\theta\Psi(\theta)}{\lambda\left(1-e^{-\theta}\right)} \tag{18}$$

The Ψ(θ) values are given in Table 1. It can be noticed that $\lim_{\theta\to 0}E(X)\to\frac{1}{\lambda}$ which is the expected value of the exponential distribution.

The quantile function $Q(p)$ and the median $Q(0.5)$ of the UE distribution are defined as

$$Q(p)=-\frac{1}{\lambda}\ln\left\{1+\frac{\ln\left[1+p\left(e^{-\theta}-1\right)\right]}{\theta}\right\} \tag{19}$$

Table 1. The Ψ(θ)values

θ	0	0.1	0.2	0.3	0.4	0.5	0.6
Ψ(θ)	1.000	0.9280	0.8617	0.8006	0.7443	0.6924	0.6445
θ	0.7	0.8	0.9	1.0	1.5	2.0	2.5
Ψ(θ)	0.6003	0.5594	0.5216	0.4867	0.3448	0.2344	0.1140

$$Q(0.5)=-\frac{1}{\lambda}\ln\left\{1+\frac{\ln\left[1+0.5\left(e^{-\theta}-1\right)\right]}{\theta}\right\} \tag{20}$$

respectively.

3. Reliability Analysis for UE distribution

In this section, the UE distribution is applied to the well-known reliability procedures.

3.1. Reliability Function

The reliability function means, the probability over duration, based on the time. The reliability function is also known as the survival function. The term reliability indicates the systems or devices in the engineering problems whereas survival is a term used for humans or animals in actuarial analysis. The reliability function is monotonically decreasing and right continuous.

The reliability function for UE distribution is defined by:

$$R(t)=1-F(t)=\frac{e^{-\theta}-e^{-\theta\left(1-e^{-\lambda t}\right)}}{e^{-\theta}-1}. \tag{21}$$

Figure 2 shows the reliability functions of UE distribution for some representative θ values. It should be also noted that $\lim_{\theta\to 0}R(t)\to e^{-\lambda t}$, which is the reliability function of standard exponential distribution.

3.2. Hazard Function

The hazard rate means instantaneous rate of occurrence of the event and is also known as the failure rate. The hazard function of UE

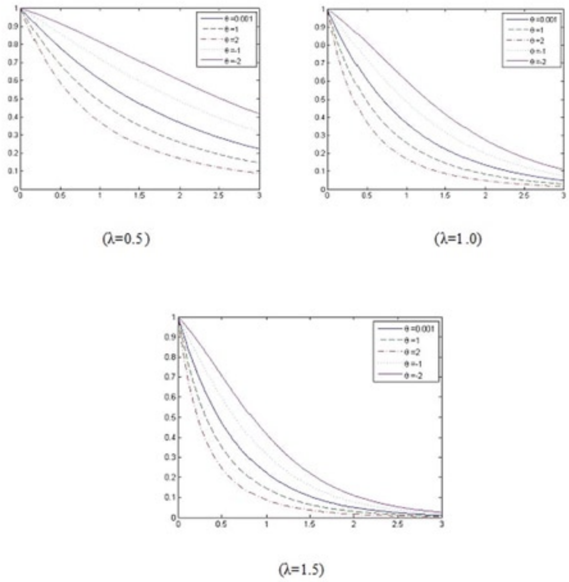

Fig. 2. The reliability functions of the UE distribution

distribution can be determined as follows

$$h(t)=\frac{\theta\lambda e^{-\lambda t}e^{-\theta\left(1-e^{-\lambda t}\right)}}{e^{-\theta\left(1-e^{-\lambda t}\right)}-e^{-\theta}}. \tag{22}$$

Taking $\lim_{\theta\to 0}h(t)\to\lambda$ into account, we can obtain the failure rate of exponential distribution. Figure 3 shows the hazard functions of UE distribution for some θ values. It is clear from the figure, when the parameter θ is greater than 0, then the failure rate function has a decreasing form. On the other hand, if the parameter θ is less than 0, then the failure rate function becomes increasing.

