16TH INTERNATIONAL GEOMETRY SYMPOSIUM

JULY 4-7, 2018

MANISA, TURKEY

ABSTRACTS BOOK

MANİSA CELAL BAYAR UNIVERSITY

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16TH INTERNATIONAL GEOMETRY SYMPOSIUM

ABSTRACTS BOOK

Proceedings of the 16th International Geometry Symposium

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Jointly Organized By
Manisa Celal Bayar University

FOREWORD

Hosted by Manisa Celal Bayar University between July 4-7, 2018, the 16th International Geometry Symposium was held in Manisa, a city of learning throughout history. Undergraduate students aiming to do scholarly studies as well as new researchers had a great opportunity of getting together with highly experienced researchers. In light of scientific developments in Geometry, presentations were made, and discussions were held, thus paving the way for new research. All the studies in this booklet were peer-reviewed, and then brought up to the attention of the audience. Through their presentations, the keynote speakers helped the researchers explore some new ways of thinking.

In making our event happen, special thanks go to the following: Office of the Rector of Manisa Celal Bayar University for letting us use its facilities, office of the Governor of Manisa, Yunusemre Municipality, Şehzadeler Municipality, TÜBİTAK (2223-B), Ziraat Bankası A.Ş., Pegem Akademi and all our collagues and students who worked with us to make this symposium a success.

Prof. Dr. Mustafa KAZAZ

Head of the Organizing Committee

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Invited Speakers

Modern Topics in The Geometry of Einstein Spaces

Adela Mihai

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Abstract

Given a compact C^∞-differentiable manifold M, dim $M = n$, the following question arises (René Thom, Strasbourg Math. Library, 1958, see [1]):

Are there any best (or nicest, or distinguished) Riemannian structures on M?

A good candidate for such a privileged metric on a given manifold is an Einstein metric, if one considers the *best metrics* those of constant sectional curvature. More precisely, if the dimension of the manifold is greater than 2, a good generalization of the concept of constant sectional curvature might be the notion of *constant Ricci curvature* [1].

A Riemannian manifold (M, g) of dimension $n \geq 3$ is called an *Einstein space* if $Ric = \lambda \cdot id$, where trivially $\lambda = \kappa$, with k the (normalized) *scalar curvature*; in this case one easily proves that $\lambda = \kappa = constant$.

We recall the fact that any 2-dimensional Riemannian manifold satisfies the relation $Ric = \lambda \cdot id$, but the function $\lambda = \kappa$ is not necessarily a constant. It is well known that any 3-dimensional Einstein space is of constant sectional curvature. Thus, the interest in Einstein spaces starts with dimension $n = 4$.

Singer and Thorpe [4] discovered a symmetry of sectional curvatures which characterizes 4-dimensional Einstein spaces. Later, this result was generalized by B.Y. Chen e.a., in [2], to Einstein spaces of even dimensions $n = 2k \geq 4$. We established in [3] curvature symmetries for Einstein spaces of arbitrary dimension $n \geq 4$.

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Some Characterization Theorems for Lightlike Hypersurfaces of Semi-Riemannian Manifolds Admitting A Semi-Symmetric Non-Metric Connection

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Abstract

We study lightlike hypersurfaces of semi-Riemannian manifolds admitting a semisymmetric non-metric connection whose structure vector field is tangent to the hypersurface. We obtain conditions for the induced Ricci type tensor of a lightlike hypersurface of such semi-Riemannian manifolds to be symmetric, which in general is not symmetric and find a characterization theorem for a lightlike hypersurface to be screen conformal. We also find conditions for a lightlike hypersurface of a semi-Riemannian space form to be Ricci flat and show that the null sectional curvature of lightlike hypersurface also vanishes. Finally, we obtain Chen-like inequalities on lightlike hypersurfaces of a semi-Riemannian manifold admitting semi-symmetric nonmetric connection.

Branched Covering Surfaces

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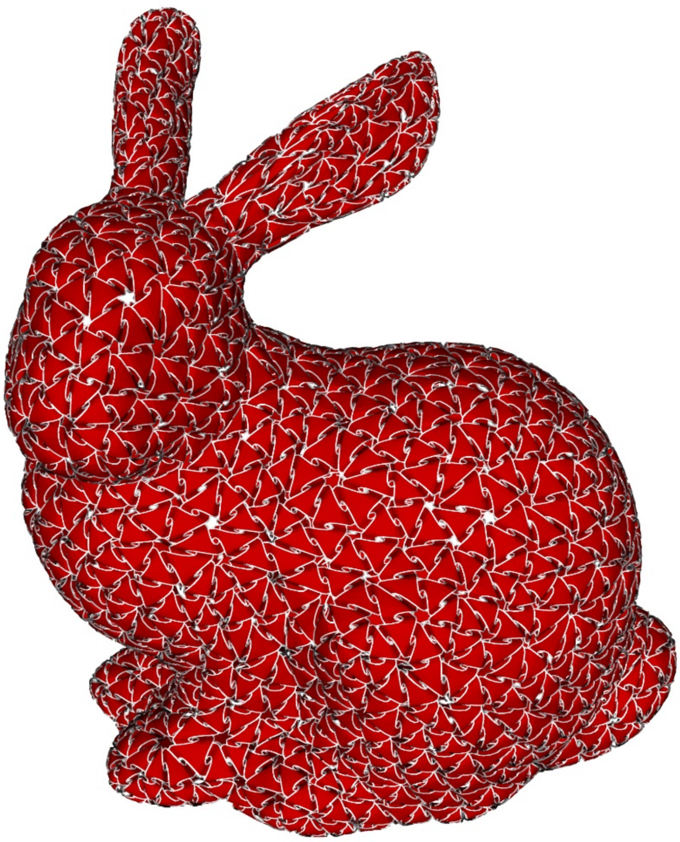
Abstract

