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Integral Input-to-State Stability of Traffic Flow with Variable Speed Limit

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Abstract: In this paper, a variable speed limit controller is proposed to ensure integral input-to-state stability of the traffic state by using saturated feedback. The ordinary differential equations model of traffic flow is used with a two-phase fundamental diagram to design the controller. A common Lyapunov function is found for both phases which ensures the integral input-to-state stability of the closed-loop system. Proposed controller is tested with numerical examples to show the robustness properties particularly bounded energy frequently bounded state and bounded energy weakly converging state which are the characterizations of the integral input-to-state stability.

Keywords: integral input-to-state stability, switched systems, variable speed limit, intelligent transportation systems.

1. INTRODUCTION

Intelligent Transportation Systems (ITS) is an important component of the traffic control and management that aims to offer a better service on the available infrastructure without any change. Variable Speed Limit (VSL) is a tool in ITS to improve qualitative measures of traffic flow. Various VSL approaches are designed to improve the traffic state, mobility, and impact on environment in Carlson et al. (2011, 2013); Iordanidou et al. (2014); Jin and Jin (2015); Khondaker and Kattan (2015); Muralidharan and Horowitz (2015); Li et al. (2017); Du and Razavi (2019); Frejo et al. (2019). We refer to the studies of Zhang and Ioannou (2016, 2017, 2018) for VSL controllers developed by using feedback controllers that we also consider in this study.

From system modeling point of view, there are three ways to design feedback controllers:

- Designing feedback controllers using Partial Differential Equation (PDE) models such as the LWR (see Lighthill and Whitham (1955); Richards (1956)), Burgers (see Musha and Higuchi (1978); Burgers (1995)), and etc., to obtain continuous-time control laws and discretizing them to apply it.
- Using state space discretized Ordinary Differential Equation (ODE) models to design continuous-time feedback controllers and discretizing them (see for example Daganzo (1994, 1995); Celikoglu (2014); Celikoglu and Silgu (2016); Agarwal et al. (2015); Zhang and Ioannou (2016, 2017, 2018)).
- Directly using state and time discretized systems such as METANET (see Kotsialos et al. (2002)) to obtain discrete-time control laws.

There are advantages and disadvantages of the three ways of designing feedback controllers. Even though it is more accurate to use PDE models to represent the traffic flow phenomena, the infinite-dimensional nature of such models makes it difficult to design feedback controllers because the control input is finite-dimensional. On the other hand, using state and time discretized systems brings out discretization errors. They also require discrete-time control synthesis tools that is an area of active research. Thus, we use ODE model of traffic flow, which is derived by state space discretization of the LWR equation to use powerful tools such as Input-to-State Stability (ISS) and its variants to design feedback controllers.

ISS is a powerful tool in the analysis of the stability of nonlinear systems under inputs developed by Sontag (1989); Sontag and Wang (1995, 1996). The ISS property ensures that, the dynamical system evolves properly in the absence of disturbances and this nominal behavior is preserved up to a steady state error depending on the strength of the applied disturbance in the presence of perturbations. A weaker but still very useful property is called Integral Input-to-State Stability (IISS) proposed by Sontag (1998); Angeli et al. (2000). Rather than measuring the impact of the input magnitude on the system behavior, IISS rather focuses on measuring the impact of the energy of the input. In other words, IISS ensures to obtain a finite gain using an integral norm.

In this study, we design a VSL contoller using feedback control methods in order to ensure the IISS of the ODE model of traffic flow under a two-phase fundamental diagram. The two-phase fundamental diagram brings out a state-dependent switching structure in the ODE model that can reveal undesirable behavior in the traffic state. In such a state-dependent switching structure, we know from the counterexample in Section 2.1.5. of Liberzon

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(2003) that even two stable subsystems may not share a common Lyapunov function indicating that the whole system may be unstable. In this context, we propose a VSL controller to guarantee the IISS by finding a common Lyapunov function in the present study.

