DE GRUYTER DOI 10.1515/secm-2012-0017 — Sci Eng Compos Mater 2012; 19(4): 373–380

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Estimation of fracture energy of high-strength steel fibre-reinforced concrete using rule-based Mamdani-type fuzzy inference system

Abstract: In this study, we worked to estimate the fracture energy of steel fibre-reinforced concrete (SFRC) according to the water/cement ratio (w/c), tensile strength of steel fibre, steel fibre volume fraction and flexural strength of concrete sample as inputs using the Mamdani-type fuzzy inference system (FIS). In the study, the values obtained from the model and experimental divided three groups (each group has six experimental results) according to the w/c ratios to evaluate the fuzzy logic (FL) model approximate reasoning ability. As a result, the Mamdani-type FIS has shown a satisfying relation with the experimental results and suggests an alternative approach to evaluate the fracture energy estimation using related inputs.

Keywords: fracture energy; fuzzy logic; modelling; steel fibre-reinforced concrete.

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1 Introduction

Concrete is the most widely used construction material. A variety of types of concrete exist. It is a heterogeneous material consisting of cement, water, sands and aggregates [1–6]. While the heterogeneous structure of concrete can produce some undesirable effects [7, 8], concrete remains an indispensable construction material and allows engineers to incorporate many materials into it [9].

Concrete is a cement-based composite material and shows a semi-brittle behaviour under a certain threshold loading. Fracture energy is one of the most important mechanical properties in understanding the ductility or brittleness of concrete and determining the design criteria of large concrete structural elements. Civil engineering structures are generally exposed to static and live loading and also dynamic loadings such as impact, earthquake, explosions and hazards. Fracture energy for concrete structures and bearing elements is important as well as strength in assessing the safety of structures. Fracture energy capacity of a structural system and its elements is a significant parameter to be taken into account in the design of earthquake-resistant buildings. Therefore, fracture energy capacity or energy to be dissipated up to the failure under any type of loading must be truly known or determined for the concrete elements.

The fracture energy (G_f) is defined as the area under the load-deflection curve per unit fractured surface area under bending. A method recommended by RILEM [10] and Petersson [11] is generally used for the determination of G_f using simple three-point bending test. Softening part of the load-deflection curve is the most critical one and must be determined carefully in obtaining fracture energy. Besides, controlling the behaviour of concrete, after a crack exists, is not easy and need to close-loop the deformation-controlled testing machines. Linear voltage displacement transducers (LVDTs) with high sensitivity and data recorder or software are required to record the load and displacement simultaneously. Those types of testing machines and accessories are very expensive. Specimens used for bending tests given in related standards are at least 15×15×60 cm^3 prisms. Therefore, it can be stated that the test setup and determination of fracture energy of concrete is an exacting task for researcher. Fibres have been used for both polymer reinforcement [12] and in fibre-reinforced concretes [13, 14].

In general, reinforced concrete performance depends on formulations as well as the fibre characteristics, including type, geometry, distribution, orientation and concentration [15]. Many different kinds of fibres, such as metallic, polymeric, coated, uncoated or modified by irradiation, have been used in concrete engineering for their specific advantages [16–24].

Steel fibres are used in concrete to improve some mechanical and durability properties of concrete [25–28]. The most significance improvement is shown on the fracture energy of concrete [28, 29]. Steel fibres added to concrete are randomly oriented in the concrete matrix and show its effect after matrix cracking by delaying crack formation and limiting crack propagation by reducing the crack tip opening displacement [30, 31]. This is known as *crack bridging mechanism*. The performance of fibre in the matrix depend on fibre type, orientation of fibres in the matrix, aspect ratio (length/diameter), volume fraction of fibres, tensile strength of the fibre and matrix strength or water/cement ratio [32–34]. The main objective in this research is to estimate the fracture energy, experimentally obtained, depending on independent variables of water/cement ratio, fibre tensile strength, fibre volume fraction and dependent variable of flexural strength of concrete using Mamdani-type (using Fuzzy Logic (FL) Toolbox) FIS.

2 Fuzzy Logic

2.1 Theory of Fuzzy Logic

FL was used for the first time in 1965 by L. A. Zadeh [35]. In this approach, Zadeh developed a new consideration instead of Aristotelian logic, which contains two definite and two different possibilities only (1 or 0). It needs only to set a simple controlling method based on engineering experience. Therefore, it is particularly useful in complicated structural systems. FL has been developing since 1965 and become most successful in application [36]. In the Aristotelian logic, all systems such as mathematic or stochastic have three components. These are input, system behaviour and output [37].