Multivalued functions and differential forms naturally lead to the concept of branched covering surfaces and more generally of branched covering manifolds in the spirit of Hermann Weyl's book "Die Idee der Riemannschen Fläche" from 1913. This talk will illustrate and discretize basic concepts of branched (simplicial) covering surfaces starting from complex analysis and surface theory up to their recent appearance in geometry processing algorithms and artistic mathematical designs. Applications will touch differential based surface modeling, image and geometry retargeting, global surface and volume remeshing, and novel weaved geometry representations with recent industrial applications.



The Principle of Transference Between Real and Dual Lorentzian Spaces and Dual Lorentzian Angles

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Abstract

Dual number algebra is powerful mathematical tool for the computer-aided geometrical design (CAGD), the kinematic and dynamic analysis of spatial mechanism, robotic and human body motion analysis.

In this study we define the fundamental concepts of the dual Lorentzian space, the principle of transference and the eight dual Lorentzian angles between oriented timelike, spacelike and lightlike lines in the 3-dimensional Lorentz space R_1^3. According to this theory; $\tilde{H}_0^2$ is the new dual model of the hyperbolic geometry from non–Euclidean geometries in the dual Lorentzian space D_1^3 and it contains the real unit hyperbolic sphere H_0^2 (real hyperbolic plane or real hyperboloid model). The unit dual Lorentzian sphere $\tilde{S}_1^2$ is dual de-Sitter Space-Time containing the real de-Sitter Space-Time S_1^2 in the 3- dimensional Lorentz space R_1^3. Also, the dual lightlike cone $\tilde{\Lambda}^2$ contains real lightlike cone Λ^2. The dual Lorentzian space and the principle of transference are power tools for the geometries of the curves (Lorentzian ruled surfaces) on the dual Lorentzian quadrics $\tilde{H}_0^2$, $\tilde{S}_1^2$, $\tilde{\Lambda}^2$; the computer aided Lorentzian geometric design (CALGD), the dual Lorentzian spherical kinematic (DLSK), dual hyperbolic spherical kinematic (DHSK), dual lightlike cone kinematic (DLCK), dual Lorentzian spatial kinematic (DLSK), the dual Lorentzian spatial mechanism (DLSM), Dual Lorentzian robotic (DLR), and workers on the theories of special and general relativity.

We hope that this work not only concerns Lorentz geometry and Relativity workers but also directly related to astronomy with many fields of engineering.

Keywords: Dual Lorentzian space; The Principle of Lorentzian Transfer; Unit dual Lorentzian sphere; Unit dual hyperbolic Sphere; Dual Lightlike cone; Dual Lorentzian Angles.

Split Quaternions and Hyperbolic Spinor Representation of Transformations

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Abstract

In this study, a historical continuum of development of spinor theory is summarized. Although the name of spinor was first used by physicists, its mathematical form was firstly introduced by Cartan in 1913. From past to present the literature on spinor becomes substantially extensive since it has applications to electron spin, quantum mechanics, electronic magnetic field and electric transmission lines.

After revisiting the geometrical and analytical description of a spinor, the equations for the rotation of spinors are given. The spinors have an essential role in numerous scientific area instead of the vectors. In this regard, one of the aim of this study is to give the hyperbolic spinor representations of Frenet, Bishop and Darboux equations for a non-null curve in $\mathbb{R}_1^3$, since these equations have vital importance in differential geometry.

Moreover, a different perspective is developed for the relationship between the rotations in $\mathbb{R}_1^3$ and their corresponding split quaternions in $\mathbb{E}_2^4$ making use of hyperbolic spinors. As a consequence, this treatment provides a natural extension of split quaternions and allows us to compare well known concepts of Lorentzian Kinematics with newly introduced.