Our main motivation to investigate IISS of the model under consideration originates from the saturated nature of VSL control design. Existing studies (including Zhang and Ioannou (2016, 2017, 2018)) consider designing VSL controllers, which are not directly applicable to the study field. However in practice, VSL commands never exceed the legal speed limit whereas they are bounded from below by a certain operating limit for practical reasons. That is why, most common practice is to design the VSL controller to a certain aim, then make it ready for implementation by bounding it from above and below. In contrast to those studies, we directly consider saturated VSL controller and show that the IISS is guaranteed for the closed-loop system (CLS). One another main motivation to investigate IISS comes from the asymptotic characterization of the IISS. We mainly focus on the behavior of the state trajectories under bounded energy. Since IISS owns bounded energy frequently bounded state and bounded energy weakly converging state properties (see Angeli et al. (2004)), we conduct our analysis on IISS.

We adopt a standard notation throughout the paper. $|\cdot|$ is the Euclidean norm. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathscr{PD}$ if it is continuous, and satisfies $\alpha(0) = 0$ and $\alpha(s) > 0$ for all $s > 0$. $\alpha \in \mathscr{K}$ if $\alpha \in \mathscr{PD}$ and it is increasing. $\alpha \in \mathscr{K}_\infty$ if $\alpha \in \mathscr{K}$ and it is unbounded. $\alpha \in \mathscr{L}$ if it is continuous non-increasing and tends to zero as its argument tends to infinity. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathscr{KL}$ if $\beta(\cdot,t) \in \mathscr{K}$ for each $t \in \mathbb{R}_{\geq 0}$ and $\beta(s,\cdot) \in \mathscr{L}$ for each $s \in \mathbb{R}_{\geq 0}$. Inside the matrix blocks, "*" stands for the corresponding conjugate term or block of the upper two-phase matrix. $\text{sat}(\cdot\,; a, b)$: $\mathbb{R}_{\geq 0} \to [a,b]$ is the scalar saturation function to be used in VSL operation defined as $\text{sat}(s;a,b) = a + \frac{2\cdot(b-a)}{\pi}\tan^{-1}(s)$ where $a, b > 0$ are the minimum and maximum operating values with $a < b$.

The rest of the paper is organized as follows. We give the preliminaries and problem description including the definitions and Lyapunov characterizations of IISS, the presentation of the ODE model of traffic flow with the fundamental diagram setting and VSL formulation in Section 2. The state feedback design and the CLS are presented in the same section as well. The main result including the proof of the IISS of the CLS is presented in Section 3. We present some numerical examples to demonstrate the validity of our main result in Section 4. Section 5 concludes the paper.

2. PRELIMINARIES AND PROBLEM DESCRIPTION

2.1 IISS Background

In order to introduce the *integral input-to-state stability (IISS)*, we take the corresponding nonlinear systems of the form

$$\dot{\mathbf{x}} = f_p(\mathbf{x},\mathbf{u}),\ \forall p \in \mathscr{P}, \tag{1}$$

where $\mathbf{x}(t)$ is the current value of the state vector evolving in $\mathbb{R}^n$ and $\mathbf{u} : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ is the measurable and locally essentially bounded input vector. Here, the family of locally Lipschitz vector fields $\mathscr{F} = \{f_p : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n,\ p \in \mathscr{P}\}$ are defined on the index set $\mathscr{P}$. Each element of $\mathscr{F}$ assumed to satisfy $f_p(0,0) = 0$, $\forall p \in \mathscr{P}$. Now, we introduce IISS for (1), which are uniform over the set $\mathscr{P}$. Note that, the following definition is a natural extension of the definition in Sontag (1998); Angeli et al. (2000) for autonomous systems.

Definition 1. (IISS). The family of systems (1) is said to be IISS, if there exist functions $\alpha \in \mathscr{K}_\infty$, $\beta \in \mathscr{KL}$ and $\gamma \in \mathscr{K}_\infty$, for every initial state $\mathbf{x}(0)$, every input $\mathbf{u}$ and $p \in \mathscr{P}$ such that the estimate

$$\alpha(|\mathbf{x}(t)|) \leq \beta(|\mathbf{x}(0)|,t) + \int_0^t \gamma(|\mathbf{u}(s)|)ds, \tag{2}$$

holds along all solutions of (1).

A first result of this property is that since $t \mapsto \int_0^t \gamma(|\mathbf{u}(s)|)ds$ is bounded on any bounded time interval, (2) ensures that no finite escape-time can occur, which ensures that solutions of (1) exist at all positive times.

Another consequence of IISS is that the system is globally uniformly asymptotically stable in the absence of inputs.