The main process of a general FIS includes four activities called fuzzification, fuzzy rule base, fuzzy inference engine and defuzzification [38] (Figure 1). These parts are detailed below.

- Input: It contains all input parameters and information about them.

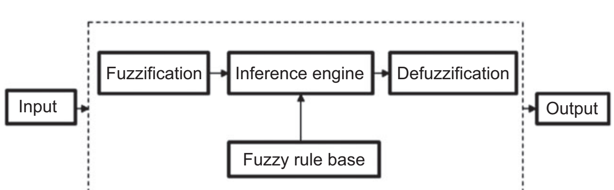

Figure 1 Basic elements of FL [38].

- Fuzzification: It converts each input data to degrees of membership by a lookup in one or more several membership functions.
- Fuzzy rule base: This contains rules that include all possible fuzzy relation between input and outputs using the IF-THEN format.
- Fuzzy interference engine: Collects all fuzzy rules in the fuzzy rule base and learns how to transform a set of inputs to related outputs.
- Defuzzification: This converts the resulting fuzzy outputs from the fuzzy interference engine to a number [39].

In recent years, the number and variety of applications of FL have increased significantly. The applications range from consumer products such as cameras, camcorders, washing machines and microwave ovens to industrial process control, medical instrumentation, decision-support systems and portfolio selection [40]. When the applications of FL is analysed for civil engineering, FL has been extensively used in the fields of civil engineering applications especially in cement and concrete properties estimation [36, 37, 39, 41–54].

Fuzzy inference is the real process of mapping from a given set of input variables to an output relied upon a set of fuzzy rules. There are two types of FIS that can be implemented in the MATLAB's FIS toolbox: Mamdani type and Sugeno type. Mamdani's method is the most commonly seen fuzzy methodology, and it expects the output MFs to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification [55].

2.2 Modelling fuzzy expert systems

Fuzzy expert system modelling can be pursued using the following steps.

- Select the relevant input and output variables. Determine the number of linguistic terms associated with each input/output variable. Also, choose the appropriate family of membership functions, fuzzy operators, reasoning mechanism, and so on.
- Choose a specific type of FIS (for example, Mamdani, Takagi-Sugeno, etc.). In most cases, the inference of the fuzzy rules is carried out using the "min" and "max" operators for fuzzy intersection and union.
- Design a collection of fuzzy *if-then* rules (knowledge base). To formulate the initial rule base, the input space is divided into multidimensional partitions, and then, actions are assigned to each of the partitions [56].

Type	Length l (mm)	Diameter (d), mm	Aspect ratio (l/d)	Density (g/cm³)	Tensile strength, f_{su} (N/mm²)
Dramix RC 80/60 BN (with low carbon)	60	0.75	80	7.85	1050
Dramix RC 80/60 BP (with high carbon)	60	0.71	85	7.85	2000

Table 1 Properties of hooked-end steel fibres used in this study.

3 Experimental study

3.1 Details of the experimental

CEM I 42.5R Portland Cement as binder, silica fume as mineral admixture, crushed limestone fines (0–4 mm) and crushed limestones (4–12 mm and 12–19 mm) as aggregate were used in the production of SFRC specimens. A high-range water-reducing admixture was used to control the water demand of fresh concrete. Cold drawn and hooked-ends steel fibres having different tensile strengths of 1050 and 2000 MPa were used in the mixtures. Water/cement ratios of 0.35, 0.45 and 0.55 and steel fibre volume fractions (V_f) of 0.33%, 0.67% and 1% were also the independent variables used in the design of SFRC mixtures as steel fibre tensile strength. The properties of steel fibre are given in Table 1. To obtain a uniform mixture during the fresh concrete state, cement, silica fume and all aggregates were mixed and blended first, and then, a mixture of water and high-range water-reducing admixture was added to the mixture. Finally, steel fibres were scattered in the mixture. Specimens were demoulded after 24 h and kept in standard water curing for 28 days. A standard cylinder and disc specimens were produced for mechanical strengths as compressive and splitting tensile, respectively. In determining the fracture energy and flexural tensile strength of SFRCs, the beam test according to EN 14651 [57] standard, at which load is applied at one-third points of the specimen, was performed on 150×150×700 mm³ prismatic notched specimens. For each series of SFRC, two prismatic centrally notched specimens were tested using a feedback deflection-controlled loading frame with a capacity of 250 kN. During the flexural test, the loading rate was 0.2 mm/min, and both the load and the mid-span deflection of the beam specimens were simultaneously recorded. Also, the load-deflection curves for each SFRC specimens were also obtained graphically during the test. The flexural test setup is given in Figure 2.