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Abstracts
of
Oral Presentations

Geometric Characterization of Surfaces on Time Scales

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Abstract

The mimetic discretization of differential operators is a process that maintains the fundamental properties of continuous differential operators. The main goal in this process is to ensure that the protected properties are maximized and, if not, to give up most of the properties. Geometric partial differential equations are very important to find discrete analogues of differential geometric operators such as mean curvature, Gaussian curvature and Laplace Beltrami, which are defined by surface normal. The de facto methods, such as finite differences and finite elements, are directly related to the discretization of the equation system. A disadvantage of this method is that the selected discretization process may have little connection to the underlying physical problem. In this study, we use mimetic discretization property of time scale calculus to obtain dynamic surfaces which involve discrete and continuous parts together. By introducing the concept of symmetric dynamic differentiation, we obtain the normal fields and study the Gaussian curvature of such surfaces.

Keywords: Time Scale Calculus; Discrete Manifolds; Normal Field Approximation; Discrete Gaussian Curvature.

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Curvatures of Clusters in Complex Networks

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Abstract

Networks are mathematical structures that are universally used to describe a large variety of complex systems such as the brain or the Internet. Characterizing the geometrical properties of these networks has become increasingly relevant for routing problems, inference and data mining. In real growing networks, topological, structural and geometrical properties emerge spontaneously from their dynamical rules. In this study, we present a framework to determine Ricci and constant curvatures of the granular structure of networks which are dense cluster of complex networks.

Keywords: Discrete Manifolds; Network Analysis; Geometric Computation.

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The Weighted Ricci Curvature and Compactness on Finsler Manifolds

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Abstract

Let (M,F) be a forward complete and connected Finsler manifold of dimensional $n \geq 2$. By using the weighted Ricci curvature, we prove some Cheeger-Gromov-Taylor type compactness theorems. To obtain these results, we use the index form of a minimal unit speed geodesic segment, Bochner-Weitzenböck formula and Hessian comparison theorem.

Keywords: Diameter estimate; Distortion; S-curvature; Finsler manifold; Weighted Ricci curvature.

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Horizontal Lifts of Vector Fields to the Semi-tensor Bundle

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Abstract

Using the fiber bundle M over a manifold B, we define a semi-tensor (pull-back) bundle tB of type (p,q) [6]. (For definition of the pull-back bundle, see for example [[1],[2],[3],[4],[5],[7],[8],[9]]). We consider horizontal lifting problem of projectable geometric objects on M to the semi-tensor (pull-back) bundle tB of type (p,q). We note that semi-tensor bundle were examined in ([2],[12],[14]). The main purpose of this paper is to study the behaviour of horizontal lifts of projectable vector fields and some operators for semi-tensor (pull-back) bundle tB of type (p,q) [10],[11],[13],[15],[16].

Keywords: Vector field; Horizontal lift; Pull-back bundle; Semi-tensor bundle.

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Diagonal Lift Problems in the Semi-tangent Bundle

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Abstract

The main purpose of this paper is to study the behavior of diagonal lifts of affinor (tensor of type (1,1)) fields on cross-sections for pull-back (semi-tangent [7]) bundle t*B. (For definition of the pull-back bundle, see for example [[1],[2],[3],[4],[5]]). In this context cross-sections in semi-tangent (pull-back) bundle tM of tangent bundle TM by using projection (submersion) of the cotangent bundle T*M can be also defined [6],[8]. Also, a new example for good square [9] presented in this paper.

Keywords: Vector field; Complete lift; Diagonal lift; Pull-back bundle; Cross-section; Semi-tangent bundle.

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Semi-Tensor Bundle and the Complete Lift of Vector Fields

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Abstract

Using the fiber bundle M over a manifold B, we define a semi-tensor (pull-back) bundle tB of type (p,q). We consider complete lifting problem of projectable vector fields on M to the semi-tensor bundle tB of type (p,q).

Keywords: Vector field; Complete lift; Pull-back bundle; Semi-tensor bundle.

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A Study on Mannheim Offsets of Ruled Surfaces

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Abstract

From past to present, the geometers have defined some different offsets of curves (or surfaces) for example the involute-evolute, Bertrand, Mannheim and Smarandache. Offsets of curves (or surfaces) generally more complicated than their progenitor curve (or surfaces). These curves also have many applications in gear industry and business. Moreover, the frame fields constitute an important subject while examining the differential properties of curves and surfaces. In this study, using Darboux frame $\{\mathbf{T},\mathbf{g},\mathbf{n}\}$ of ruled surface $\varphi(s,v)$, Mannheim offsets $\varphi^*(s,v)$ with Darboux frame $\{\mathbf{T}^*,\mathbf{g}^*,\mathbf{n}^*\}$ of $\varphi(s,v)$ are identified. Characteristic properties of Mannheim offsets $\varphi^*(s,v)$ as a striction curve, distribution parameter and orthogonal trajectory are investigated according to Darboux frame of $\varphi(s,v)$. The distribution parameters of ruled surfaces $\varphi_{\mathbf{T}^*},\varphi_{\mathbf{g}^*}$ and $\varphi_{\mathbf{n}^*}$ are given.

Keywords: Mannheim; Ruled surface; Darboux frame.