3.3. Mean Time to Failure

Mean time to failure (MTTF) is a measure of the length of the time a system is failed. It is usually used for nonrepairable systems. MTTF of UE is defined

$$MTTF=\int_0^{\infty}tf(t)dt=\frac{\theta\,\ddot{}\,(\theta)}{\lambda\left(1-e^{-\theta}\right)}. \tag{23}$$

It should be remembered that the Ψ(*θ*) values are given in Table 1 for some representative *θ* values.

3.4. Censoring

Censoring is a condition in which some data cannot be observed precisely due to various reasons. There are many censoring schemes in literature. The most frequently encountered censoring is Type II. In the context of Type II censored data, the smallest r observations can-

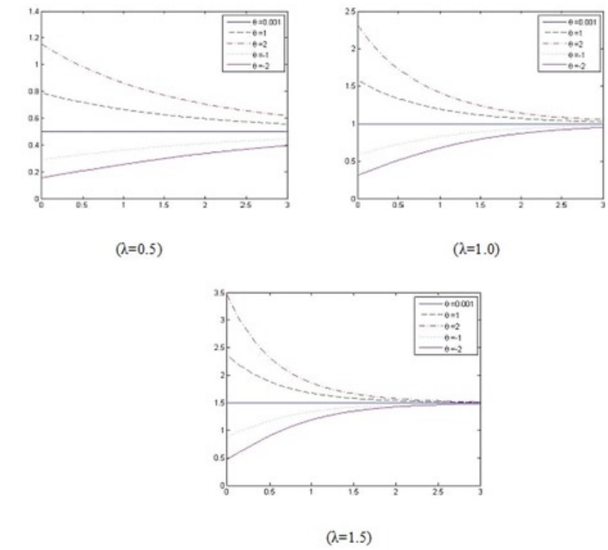

Fig. 3. The hazard functions of the UE distribution.

not be observed. Therefore, we have $n-r$ observations, where n is the number of observations. Let $X_1, X_2, \ldots, X_n$ be lifetimes of the sample. By ordering the sample we get $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$. In type II censoring, we only have $X_{(1)}, X_{(2)}, \ldots, X_{(r)}$. ML estimation method is based on maximizing the likelihood function. The likelihood function of Type II censored data is:

$$L = \frac{n!}{(n-r)!}\prod_{i=1}^{r} f(x_i)\left[1-F(x_i)\right]^{n-r}$$

$$= \frac{n!}{(n-r)!}\prod_{i=1}^{r}\frac{\theta\lambda e^{-\lambda x_i} e^{-\theta\left(1-e^{-\lambda x_i}\right)}}{\left(1-e^{-\theta}\right)}\left[\frac{e^{-\theta\left(1-e^{-\lambda x_r}\right)}-1}{e^{-\theta}-1}\right]^{n-r}. \tag{24}$$

The log-likelihood function can be obtained by taking natural logarithm of (24) as:

$$\ln L = r\ln(\theta) + r\ln(\lambda) - r\ln\left(1-e^{-\theta}\right) - \lambda\sum_{i=1}^{r} x_i - \theta\sum_{i=1}^{r}\left(1-e^{-\lambda x_i}\right)$$

$$+(n-r)\ln\left[e^{-\theta} - e^{-\theta\left(1-e^{-\lambda x_r}\right)}\right] - (n-r)\ln\left(e^{-\theta}-1\right). \tag{25}$$

By differentiating the log-likelihood function with respect to the unknown parameters, we obtain the following likelihood equations

$$\frac{\partial \ln L}{\partial \lambda} = \frac{r}{\lambda} - \sum_{i=1}^{r} x_i - \theta\sum_{i=1}^{r}\left(x_i e^{-\lambda x_i}\right) + \frac{(n-r)e^{-\theta\left(1-e^{-\lambda x_r}\right)}\theta e^{-\lambda x_i} x_r}{e^{-\theta} - e^{-\theta\left(1-e^{-\lambda x_r}\right)}} \tag{26}$$

$$\frac{\partial \ln L}{\partial \theta} = \frac{r}{\theta} - \frac{re^{-\theta}}{1-e^{-\theta}} + \frac{(n-r)e^{-\theta}}{e^{-\theta}-1} - \sum_{i=1}^{r}\left(1-e^{-\lambda x_i}\right) + \frac{(n-r)\left[e^{-\theta\left(1-e^{-\lambda x_r}\right)}\left(1-e^{-\lambda x_i}\right) - e^{-\theta}\right]}{e^{-\theta} - e^{-\theta\left(1-e^{-\lambda x_r}\right)}}.$$