3.2 Evaluation fracture energy of SFRC

Normally, the uniaxial tensile test is an ideal test to measure the fracture energy of concrete. Owing to the difficulty in performing stable and representative direct tensile tests, researchers generally prefer flexural tensile test in which three-point bend notched beams are used to measure the total work of the fracture. In literature, there are some methods to evaluate the fracture energy of SFRCs. In this study, the fracture energy, G_f, was calculated by following the equation Eq. (1) recommended by RILEM TC-50 FMC [10]:

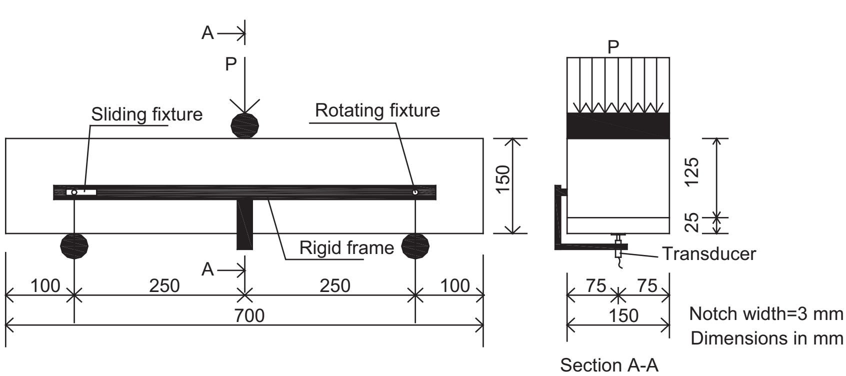

Figure 2 Flexural test setup.

$$G_f = \frac{W_0 + mg\delta_0}{B(D\text{-}a)} \quad (1)$$

where, m is the mass of the specimen, g is the acceleration due to gravity, δ_0 is the maximum deflection at the final fracture, W_0 is the energy represented by the area under the load-deflection curve, $mg\delta_0$ is the energy supplied from the weight of the beam itself, a, B and D are the notch depth, thickness and depth of beam, respectively.

4 Application of rule-based Mamdani FIS approach and results

In this presented study, we worked to develop a rule-based fuzzy model for the prediction of fracture energies of concretes (N/m) using w/c, tensile strength of steel fibre (N/mm^2), steel fibre content (%) and flexural strength of concrete samples (N/mm^2) as inputs. A flow diagram for this study is given in Figure 3.

In this flow diagram, Figure 4 illustrates the membership functions of inputs and outputs of the model, Figure 5 illustrates the defuzzification monitor of the model, Figures 6–8 illustrate the comparison of the experimental and FL results according to the sample numbers for SET I–SET II and SET III.

Rule-based fuzzy model was chosen because it is based on natural language, flexible and conceptually easy to understand [58]. Besides, it needs only to set a simple controlling method based on engineering experience. The developed model has four inputs and an output (Figure 9).

The inputs were w/c, tensile strength of steel fibre (N/mm^2), steel fibre content (%) and flexural strength of concrete samples (N/mm^2), and the output was fracture energies of concretes. In the model, the membership functions were selected as triangular membership functions (trimf) for all inputs and the output (Figure 4). Their numerical ranges are given in Table 2.

After determining the membership functions details, 216 rules were formed using the experimental results and experiences (by Köksal and Şahin's experimental experiences). Some of formed rules are given blow:

- IF w/c is "small" and tsf is "tsf1" and sfc is "small" and fs is "fs10" THEN fracture energy is "fe10"
- IF w/c is "small" and tsf is "tsf1" and sfc is "small" and fs is "fs11" THEN fracture energy is "fe1"

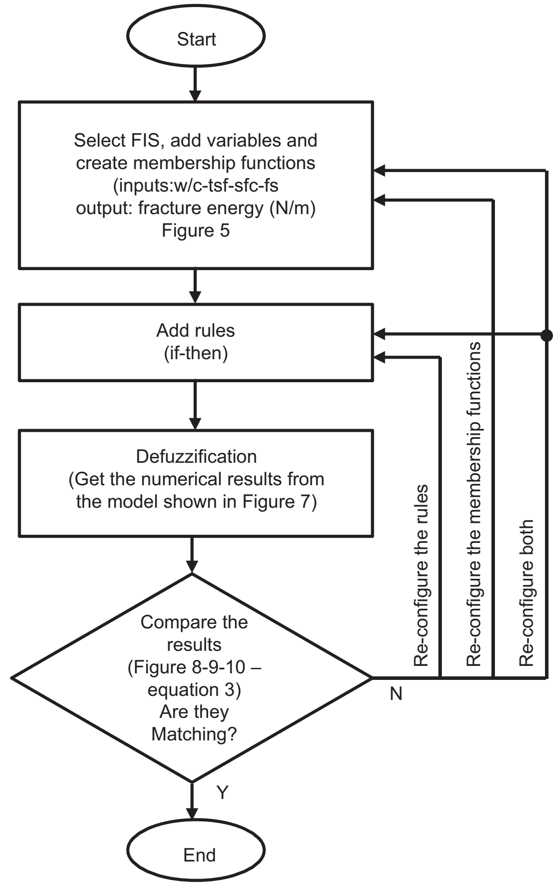

Figure 3 Flow diagram for this study.