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Inverse of Dual Quaternion Matrices and Matlab Applications

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Abstract

Majernik [2] introduced the dual quaternions which form an algebraic ring possible suitable for expressing the Galilean transformation. Besides some important algebraic properties of the dual quaternions are expressed in [2]. Moreover, matrix representation of dual quaternions, Euler and De-Moivre formulas for dual quaternions are examined in [3]. The inverse of real and complex matrices is great importance. Cohen and Leo [1] saw that it is very difficult to generalize the inverse method of adjoint matrix to quaternion matrices. In this study, we investigated inverse of dual quaternion matrices in three different ways. One of them is by using conjugate of the dual quaternion matrix and inverse of the real part of the dual quaternion matrix. Other methods are the real matrix representation of dual quaternion matrix and the adjoint matrix. In calculating the inverse of real and complex matrices, adjoint matrix is important. Many authors failed to generalize this method to quaternion matrices. But we generalized this method for dual quaternion matrices. Besides, we found the inverse of dual quaternion matrix by using first method with Matlab. Finally, we obtained the same results by applying the inverse we defined in three different ways to the same matrices.

Keywords: Dual quaternion matrices; Inverse; Adjoint matrix.

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Determinant of Dual Quaternion Matrices and Matlab Applications

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Abstract

Several authors worked on algebraic properties of quaternion matrices [1-3]. In 1997, Zhang [4] gave a brief survey on quaternions and matrices of quaternions. He studied in this subject by converting a matrix of quaternions into a pair of complex matrices. Besides, he defined q-determinant of a quaternion matrix by the determinant of complex adjoint matrix. When the literature is examined in a wide scale, no study has been found on dual quaternion matrices. In this study, we investigated determinant of dual quaternion matrices in three different ways. Firstly, we defined the determinant of dual quaternion matrices. This determinant is the same as the usual determinant and has the same properties of the usual determinant. But it was difficult to directly calculate the determinant. Therefore, we found two original new methods to directly calculate the determinant. One of them is by using trace of real matrices and the other is by using the determinant of real matrices. So, we used Matlab and easily calculated the determinant of dual quaternion matrices by these two methods. Finally, we obtained the same results by applying the determinant we defined in three different ways to the same matrices.

Keywords: Dual quaternion matrices; Determinant; Trace.

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Reidemeister Torsion of Orientable Punctured Surfaces

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Abstract

Let Σg,n,b denote the orientable surface obtained from the closed orientable surface Σg of genus $g \geq 2$ by deleting the interior of $n \geq 1$ distinct topological disks and $b \geq 1$ points. Using the notion of symplectic chain complex, we establish a formula for computing Reidemeister torsion of the surface Σg,n,b in terms of Reidemeister torsion of the closed surface Σg, Reidemeister torsion of disk, and Reidemeister torsion of punctured disk.

Keywords: Reidemeister torsion; Symplectic chain complex; Orientable punctured surfaces.

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Moving Coordinate System and Euler-Savary Formula under One-Parameter Planar Homothetic Motions in Generalized Complex Number Plane C_J

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Abstract

In this study, we firstly give the basic notations of the generalized complex number plane ($\mathfrak{p}$-complex plane) $C_{\mathfrak{p}}$. Then, we introduce the one-parameter planar homothetic motions C_J / C'_J in $\mathfrak{p}$-complex plane C_J such that $C_J \subset C_{\mathfrak{p}}$ by examining the velocities, accelerations and pole points. Besides, we discuss the relations between absolute, relative, sliding velocities (accelerations) and pole curves under these motions. Moreover, three generalized complex number planes, of which two are moving and the other one is fixed, are considered and a canonical relative system for one-parameter planar homothetic motion in C_J is defined. Euler-Savary formula, which gives the relationship between the curvatures of trajectory curves, during the one-parameter homothetic motions, is obtained with the aim of this canonical relative system.

Keywords: Generalized complex number plane; One-parameter planar homothetic motion; Kinematics; Euler-Savary formula.

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Pythagorean Hodograph λ μ - Bezier Like Curve with Two Shape Parameters

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Abstract

Pythagorean Hodographs (PH) curves have an important role in geometric modeling. Also offset curves are widely used in applications, such as textile industry, shoe industry and car body industry etc. Additionally, Bezier curves are very important for computer aided design and PH Bezier curves have no real inflection point. But, Bezier curves cause some difficulties in obtaining the desired shape because of their polynomial nature. For overcoming this problem, λμ-Bezier like curves with two shape parameters have been developed as alternatives to Bezier and B-splines curves. Therefore λμ-Bezier like curves with two shape parameters are suitable for computer-aided geometric design applications.

The purpose of our talk is to introduce PH λμ-Bezier like curves with two shape parameters which are similar to PH Bezier curves in [3]. Considering to implications of PH property for exponential Bezier basis function, we give more detailed analysis of these curves and we mainly obtain coordinates of control points of PH λμ-Bezier like curves with two shape parameters starting for an orijinal λμ-Bezier like curve. Also, we investigate two λμ-Bezier like curves with two shape parameters which are connected to G^2 end conditions.