Because of the intractable functions in the likelihood equations, it is not possible to find the closed form expressions for the ML estimators. Therefore, we have to resort to iterative methods to solve them numerically.

3.5. Stress-Strength Probability

The stress-strength probability $R = P(Y < X)$ is a measure of the system reliability with strength X and stress Y. In a stress-strength model the system fails, when the applied stress to the system is greater than its strength. Several distributions have been applied to the stress-strength reliability models, see [6, 10, 13, 16 and 17].

Let X be the strength of a system distributed UE with the parameters (λ_1, θ_1) and Y be the stress, whose distribution is UE with the parameter (λ_2, θ_2). Therefore, stress-strength probability can be derived as:

$$P(Y<X) = \int P(Y<X) f_X(x)dx = \int F_Y(x) f_X(x)dx$$

$$= \int_0^\infty \frac{\left(e^{-\theta_2\left(1-e^{-\lambda_2 x}\right)}-1\right)}{e^{-\theta_2}-1}\frac{\theta_1\lambda_1 e^{-\lambda_1 x}e^{-\theta_1\left(1-e^{-\lambda_1 x}\right)}}{1-e^{-\theta_1}}dx \tag{27}$$

$$= \int_0^\infty \frac{e^{-\theta_2\left(1-e^{-\lambda_2 x}\right)}\left(\theta_1\lambda_1 e^{-\lambda_1 x}e^{-\theta_1\left(1-e^{-\lambda_1 x}\right)}\right)}{e^{-\theta_2}-1\left(1-e^{-\theta_1}\right)}dx - \frac{1}{e^{-\theta_2}-1}.$$

By taking $u = -e^{-\lambda_1 x}$, we get:

$$\frac{\theta_1\left(e^{-\theta_2-\theta_1}\right)}{e^{-\theta_2}-1\left(1-e^{-\theta_1}\right)}\int_{-1}^{0} e^{\theta_2(-u)^{\frac{\lambda_2}{\lambda_1}}} e^{-\theta_1 u}du - \frac{1}{e^{-\theta_2}-1}. \tag{28}$$

Then taking $t = \theta_1 u$, the integral becomes:

$$\frac{\left(e^{-\theta_2-\theta_1}\right)}{e^{-\theta_2}-1\left(1-e^{-\theta_1}\right)}\int_{-\theta_1}^{0} e^{-\left(\frac{t\theta_2}{\theta_1}\right)^{\frac{\lambda_2}{\lambda_1}}} e^{-t}dt - \frac{1}{e^{-\theta_2}-1}. \tag{29}$$

Using the property,

$$e^{-\left(\frac{t\theta_2}{\theta_1}\right)^{\frac{\lambda_2}{\lambda_1}}} = \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{t\theta_2}{\theta_1}\right)^{\frac{\lambda_2}{\lambda_1}i}\right] \tag{30}$$

we get:

$$\left\{\frac{\left(e^{-\theta_2-\theta_1}\right)}{e^{-\theta_2}-1\left(1-e^{-\theta_1}\right)}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{t\theta_2}{\theta_1}\right)^{\frac{\lambda_2}{\lambda_1}i}\right]\int_{-\theta_1}^{0} t^{\frac{\lambda_2}{\lambda_1}i}e^{-t}dt\right\} - \frac{1}{e^{-\theta_2}-1} \tag{31}$$

and by the help of below expansion:

$$e^{-t} = \sum_{k=0}^{\infty}\frac{(-1)^k t^k}{k!} \tag{32}$$

we finally find the reliability as:

$$\left\{\frac{\left(e^{-\theta_2-\theta_1}\right)}{e^{-\theta_2}-1\left(1-e^{-\theta_1}\right)}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{t\theta_2}{\theta_1}\right)^{\frac{\lambda_2}{\lambda_1}i}\right]\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\frac{(-\theta)^{\frac{\lambda_2}{\lambda_1}i+k+1}}{\left(\frac{\lambda_2}{\lambda_1}i+k+1\right)}\right\}-\frac{1}{e^{-\theta_2}-1}. \tag{33}$$