- IF w/c is "small" and tsf is "tsf2" and sfc is "middle" and fs is "fs11" THEN fracture energy is "fe14"

Fracture energy value variation is a function of inputs in the model according to the formed rules displayed in Figure 10A and B. These figures illustrate the relationship between inputs and output.

There are many kinds of defuzzification method that are being used for different applications. The most commonly used technique is the centroid defuzzification technique. This technique was used in order to determine the crisp values of the outputs for this study. The centroid defuzzification technique can be expressed as Eq. (2) where x^* is the defuzzified output, $\mu_i(x)$ is the aggregated membership function and x is the output variable. As the final stage, after creating the model, the model results were obtained from the defuzzification monitor of the model (Figure 5).

$$x^* = \frac{\int \mu_i(x)\,x\,dx}{\int \mu_i(x)\,dx} \quad (2)$$

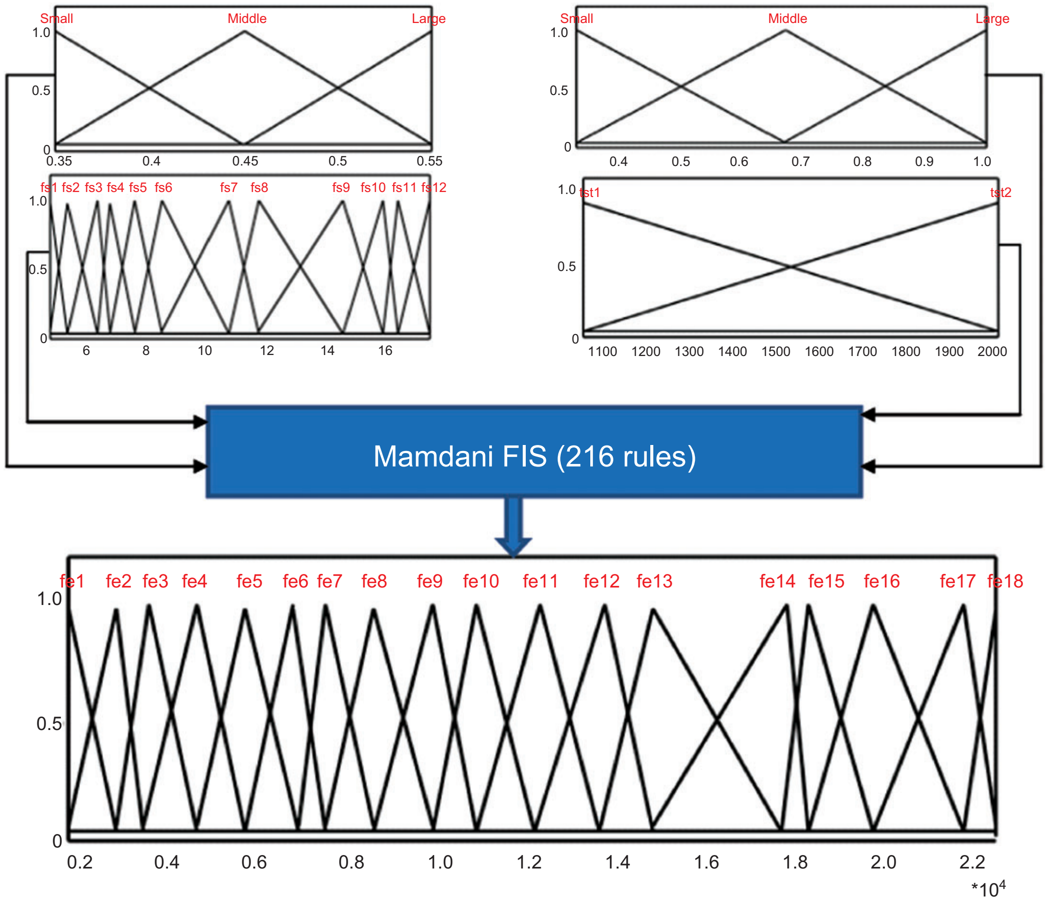

Figure 4 Membership functions of inputs and outputs of the model.