Keywords: Pythagorean hodographs; Hodographs; λμ - Bezier like curve with two shape parameters, G^2 end conditions.

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Cheng-Yau Operator and Gauss Map of Rotational Hypersurfaces in the Four Dimensional Euclidean Space

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Abstract

We consider rotational hypersurface in the four dimensional Euclidean space E^4. We study the Gauss map G of rotational hypersurface in E^4 with respect to the so-called Cheng-Yau operator L_1 acting on the functions defined on the hypersurfaces. We obtain the classification theorem that the only rotational hypersurface with Gauss map G satisfying $L_1 G = AG$ for some 4×4 matrix A.

Keywords: Euclidean spaces; Cheng-Yau Operator; Finite type mappings; Rotational hypersurfaces; L_k-operators.

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Helicoidal Hypersurfaces in the Four Dimensional Minkowski Space

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Abstract

Helicoidal hypersurfaces in the four dimensional Minkowski space are defined. There are three types, depending on the axis of rotation. Equations for the Gaussian and mean curvature are derived and many examples of the various types of hypersurfaces are given. A theorem classifying the helicoids with timelike axes and ΔH=AH is obtained.

Keywords: Helicoidal hypersurface; Laplace-Beltrami operator; Gaussian curvature; mean curvature; Minkowski 4-space.

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Surface Growth Kinematics in Galilean Space

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Abstract

In this study, we investigate the mathematical framework to model the kinematics of the surface growth of objects such as some crustacean creatures in Galilean 3-Space. For this, the growth kinematics of the different species of these creatures is obtained by applying a method with a system of differential equations. Using the analytical solutions of this system, various surface examples, including some seashells are provided and the shapes of these surfaces are illustrated.

Keywords: Alternative moving frame; Accretive growth; Darboux vector; General helix.

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Möbius-Type Hypersurface in 4-Space

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Abstract

Möbius-type hypersurface in the four dimensional Euclidean space is defined. Equations for the Gaussian and mean curvature are derived and some examples of hypersurface are given.

Keywords: Möbius-type hypersurface; Gaussian curvature; mean curvature; Euclidean 4-space.

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Mappings for Generating Rational Helices

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Abstract

In this study, we define a mapping that generates rational helices in n-dimensional Euclidean space from general helices in n-dimensional Euclidean space. Also, we define another mapping that generates rational helices in (n+1)-dimensional Euclidean space from general helices in n-dimensional Euclidean space.

Keywords: Mapping; Rational Helix; Involution.

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A Note on Isometric Immersions into $\mathbb{S}^n \times \mathbb{R}$

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Abstract

Let $f: M^n \hookrightarrow \mathbb{S}^m \times \mathbb{R}$ be an isometric immersion and ∂_t denote the unit vector field tangent to the second factor. Then, the equation

$$\partial_t = f_*(T) + \eta$$

a tangent vector field T on M and a normal vector field η along f. f is said to belong to class $\mathcal{A}$ if T is a principle direction of all of its shape operators, [2, 3]. In this work, after we give a short survey on immersions which belongs to class $\mathcal{A}$, we give a result about immersions belong to class $\mathcal{A}$ satisfying a restriction on η. We also would like to discuss a natural generalization of class $\mathcal{A}$ immersions.

Keywords: Principle directions; Class $\mathcal{A}$ immersions; Product spaces.

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On Geodesics of the Tangent and Normal Surfaces Defined by TN-Smarandache Curve According to Frenet Frame

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Abstract

In this talk, we first obtain tangent and normal surfaces defined by TN-Smarandache curve according to Frenet frame in Euclidean space. We then investigate the geodesic equations for such surfaces by calculating Christoffel symbols. We also give examples to illustrate our results. Furthermore, we examine similar problems for other ruled surfaces defined by TN-Smarandache curve.

Keywords: Christoffel symbols; Geodesic equations; Frenet frame; Euclidean space.

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Some Properties of Bicomplex Tribonacci and Tribonacci-Lucas Numbers

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Abstract

In this present paper, by using the well-known identities related to the Tribonacci and the Tribonacci-Lucas numbers to obtain the relations between a new generation of the bicomplex Tribonacci and Tribonacci-Lucas numbers we present a detailed study of the bikomplex Tribonacci and Tribonacci-Lucas numbers and obtain some properties of them.

Keywords: Tribonacci and Tribonacci-Lucas numbers; Bicomplex Tribonacci and Tribonacci-Lucas numbers.

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Real Matrix Representations for Tessarine Numbers

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Abstract

In this present paper, a tessarine number is described in four- dimensional space and its a variety of algebraic properties is presented. In addition, Pauli-spin matrix elements corresponding to base the real matrices forms of tessarine numbers are obtained. Like i and j in two different spaces are defined terms of Euler's formula. Also, the paper gives some formula and facts about the concepts of conjugate and norm which are not generally known for tessarine numbers.