Definition 2. (0-GUAS). The family of systems (1) is said to be *globally uniformly asymptotically stable in the absence of inputs* (0-GUAS) if there exists $\beta \in \mathscr{KL}$ such that, for all $\mathbf{x}(0) \in \mathbb{R}^n$ and $p \in \mathscr{P}$, the solution of the input-free family of systems $\dot{\mathbf{x}} = f_p(\mathbf{x},\mathbf{0})$ satisfies

$$|x(t)| \leq \beta(|\mathbf{x}(0)|,t), \qquad \forall t \geq 0.$$

IISS goes beyond the stability property of the nominal system. It also ensures some robustness properties known as *bounded energy frequently bounded state (BEFBS)* and *bounded energy weakly converging state (BEWCS)*. Please note that we decided not to emphasize uniformity in these robustness properties in order to prevent the confusion with uniform bounded energy bounded state (UBEBS) property which is a similar property rephrased with function class formalism.

Definition 3. (BEFBS, BEWCS). The family of systems (1) is said to have the BEFBS property if there exists $\gamma \in \mathscr{K}_\infty$ such that, for all $\mathbf{x}(0) \in \mathbb{R}^n$, all measurable and locally essentially bounded input vectors $\mathbf{u} : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ and all $p \in \mathscr{P}$, the solution of the system (1) satisfies

$$\int_0^\infty \gamma(|\mathbf{u}(s)|)ds < \infty \quad \Rightarrow \quad \liminf_{t\to\infty} |\mathbf{x}(t)| = 0. \tag{3}$$

It is said to have the BEWCS property if there exists $\gamma \in \mathscr{K}_\infty$ such that, for all $\mathbf{x}(0) \in \mathbb{R}^n$, all measurable and locally essentially bounded input vectors $\mathbf{u} : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ and all $p \in \mathscr{P}$, the solution of the system (1) satisfies

$$\int_0^\infty \gamma(|\mathbf{u}(s)|)ds < \infty \quad \Rightarrow \quad \liminf_{t\to\infty} |\mathbf{x}(t)| < \infty. \tag{4}$$

The strength of IISS lies in the fact that it guarantees not only 0-GAS, but also BEFBS and BEWCS, as proved in Angeli et al. (2004). This implication also holds true in switched systems (Haimovich and Mancilla-Aguilar, 2019).

One of the major reasons for the success of IISS lies on its Lyapunov characterization. Here is the necessity and sufficient condition for IISS in switched systems (Liberzon, 1999; Haimovich and Mancilla-Aguilar, 2019).

Proposition 4. (IISS Lyapunov Characterization). The family of systems (1) is IISS if and only if there exists a differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ and $\alpha \in \mathscr{PD}$ and $\alpha_1, \alpha_2, \gamma \in \mathscr{K}_\infty$ satisfying

$$\alpha_1(|\mathbf{x}|) \leq V(\mathbf{x}) \leq \alpha_2(|\mathbf{x}|) \tag{5a}$$

$$\dot{V}|_{(1)} := \nabla V \cdot f_p(\mathbf{x},\mathbf{u}) \leq -\alpha(|\mathbf{x}|) + \gamma(|\mathbf{u}|), \tag{5b}$$

for all $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$ and $p \in \mathscr{P}$.

2.2 ODE Model of Traffic Flow

In this study, we consider the ODE model of traffic flow. As its analogous model hydrodynamics, ODE model of traffic flow is also formulated based on vehicle conservation assumption. This assumption indicates that the change in the amount of vehicles should be equal to the difference of cumulative inflows and outflows of the corresponding road segment. Concerning the vehicle conservation, we have the following continuous-time ODE for a given road segment:

$$\dot{\rho} = \frac{1}{L}[q_1 - q_2], \tag{6}$$

The variables and the constants in Equation (6) are defined as follows:

- ρ denotes the number of vehicles at a given length of the road segment,
- q_1 represents the number of vehicles coming from the mainstream to continue into the mainstream of the road segment within a given time interval,
- q_2 denotes the number of vehicles leaving the mainstream to continue their travel on the mainstream of the road segment within a given time interval, and
- L is the length of the road segment.

2.3 Fundamental Diagram Setting

There are many relationships that are used to link traffic variables. Based on empirical findings, different types of relationships are defined between density-speed, density-flow, flow-speed, and etc.