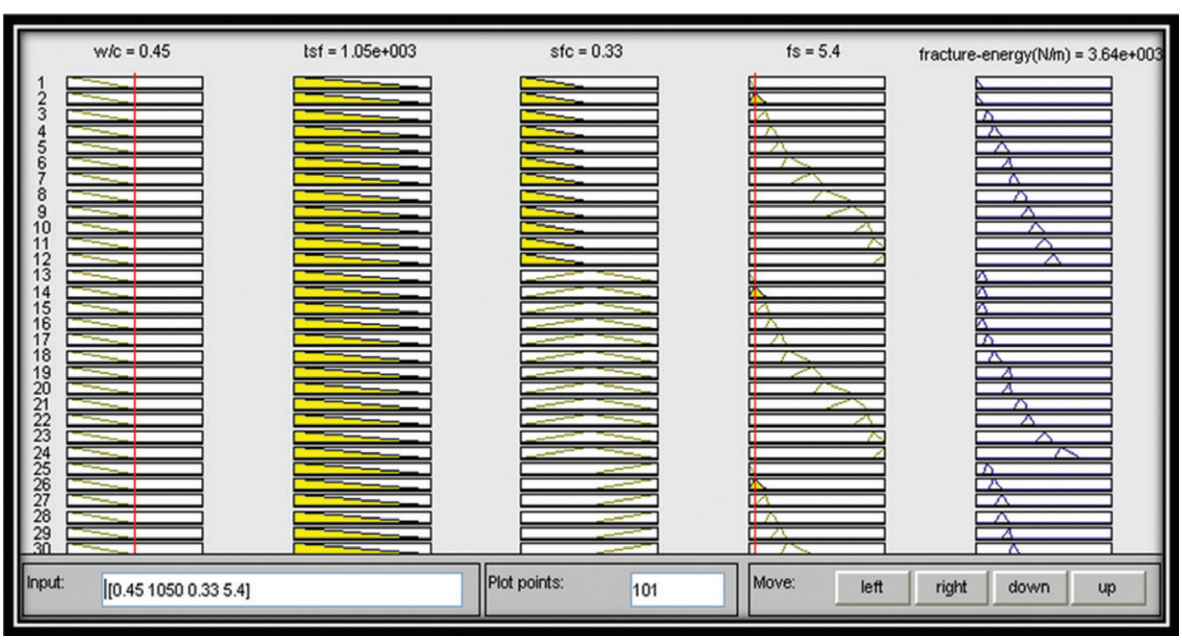

Figure 5 Defuzzification monitor of the model.

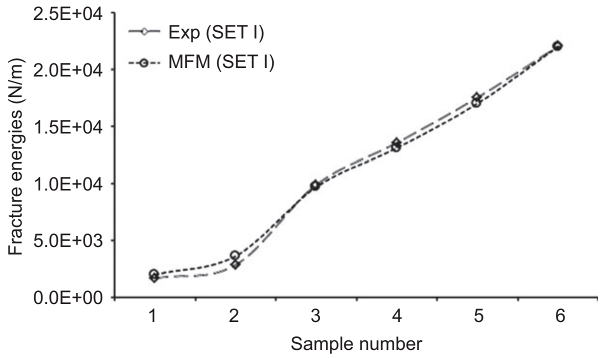

Figure 6 Comparison of the experimental and FL results according to the sample numbers (SET I).

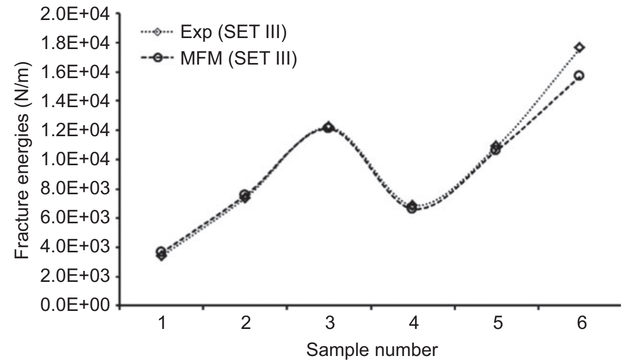

Figure 8 Comparison of the experimental and FL results according to the sample numbers (SET III).

The values obtained from the model and experimentally divided three groups according to the w/c ratios to evaluate FL model predictability. The adequacy of the developed FL model was evaluated by considering the parameter of coefficient of determination (R^2) Eq. (3) and matching figures (Figures 6–8).