Keywords: Tessarine numbers; Tessarine matrices; Euler's formula; Pauli-spin matrix elements.

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Some Characterizations for Ruled Surface Pair Generated by Natural Lift Curve in Dual Space

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Abstract

In this study, firstly, the Frenet vector fields $\overline{T}, \overline{N}, \overline{B}$ of the natural lift $\bar{\alpha}$ of a curve α are calculated in dual space. Secondly, we obrained striction lines and distribution parameters of ruled surface pair generated by the natural lift $\bar{\alpha}$. Finally, those notions are compared with each other for $\bar{\alpha}$ in dual space.

Keywords: Natural Lift; Striction Line; Distribution Parameter; Ruled Surface.

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Approximating the Definite Integral Computation: A Novel Method

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Abstract

This work uses Bezier curves to form an approximation of the integrals whose solution cannot be fully computed. For approximate solutions of such integrals, there are some methods: "Midpoint rule", "Trapezoidal rule" and "Simpson's rule". In this work, the method that gives the best approach among these rules is examined. So, the Bezier approach is shown as the best method. Results are illustrated via the tables.

Keywords: Bezier curves; Mean Value Theorem; Blossoming.

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Spherical Bézier Curves and Ruled Surfaces

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Abstract

This paper considers a kind of design of a ruled surface. The design interconnects some concepts from the fields of computer aided geometric design (CAGD) and kinematics. A dual unit spherical Bézier-like curve on the dual unit sphere (DUS) is obtained by a novel method with respect the control points. A dual unit spherical Bézier-like curve corresponds to a ruled surface by using transference principle Study [19] and closed ruled surfaces are determined via control points and also, integral invariants of these surfaces are investigated. Finally, the results are illustrated by several examples.

Keywords: Kinematics; Bézier curves; E. Study's map; Spherical interpolation.

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Recent Developments on Magnetic Curves

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Abstract

This presentation includes a brief selection of results we obtained so far in the study of magnetic curves as well as some future work. As the study of magnetic curves was intensively developed in Kaehler manifolds, where the Kaehler 2-form is closed and hence defines a magnetic field, we investigate the magnetic curves in Sasakian and cosymplectic manifolds [1] and [2]. In particular, we generalized the classification of magnetic curves in product spaces of type M2xR, as it was already done in S2xR and Euclidean space E3. See also [6]. For higher dimensions, we showed in [4] that magnetic curves in R2N+1 have order 5. The properties of closeness and periodicity were investigated in [5] for magnetic curves on the 3-torus. Concerning the magnetic curves in quasi-Sasakian manifolds, we approached the problem only in the 3-dimnesional case [3], as it is well known that the geometry of quasi-Sasakian 3-manifolds is rather special, the arbitrary dimensions case remaining still open.

Keywords: Magnetic curve; Periodic magnetic curve; Sasakian manifold.

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Conformal Riemannian Maps in Complex Geometry

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Abstract

Conformal semi-invariant Riemannian maps from Kaehler manifolds to Riemannian manifolds are investigated. We give some examples and study the geometry of leaves of certain distributions. Also, we investigate certain conditions for such maps to be horizontally homothetic map. Moreover, we introduce special pluriharmonic maps and obtain characterizations by using this notion.

Keywords: Riemannian maps; Conformal Riemannian maps; Conformal semi-invariant Riemannian maps.

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On Grassmann Images of Rotational Surfaces in E^4

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Abstract

In the present study we consider rotational surfaces in 4-dimensional Euclidean space E^4. We obtain necassary and sufficent conditions for these kind surfaces whose Grassmann images are isometric to product of two spheres.

Keywords: Spherical Products; Rotational Surfaces; Grassmann image.

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On Rotational Submanifolds in Euclidean Spaces

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Abstract

The rotational embedded submanifold was first studied by N. Kuiper as a submanifold in E^{n+d}. The generalized Beltrami submanifolds and toroidals submanifold are the special examples of these kind of submanifolds. In the present article, we consider 3-dimensional rotational embedded submanifolds in Euclidean 5-space E^5. We give some basic curvature properties of this type of submanifolds. Further, we obtained some results related with the scalar curvature and mean curvature of these submanifolds. As an application, we give an example of rotational submanifold in E^5.

Keywords: Rotational submanifolds; Scalar curvature; Mean curvature.

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A Study of Wintgen Like Inequality for Submanifolds in Statistical Warped Product Manifolds

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Abstract

In this paper, we study statistical manifolds and their submanifolds. We first construct two new examples of statistical warped product manifolds and give a method how to construct Kenmotsu-like statistical manifold and cosymplectic-like statistical manifold based on the existence of Kaehler-like statistical manifold. Then we obtain the general Wintgen inequality for statistical submanifolds of statistical warped product manifolds.

Keywords: Statistical manifold; Warped product manifold; The generalized Wintgen inequality.