In this paper, we use the triangular fundamental diagram for flow-density relationship. We adopt a speed-density relationship that is generally represented by a piecewise function:

$$v(\rho) = \begin{cases} v_f & , \quad \rho < \rho_{cr} \\ C\left(\frac{1}{\rho} - \frac{1}{\rho_{max}}\right) & , \quad \rho \geq \rho_{cr} \end{cases}$$

where v_f denotes the free-flow speed, ρ_{cr} represents the critical density and C is a sensitivity parameter. The corresponding flow-density relationship of the traffic flow can be obtained by applying $q = v\rho$

$$q(\rho) = \begin{cases} v_f \rho & , \quad \rho < \rho_{cr} \\ C\left(1 - \frac{\rho}{\rho_{max}}\right) & , \quad \rho \geq \rho_{cr}. \end{cases}$$

With continuity assumption of $v(\cdot)$ and $q(\cdot)$, one may take the sensitivity parameter as

$$C = \frac{v_f \rho_{cr} \rho_{\max}}{\rho_{\max} - \rho_{cr}}.$$

It is now possible to express the mainstream outflow dynamics as:

$$q_2(\rho) = \begin{cases} v_f \rho & , \quad \rho < \rho_{cr} \\ C\left(1 - \frac{\rho}{\rho_{max}}\right) & , \quad \rho \geq \rho_{cr}. \end{cases} \tag{7}$$

2.4 Variable Speed Limit Formulation

In order to apply VSL, let us first define the VSL rate $u_{VSL} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. VSL rate is reflected by altering the static free-flow speed as follows:

$$v_f(u_{VSL}) = v_f^* \cdot u_{VSL} \tag{8}$$

where v_f^* is the free-flow of the non-VSL case.

Summing up, we have the following state space representation for (6) by the fundamental diagram setting for the mainstream outflow dynamics (7) and VSL formulation (8):

$$\dot{\rho} = \begin{cases} -\frac{v_f^*}{L}\rho \cdot u_{VSL} + \frac{1}{L}q_1 & , \rho < \rho_{cr} \\ -\frac{C^*}{L}\left(1 - \frac{\rho}{\rho_{max}}\right) \cdot u_{VSL} + \frac{1}{L}q_1 & , \rho \geq \rho_{cr} \end{cases} \tag{9}$$

where

$$C^* = \frac{v_f^* \rho_{cr} \rho_{\max}}{\rho_{\max} - \rho_{cr}}.$$

2.5 State Feedback Controller Design

In this study, we consider the state feedback controller of the form

$$u_{VSL}(\rho) = \begin{cases} 1 & , \rho < \rho_{cr} \\ \text{sat}\left(\rho\left(1 - \frac{\rho}{\rho_{max}}\right); \frac{v_{min}}{v_f^*}, 1\right) & , \rho \geq \rho_{cr} \end{cases} \tag{10}$$

where $\text{sat}(\cdot; a, b)$ is the saturation function to be used in VSL operation as defined before. For sake of simplicity, we drop a and b in the defined saturation function and use it as $\text{sat}(\cdot) := \text{sat}(\cdot; a, b)$ throughout the paper.

The selection of the state feedback of the form (10) for the congested flow case ($\rho \geq \rho_{cr}$) roots back to the study of Gutman (1981). In that paper, it is shown that a quadratic state feedback of a form that turns the dissipation into a full quartic form stabilizes the CLS of the bilinear system. To that aim, we consider a quadratic feedback with a difference in input saturation. Even though the input saturation deforms the dissipation inequality to ensure ISS, we will show that we still can guarantee IISS by having a class $\mathcal{PD}$ dissipation in Section 3.

The corresponding CLS is therefore

$$\dot{\rho} = \begin{cases} -\frac{v_f^*}{L}\rho + \frac{1}{L}q_1 & , \rho < \rho_{cr} \\ -\left(\frac{C^*}{L}\right)\left(1 - \frac{\rho}{\rho_{max}}\right) \text{sat}\left(\rho\left(1 - \frac{\rho}{\rho_{max}}\right)\right) + \frac{1}{L}q_1 & , \rho \geq \rho_{cr}. \end{cases} \tag{11}$$

As our aim is to design a state feedback controller of the form (10) such that the CLS (11) is IISS, we express our main result in the present paper as in the following.