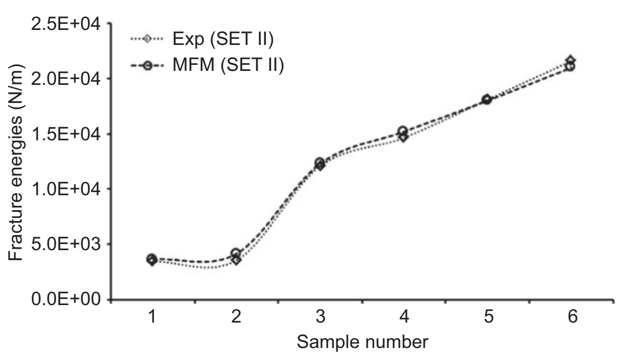

Figure 7 Comparison of the experimental and FL results according to the sample numbers (SET II).

$$R^2 = 1 - \left\{ \frac{\left[\sum_{i=1}^{n} \left(y_{i(m)} - y_{i(p)} \right)^2 \right]}{\left[\sum_{i=1}^{n} \left(y_{i(m)} - y_{i(mean)} \right)^2 \right]} \right\} \quad (3)$$

Here, *m* is the measured value, *p* is the predicted value, *mean* is the average measured value and *n* is the number of data.

5 Conclusions

The potential of the rule-based Mamdani-type FL model for the estimation of the fracture energy of SFRC according to the w/c, tensile strength of steel fibre, steel fibre volume fraction and flexural strength of concrete sample as inputs using Mamdani-type FIS has been investigated in this research. Experimental data were used while developing the model. After the modelling process, the results obtained from the developed model were compared with

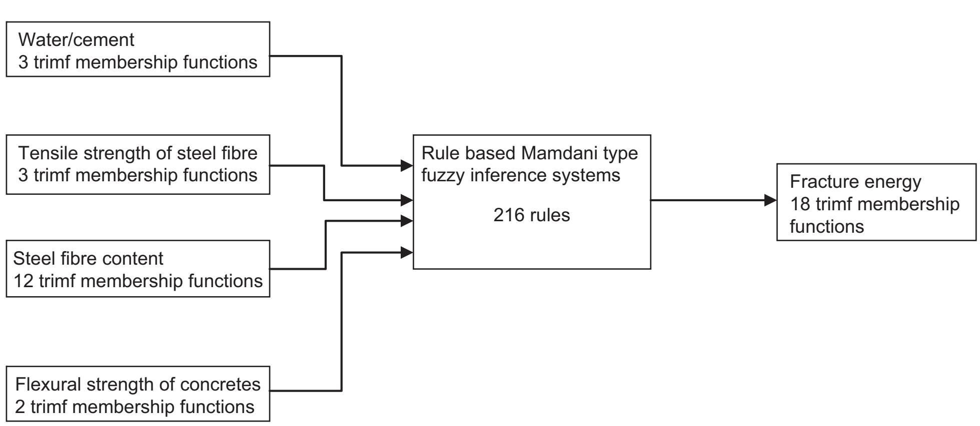

Figure 9 General structure of the model.

Parameters	Membership function details
Input – w/c	3 trimf – range 0.35–0.55
Input – tensile strength of steel fibre (tsf) (N/mm^2)	2 trimf – range 1050–2000
Input – steel fibre content (sfc) (%)	3 trimf – range 0.33–1
Input – flexural strength of concrete samples (fs) (N/mm^2)	12 trimf – range 4.8–17.3
Input – fracture energies of concretes (fe)	18 trimf – range 1700–2.22e+004

Table 2 Details of chosen parameters for FL modelling.