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Yamabe Solitons on Three-Dimensional Normal Almost Paracontact Metric Manifolds

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Abstract

We study Yamabe solitons on three-dimensional para-Sasakian, paracosymplectic and para-Kenmotsu manifolds. Finally, we construct examples to illustrate the obtained results.

Keywords: Para-Sasakian manifold; Paracosymplectic manifold; Para-Kenmotsu manifold.

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Almost Cosymplectic Statistical Manifolds

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Abstract

This paper is a study of almost contact statistical manifolds. Especially this study is focused on almost cosymplectic statistical manifolds. We obtained basic properties of such manifolds. It is proved a characterization theorem and a corollary for the almost cosymplectic statistical manifold with Kaehler leaves. We also study curvature properties of an almost cosymplectic statistical manifold. Moreover, examples are constructed.

Keywords: Statistical manifold; Kaehler statistical manifold; Sasakian statistical manifold.

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The Golden Ratio and Finite Blaschke Products of Degree Two and Three

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Abstract

The golden ratio $\alpha = \frac{1+\sqrt{5}}{2}$ has many applications in geometry and modern research areas. A Blaschke product of degree *n* for the unit disc is a function defined by

$$B(z) = \beta \prod_{i=1}^{n-1} \frac{z-a_i}{1-\overline{a_i}z}$$

where $|\beta| = 1$ and $|a_i| < 1,\ 1 \le i \le n-1.$ It is well known that every Blaschke products *B* of degree *n* with $B(0) = 0$, is associated with a unique Poncelet curve. It is well-known that the Poncelet curve associated with a Blaschke product of degree *2* and *3* are a point and an ellipse (respectively). In this study, we investigate the relationships between any Blaschke product of degree 2 (resp. 3) and the golden ratio.

Keywords: Finite Blaschke products; Golden ratio.

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Similar Cartan Null Curves in Minkowski 4-space with Variable Transformations

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Abstract

This paper is a detailed study on similar null Cartan curves in Minkowski 4-space by considering the Cartan framed vectors. To do this, we introduce families of similar null Cartan curves in Minkowski 4-space with variable transformation μ_{β}^{α}. Using new Cartan framed vectors of these curves, we obtain some new relationships between with non-zero curvatures of the similar partner null curves and we give unit tangent vector L of C satisfies a vector differential equation of fourth order in Minkowski 4- space.

This recent work gives some formulas, facts and properties about similar null Cartan curves that are obtained by using Cartan framed vectors, which are not generally known.

Keywords: Minkowski Spacetime; Cartan frame; Similar null Cartan curves; Variable transformation.

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On the Special Geometry of Calabi-Yau Moduli Spaces

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Abstract

In order to obtain a supersymmetric theory with gravity in 10-dimensions there has to exist 6-dimensional compact manifolds endowed with a Riemannian metric which also admits a covariantly constant spinor. In addition to their interesting geometric and topological properties, this requirement of supergravity theories motivates detailed studies on Ricci-flat Kähler manifolds having n complex dimensions with SU(n) holonomy, i.e. Calabi-Yau manifolds. In this study, we deal with geometric structures in moduli space of Calabi-Yau manifolds. Some relations between Kähler forms and Kähler potential will be derived and applied on a Kähler-structure modulus space. A supersymmetric three-cycle inside Calabi-Yau three-fold will be characterized as a hypersurface.

Keywords: Calabi-Yau manifolds; Kähler potential; Moduli spaces.

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Euclidean Curves with Incompressiable Canonical Vector Fields

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Abstract

In the present study we consider Euclidean curves with incompressible canonical vector fields. We investigate such curves in terms of their curvature functions. Recently, B.Y. Chen gave classification of plane curves with incompressible canonical vector fields. For higher dimensional case we gave a complete classification of Euclidean space curves with incompressible canonical vector fields. Further we obtain some results of the Euclidean curves with incompressible canonical vector fields in 4-dimensional Euclidean space E^4.

Keywords: Regular curve; Generalized helix; Salkowski curve; Canonical vector field.

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Differential Equations for a Space Curve According to the Unit Darboux Vector

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Abstract

In this paper, the differential equation of a space curve, in the Euclidean 3-space E^3, is given first according to the unit Darboux vector and then according to the normal connexion. In addition, in the case of helix of the curve, the differential equation obtained from Laplace and normal Laplace operators is given. We give the differential equations of a Frenet curve with non-zero curvatures and unit Darboux vector C as $D^3_T C = \mu_3 D^2_T C + \mu_2 D_T C + \mu_1 C$ where μ_i are the coefficiants. We also give the differential equation characterizing the Frenet curve according to the normal connexion as $D^2_T C^\perp = \lambda_2 D_T C^\perp + \lambda_1 C^\perp$ where λ_i are the coefficiants. Finally, we write the differential equations of a helix to be an example of all cases we mentioned.

Keywords: Darboux vector; Laplacian operator; Helix; Space curve; Differential equation.