3. MAIN RESULT

Our main result is the following.

Theorem 5. The CLS (11) is IISS.

Proof. Let us first define the function $V(\rho) = \ln(1+\rho^2)$. It is trivial that V satisfies (5a). In order to show that the CLS (11) is IISS, we consider two cases.

- **Case 1:** First consider the case $\rho < \rho_{cr}$. Deriving V along the solutions of (11) yields

$$\dot{V}|_{(11)} = \frac{2\rho\dot{\rho}}{1+\rho^2} = -\frac{(2v_f^*/L)\rho^2}{1+\rho^2} + \frac{1}{L}\frac{2\rho}{1+\rho^2}q_1. \tag{12}$$

Observing that $\left|\frac{2\rho}{1+\rho^2}\right| \leq 1$, we obtain

$$\dot{V}|_{(11)} \leq -\eta_1(|\rho|) + \gamma(|q_1|) \tag{13}$$

where $\eta_1(s) = \frac{(2v_f^*/L)s^2}{1+s^2}$ and $\gamma(s) = (1/L)|s|$. One can see that $\eta_1 \in \mathscr{PD}$ and $\gamma \in \mathscr{K}_\infty$ so that the CLS (11) is IISS for the case $\rho < \rho_{cr}$.

- **Case 2:** Let us consider the case $\rho \geq \rho_{cr}$. By deriving V along the solutions of (11), we have

$$\dot{V}|_{(11)} = \frac{2\rho}{1+\rho^2}\dot{\rho} = -\frac{\left(\frac{2C^*}{L}\right)\rho\left(1-\frac{\rho}{\rho_{max}}\right)sat\left(\rho\left(1-\frac{\rho}{\rho_{max}}\right)\right)}{1+\rho^2} + \frac{1}{L}\frac{2\rho}{1+\rho^2}q_1 \tag{14}$$

Again, using the same fact that $\left|\frac{2\rho}{1+\rho^2}\right| \leq 1$, one can write

$$\dot{V}|_{(11)} \leq -\eta_2(|\rho|) + \gamma(|q_1|) \tag{15}$$

where $\eta_2(s) = \frac{(2C^*/L)s(1-s/\rho_{max})\cdot sat(s(1-s/\rho_{max}))}{1+s^2}$ and $\gamma(s) = (1/L)|s|$. One can also observe that $\eta_2 \in \mathscr{PD}$ and $\gamma \in \mathscr{K}_\infty$ so that the CLS (11) is IISS for the case $\rho < \rho_{cr}$.

Now, in order to conclude that the CLS (11) is IISS for all ρ, we define $\eta(s) = \min\{\eta_1(s), \eta_2(s)\}$ for all $s \geq 0$. Note that, $\eta(0) = 0$ and $\eta(s) > 0$ as $s > 0$ by definition. Recall that, $\min : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, $\min : (x,y) \mapsto \min(x,y)$ is continuous. As we know that continuity is preserved under composition, η is continuous. Thus, $\eta \in \mathscr{PD}$ and, we have

$$\dot{V}|_{(11)} \leq -\eta(|\rho|) + \gamma(|q_1|), \tag{16}$$

for all ρ, which tells that $V(\rho) = \ln(1+\rho^2)$ is a common IISS Lyapunov functional for the CLS (11) implying that the CLS (11) is IISS. □

In the next section, we present the validity of Theorem 5 through two numerical examples.

4. NUMERICAL EXAMPLES

In this section, we present two numerical examples to illustrate the validity of Theorem 5. Hence, we analyze the behavior of the CLS (11) and compare it with (9) with $u_{VSL}(t) \equiv 1$. To this aim, we choose

$$L = 1\ km,\ \rho(0) = 0\ veh/km,\ \rho_{max} = 200\ veh/km, \\ \rho_{cr} = 20\ veh/km,\ v_f^* = 100\ km/h,\ v_{min} = 50\ km/h,\ . \tag{17}$$

We first choose

$$q_1(t) = q_{1,cap} \cdot exp((t-t_{peak})^2/2\sigma)$$

with $q_{1,cap} = 1000\ veh/h$, $t_{peak} = 10\ h$ and $\sigma = 10$ to introduce a peak demand scenario for the road segment. One can see that q_1 satisfies the bounded energy assumption:

$$\int_0^{2t_{peak}} q_1(\tau)d\tau \leq \int_{-\infty}^{\infty} q_1(\tau)d\tau \leq q_{1,cap}\sqrt{2\pi}\sigma < \infty$$