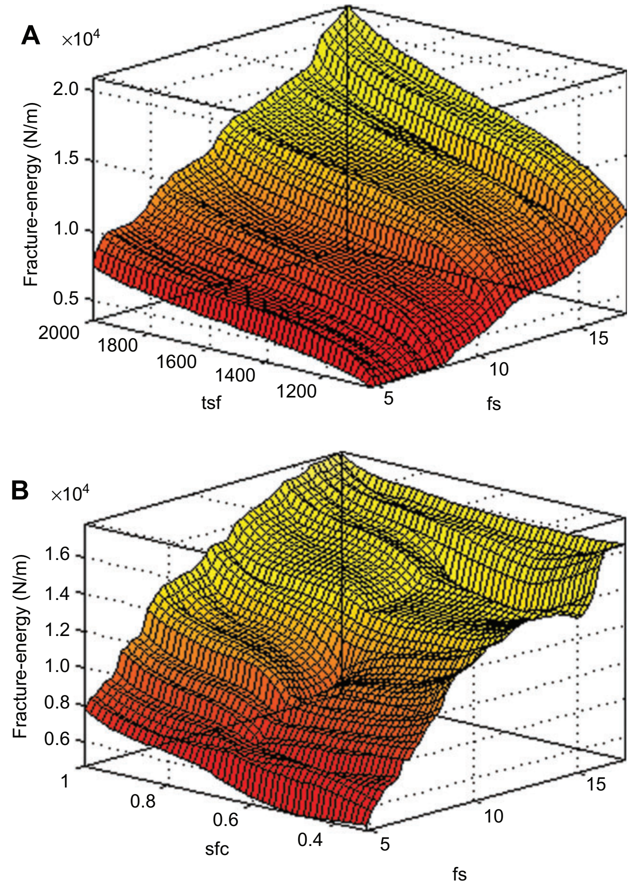

Figure 10 (A and B) The fracture energy from FL model as a function of inputs.

the experimental results. According to the experimental and modelling results, the following conclusions can be written from this investigation:

It is experimentally obtained that addition of steel fibre results in significant improvements in fracture energies and flexural strengths of SFRCs. On the other hand, steel fibres with a high tensile strength are more effective in comparison to those with low tensile strength.

When the matching figures are analysed (Figures 6–8), it can be concluded that the values are close to each other.

When the results were compared using the coefficient of determination (R^2) values, the values were found to be 0.9962 for Set I, 0.9959 for SET II and 0.9679 for SET III. These results show very acceptable relations between the developed model results and the experimental results.

As a result, it was shown that the fracture energy values of SFRC can be predicted using the newly developed rule-based FL model in a relatively short period of time. Thus, rule-based Mamdani-type FL can be an alternative approach for the evaluation of the fracture energy values of steel fibre-reinforced concretes.

Received April 1, 2012; accepted June 28, 2012; previously published online July 26, 2012.

References

[1] Gencel O. *Sci. Eng. Compos. Mater.* 2011, 18, 191–199.
[2] S. Mindess. *J. Mater. Ed.* 1982, 5, 983.
[3] M. Regoud. *J. Mater. Ed.* 1986, 9, 201.
[4] Roy DM, Scheetz BE, Silsbee MR. *J. Mater. Ed.* 1993, 15, 1.
[5] Mcphee DE, Glasser FP. *J. Mater. Ed.* 1993, 15, 33.
[6] Davidovits J. *J. Mater. Ed.* 1994, 16, 91.
[7] Aydin AC. *Constr. Build. Mater*. 2007, 21, 1149–1154.
[8] Gencel O, Ozel C, Brostow W, Martínez-Barrera G. *Mater. Res. Innov.* 2011, 15, 216–225.
[9] Gencel O, Kocabas F, Gok MS, Koksal F. *Constr. Build. Mater.* 2011, 25, 3486–3494.
[10] RILEM Draft Recommendation, 50-FMC committee fracture mechanics of concrete. *Mater. Struct.* 1985, 18, 285–290.
[11] Petersson PE. *Cement Concrete Res.* 1980, 10, 79–89.
[12] Pisanova E, Zhandarov S. In *Performance of Plastics*, Brostow W, Ed., Hanser: Munich-Cincinnati, 2000 [Chapter 19].
[13] Martinez-Barrera G, Vigueras-Santiago E, Hernandez-Lopez S, Menchaca-Campos C, Brostow W. *Polym. Eng. Sci.* 2005, 45, 1426–1431.
[14] Martinez-Barrera G, Urena-Nunez F, Gencel O, Brostow W. *Compos. Part A* 2011, 42, 567–572.
[15] Zollo RF. *Cement Concrete Compos.* 1997, 19, 107–122.
[16] Kakemi M, Hannant D. *Cement Concrete Compos.* 1996, 18, 61.
[17] Wang W, Wu S, Dai H. *Mater. Sci. Eng. A* 2006, 434, 347.
[18] Song PS, Hwang S, Sheu BC. *Cement Concrete Res.* 2005, 35, 1546.
[19] Hsiea M, Tu C, Song PS. *Mater. Sci. Eng. A* 2008, 494, 153.