If we take $\lambda_1=\lambda_2$ we can find stress strength reliability as:

$$P(Y<X)=\int P(Y<X)f_X(x)dx=\int F_Y(x)f_X(x)dx \tag{34}$$

$$=\frac{\theta_1}{e^{-\theta_2}-1\left(1-e^{-\theta_1}\right)}\left(\frac{1}{\theta_1+\theta_2}-\frac{\left(e^{-\theta_2-\theta_1}\right)}{\theta_1+\theta_2}\right)-\frac{1}{e^{-\theta_2}-1}.$$

Let $X_1,X_2,\ldots,X_{n_1}$ and $Y_1,Y_2,\ldots,Y_{n_2}$ be two independent random samples from the UE distribution with parameters (λ_1,θ_1) and (λ_2,θ_2) respectively. Assuming θ_1 and θ_2 are known. To obtain the ML estimatorsfor R, we have to find the ML estimators for λ_1 and λ_2 . The ML estimators for λ_1 and λ_2 can be obtained by using the following formulas numerically.

$$\hat{\lambda}_1=\frac{n_1}{\sum_{i=1}^{n_1}x_i+\theta_1\sum_{i=1}^{n_1}x_ie^{-\lambda_1x_i}}$$

$$\hat{\lambda}_2=\frac{n_2}{\sum_{i=1}^{n_1}y_i+\theta_1\sum_{i=1}^{n_1}y_ie^{-\lambda_2y_i}} \tag{35}$$

If θ_1 and θ_2 are not known, we can also find the ML estimators of these parameter by:

$$\frac{n_1\left(1-e^{-\theta_1}-\theta_1e^{-\theta_1}\right)}{\theta_1\left(1-e^{-\theta_1}\right)}=\sum_{i=1}^{n_1}\left(1-e^{-\lambda_1x_i}\right)$$

$$\frac{n_2\left(1-e^{-\theta_2}-\theta_2e^{-\theta_2}\right)}{\theta_2\left(1-e^{-\theta_2}\right)}=\sum_{i=1}^{n_2}\left(1-e^{-\lambda_2y_i}\right) \tag{36}$$

solving (36) iteratively. [17] proved that the ML estimators for this reliability are more efficient than the UMVUE and Bayes estimators with respect to MSE values. For this reason, we only obtain ML estimators for stress-strength reliability. One can also find the UMVUE and Bayes estimators. Table 2 shows the reliabilities and ML estimators of these reliabilities calculated from some representative distribution parameters. The MLEs are considered according to the n_1 and n_2 respectively. Table 2 shows stress-strength probabilities and their ML estimations for some representative parameters.

It should be also noted that $\lim_{\theta_1,\theta_2\to 0} R=\frac{\lambda_2}{\lambda_1+\lambda_2}$. This is also the stress-strength probability of the exponential distribution. Additionally, if the other parameters remain stable the following remarks can be obtained