In order to demonstrate the effects of the proposed VSL strategy (8) with (10), we have considered two cases. In the first case, we take (9) with $u_{VSL}(t) \equiv 1$ for all $t \in [0, 2t_{peak}]$, which represents the case that no VSL is applied to (9), namely No-VSL. In the case that we take the CLS (11), which represents the case that the VSL strategy (8) is applied with (10). According to the parameters and inputs that we introduce, the simulations show in Figure 1 that the CLS (11) is IISS, and as a consequence satisfies the BEFBS and BEWCS properties.

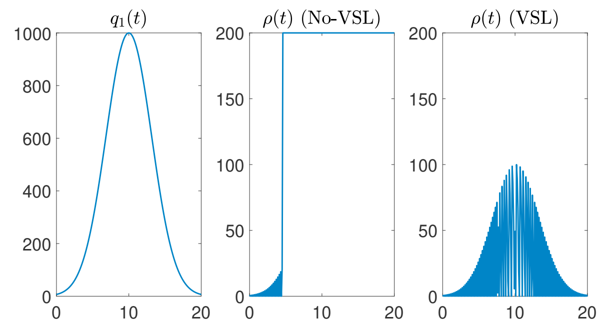

Fig. 1. Simulation Results for No-VSL and VSL Cases under the Input satisfying Bounded Energy Assumption.

As a second example, we select

$$q_1(t) = \frac{q_{1,cap}}{((t-t_{peak})^2+1)}$$

with similar network characteristics as in (17) which also satisfies the bounded energy assumption by

$$\int_0^{2t_{peak}} q_1(\tau)d\tau \leq \int_{-\infty}^{\infty} q_1(\tau)d\tau \leq q_{1,cap}\pi < \infty.$$

As in the previous selection of $q_1(\cdot)$, we consider (9) with $u_{VSL}(t) \equiv 1$ for all $t \in [0, 2t_{peak}]$ and the CLS (11). The simulations in Figure 2 demonstrate that the CLS (11) is IISS, which tells us that it also owns the BEFBS and BEWCS properties.

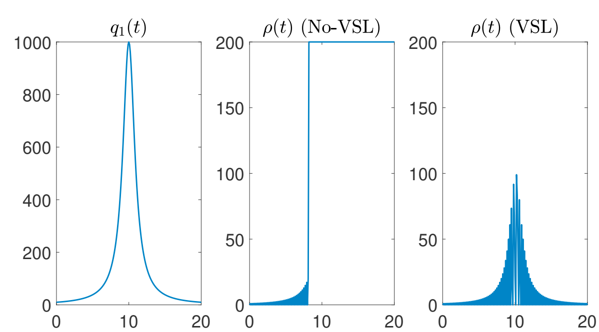

Fig. 2. Simulation Results for No-VSL and VSL Cases under the Input satisfying Bounded Energy Assumption.

5. CONCLUSIONS

In the present study, we introduced the variable speed limit controller to ensure integral input-to-state stability of the traffic state by using saturated feedback. We used the ordinary differential equations model to present the traffic state, and associated it with a two-phase fundamental diagram. Two-phase fundamental diagram implementation turned out to be a state-dependent switched ordinary differential equation, and hence, the integral input-to-state stability of the closed-loop system was guaranteed by a common Lyapunov function. Finally, we demonstrated the robustness properties bounded energy frequently bounded state and bounded energy weakly converging state which are the characterizations of the integral input-to-state stability through a numerical example.

As a future extension of our study, we aim to obtain the conditions for multiple segments. We will be dealing with a cascade structure, which is shown to preserve IISS under some growth restrictions of the dissipation inequalities of the subsystems in Chaillet and Angeli (2008). We aim to figure out the validity of the results through micro-simulation, where we expect to see the effects of the proposed controller in terms of various measures on a real network using real data. Last but not least, we will investigate as well the performance of the proposed controller under lane closure phenomena and through mixed traffic scenarios involving various setups designed Erdağı et al. (2019); Silgu et al. (2019) for connected and automated vehicles. .

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