[20] Martínez-Barrera G, Vigueras-Santiago E, Hernández-López S, Menchaca-Campos C, Brostow W. *Polym. Eng. Sci.* 2005, 45, 1426–1431.
[21] Martínez-Barrera G, Menchaca-Campos C, Hernández-López S, Vigueras-Santiago E, Brostow W. *J. Mater. Res.* 2006, 21, 484–491.
[22] Martínez-Barrera G, Menchaca-Campos C, Vigueras-Santiago E, Brostow W, e-Polymers no. 2010, 042.
[23] Gencel O, Brostow W, Tea D, Thedford M. *Compos. Interface*. 2011, 18, 169–184.
[24] Koksal F, Gencel O, Unal B, Durgun MY. *Sci. Eng. Compos. Mater.* 2012, 19, 19–27.
[25] Balaguru P, Narahari R, Patel M. *ACI Mater. J.* 1992, 89, 541–546.
[26] Tasdemir MA, Bayramov F. *ITU J/d Eng*. 2002, 1, 125–144.
[27] Balendran RV, Zhou FP, Nadeem A, Leung AYT. *Build. Environ.* 2002, 37, 1361–1367.
[28] Köksal F, Altun F, Yiğit İ, Şahin Y. *J. Constr. Build. Mater.* 2008, 22, 1874–1880.
[29] Balendran RV, Zhou FP, Nadeem A, Leung AYT. *Build. Environ.* 2002, 37, 1361–1367.
[30] Şahin Y, Köksal F. *J. Constr. Build. Mater.* 2011, 25, 1801–1806.
[31] Banthia N, Trottier JF. *ACI Mater. J.* 1995, 2, 146–154.
[32] Gopalaratnam VS, Shah SP, Batson GB, Criswell ME, Ramaksishran V, Wecharatara M. *ACI Mater. J.* 1991, 88, 339–353.
[33] Koksal F, Eyyubov C, Ozcan DM. In *5th International Congress on Advances in Civil Engineering,* Istanbul, Turkey. 2002, 169–179.
[34] Shah SP, Rangan BV. *ACI Mater. J.* 1971, 68, 126–135.
[35] Zadeh LA. *Inf. Control* 1965, 8, 338–352.
[36] Tanyildizi H. *Mater. Design* 2009, 30, 2205–2210.
[37] Başyigit C, Akkurt I, Kilincarslan S, Beycioglu A. *Neural Comput. Appl.* 2010, 19, 507–513.
[38] Kuşan H, Aytekin O, Özdemir İ. *Expert Syst. Appl.* 2010, 37, 1808–1813.
[39] Akkurt I, Başyigit C, Kilincarslan S, Beycioglu A. *J. Franklin I.* 2010, 347, 1589–1597.
[40] Işık H, Arslan S. *Expert Syst. Appl.* 2011, 38, 7342–7348.
[41] Sgambi L. *Comput. Struct*. 2004, 82, 1067–1076.
[42] Mashrei MA, Abdulrazzaq N, Abdalla TY, Rahman MS. *Eng. Struct*. 2010, 32, 1723–1734.
[43] Sarıdemir M. *Adv. Eng. Softw.* 2009, 40, 920–927.
[44] Amani J, Moeini R. *Scientia Iranica* 2012, 19, 242–248.
[45] Subaşı S, Beycioğlu A, Sancak E, Şahin İ. *Neural Comput. Appl.* DOI: 10.1007/s00521-012-0879-4, 2012
[46] Topçu İB, Saridemir M. *Constr. Build. Mater.* 2008, 22, 532–540.
[47] Topçu İB, Saridemir M. *Comput. Mater. Sci*. 2008, 42, 74–82.
[48] Topçu İB, Saridemir M. *J. Mater. Processing Technol*. 2008, 199, 108–118.
[49] Saridemir M, Topçu İB, Özcan F, Severcan MH. *Constr. Build. Mater*. 2009, 23, 1279–1286.
[50] Özcan F, Atış CD, Karahan O, Uncuoğlu E, Tanyıldızı H. *Adv. Eng. Softw.* 2009, 40, 856–863.
[51] Tanyıldızı H. *Adv. Eng. Softw.* 2009, 40, 161–169.
[52] Ünal O, Demir F, Uygunoğlu T. *Build. Environ*. 2007, 42, 3589–3595.
[53] Demir F, Korkmaz KA. *Constr. Build. Mater.* 2008, 22, 1385–1393.
[54] Tosun M, Dincer K, Baskaya S. *Expert Syst. Appl.* 2011, 38, 5553–5560.
[55] Omid M. *Expert Syst. Appl.* 2011, 38, 4339–4347.
[56] Abraham A. In *Handbook for Measurement Systems Design*, Sydenham P, Thorn R, Eds., John Wiley and Sons Ltd.: London, 2005, 909–919, ISBN 0-470-02143-8.
[57] EN 14651. Test method for metallic fibered concrete-measuring the flexural tensile strength. European Committee for Standardization, June 2005, pp. 17.
[58] MATLAB Fuzzy Logic Toolbox™ User's Guide R2011b.