Table 2. The stress-strength probabilities of UE distribution

λ_1	λ_2	θ_1	θ_2	R	MLE(R)
	0.5			0.5000	0.5008
0.5	1.0	0.5	0.5	0.6593	0.6544
	2.0			0.7901	0.7923
	0.5			0.5438	0.5465
0.5	1.0	0.5	1.0	0.6928	0.6956
	2.0			0.8132	0.8139
	0.5			0.6139	0.6175
0.5	1.0	0.5	2.0	0.7530	0.7549
	2.0			0.8539	0.8599
	0.5			0.3407	0.3388
1.0	1.0	0.5	0.5	0.5000	0.5034
	2.0			0.6593	0.6601
	0.5			0.3814	0.3786
1.0	1.0	0.5	1.0	0.5408	0.5501
	2.0			0.6928	0.7004
	0.5			0.4609	0.4692
1.0	1.0	0.5	2.0	0.6169	0.6234
	2.0			0.7530	0.7601
	0.5			0.2099	0.1982
2.0	1.0	0.5	0.5	0.3407	0.3387
	2.0			0.5000	0.5091
	0.5			0.2432	0.2387
2.0	1.0	0.5	1.0	0.3814	0.3785
	2.0			0.5408	0.5472
	0.5			0.3114	0.3006
2.0	1.0	0.5	2.0	0.4609	0.4646
	2.0			0.6169	0.6204
	0.5			0.4592	0.4633
0.5	1.0	1.0	0.5	0.6186	0.6231
	2.0			0.7568	0.7612
	0.5			0.3831	0.3756
0.5	1.0	2.0	1.0	0.5391	0.5454
	2.0			0.6886	0.6934
	0.5			0.3072	0.2994
1.0	1.0	1.0	2.0	0.4592	0.4643
	2.0			0.6186	0.6232
	0.5			0.2470	0.2388
1.0	1.0	2.0	0.5	0.3831	0.3736
	2.0			0.5391	0.5423
	0.5			0.1868	0.1776
2.0	1.0	1.0	1.0	0.3072	0.2991
	2.0			0.4592	0.4665
	0.5			0.1461	0.1387
2.0	1.0	2.0	2.0	0.2470	0.2395
	2.0			0.3831	0.3799

- If θ_1 increases the probability decreases,
- If θ_2 increases the reliability also increases,
- If λ_1 increases the probability decreases,
- If λ_2 increases the reliability also increases.

4. Numerical Example

In this section, we consider the data given by [12]. The data is about the number of million revolutions before failure for each 23 ball bearings in the life test. [8] proposed the *GE* distribution and subsequently compared this with Weibull and gamma distributions. The data are as follows: 17.88; 28.92; 33; 41.52; 42.12; 45.60; 48.40; 51.84; 51.96; 54.12; 55.56; 67.80; 68.64; 68.64; 68.88; 84.12; 93.12;

98.64; 105.12; 105.84; 127.92; 128.04; 173.40. We propose the UE distribution for this data set. We obtain the ML estimators of the parameters and calculate the log-likelihood values and AIC statistics. The results are:

$\hat{\lambda} = 0.0147 \;\; \hat{\theta} = -1.5410 \;\; \hat{\mu} = 25.998 \;\; \ln L = -112.428$ and $AIC = 230.857$

while the ML estimators, log-likelihood values and AIC statistics for Weibull, gamma and generalized exponential distributions with three parameters are obtained as:

$\hat{\alpha} = 1.5979 \;\; \hat{\lambda} = 0.0156 \;\; \hat{\mu} = 14.8479 \;\; \ln L = -112.976$ and $AIC = 231.952$

$\hat{\alpha} = 2.7316 \;\; \hat{\lambda} = 0.0441 \;\; \hat{\mu} = 10.2583 \;\; \ln L = -112.850$ and $AIC = 231.700$

and

$\hat{\alpha} = 4.1658 \;\; \hat{\lambda} = 0.0314 \;\; \hat{\mu} = 14.8479 \;\; \ln L = -112.766$ and $AIC = 231.532.$

The UE distribution has the largest log-likelihood value and the smallest AIC statistics. This indicates that UE distribution provides a much better fit and more reliable inferences than other proposed distributions.

5. Conclusion

In this paper, we propose a new lifetime distribution with both increasing and decreasing failure rates. We define the reliability function, hazard function and MTTF for this new distribution. Furthermore, Type II censoring procedure is also considered for this distribution. We obtain stress-strength probability and ML estimators of this reliability for the proposed distribution. A real data example shows the proposed distribution gets better fit and more reliable solutions from other alternatives.

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