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On a Class of Slant Curves in S-Manifolds

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Abstract

In this study, we consider a special class of slant curves in S-manifolds. Slant curves are a generalization of Legendre curves. S-manifolds are also a generalization of Sasakian manifolds. Magnetic curves are very popular in both Physics and Riemannian Geometry. Moreover, magnetic curves are considered as a generalization of geodesics. We find the parametric equations of normal magnetic slant curves in $\mathbb{R}^{2n+s}(-3s)$.

Keywords: S-manifold; Slant curve; Magnetic curve.

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Mixed Totally Geodesic Semi-Invariant Submanifolds of Trans-Sasakian Finsler Manifolds

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Abstract

Mixed totally geodesic semi-invariant submanifolds of trans-Sasakian Finsler manifolds are studied. In this regard, some structure theorems are introduced.

Keywords: Trans-Sasakian Finsler manifold; Semi-invariant submanifold.

Mathematics Subject Classification (AMS 2010): 58B20, 53C25, 58A30.

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On the Normality Conditions of Almost Kenmotsu Finsler Structures on Vector Bundles

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Abstract

Kenmotsu Finsler structures on complementary horizontal and vertical tangent bundles are introduced and integrability conditions on diffusions are discussed.

Keywords: Kenmotsu Finsler structure; Vector bundle.

Mathematics Subject Classification (AMS 2010): 58B20, 53C25, 58A30.

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On the Curvatures of Indefinite Kenmotsu Finsler Manifolds

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Abstract

Indefinite Kenmotsu Finsler manifolds are defined and Riemann curvature tensor, constant curvature, flag curvature, Ricci tensor of such kind of structures on horizontal and vertical diffusions are discussed with some results.

Keywords: Trans-Sasakian Finsler manifold, Semi-invariant submanifold.

Mathematics Subject Classification (AMS 2010): 58B20, 53C25, 58A30.

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Sasakian Lorentzian Structures on Indefinite Finsler Manifolds

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Abstract

Sasakian Lorentzian structures on indefinite Finsler manifolds are introduced and some structural theorems discussed. The fundamental relations on basic curvature tensors are given.

Keywords: Trans-Sasakian Finsler manifold; Semi-invariant submanifold.

Mathematics Subject Classification (AMS 2010): 58B20, 53C25, 58A30, 53B30.

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Contact CR-Submanifolds in Spheres

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Abstract

The notion of *CR-submanifold* in Kaehler manifolds was introduced by A. Bejancu in 70's, with the aim of unifying two existing notions, namely complex and totally real submanifolds in Kaehler manifolds. Since then, the topic was rapidly developed, mainly in two directions:

- Study CR-submanifolds in other almost Hermitian manifolds.
- Find the odd analogue of CR-submanifolds. Thus, the notion of *semi-invariant submanifold* in Sasakian manifolds was introduced. Later on, the name was changed to *contact CR-submanifolds.*

A huge interest in the last 20 years was focused on the study of CR-submanifolds of the nearly Kaehler six dimensional unit sphere. Interesting and important properties of such submanifolds were discovered, for example, by M. Antic, M. Djoric, F. Dillen, L. Verstraelen, L. Vrancken. As the odd dimensional counterpart, contact CR-submanifolds in odd dimensional spheres were, recently, intensively studied.

In this talk we focus on those proper contact CR-submanifolds, which are as closed as possible to totally geodesic ones in the seven dimensional spheres endowed with its canonical structure of a Sasakian space form. We give a complete classification for such a submanifold having dimension 4 and describe the techniques of the study. We present also the first steps concerning dimension 5 and propose further problems in this direction.

This talk is based on some papers in collaboration with M. Djoric and L. Vrancken, mainly on [1].

Keywords: (contact) CR-submanifold; Sasakian manifolds; Minimal submanifolds; (mixed) Totally geodesic CR-submanifolds.

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Contact Pseudo-Metric Structures on Indefinite Finsler Manifolds

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Abstract

Contact pseudo-metric structures on indefinite Finsler manifolds are introduced. Some structural results depending on Nijenhius tensor and second fundamental form are discussed.

Keywords: Contact pseudo-metric structure; Indefinite Finsler manifold; Vector bundle.

Mathematics Subject Classification (AMS 2010): 58B20, 53C25, 58A30.

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ε-Sasakian Structures on Indefinite Finsler Manifolds

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Abstract

ε-Sasakian structures on indefinite Finsler manifolds are defined and some results on curvature tensors regarding time-like and space-like vectors are obtained.

Keywords: **ε**-Sasakian structure; Indefinite Finsler manifold; Bundle diffusion.

Mathematics Subject Classification (AMS 2010): 58B20, 53C25, 58A30.

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The Motivation for the Space-Like Surface of Constant Breadth

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Abstract

The motivation for this study is to define an ovaloid surface on the convex closed space-like surfaces of constant breadth with which principal curvatures are continuous, non-vanishing functions, and to obtain some special geometrical properties of this ovaloid surface by using the radius of curvature, diameter of the surface.

Keywords: Minkowski space; Surfaces of constant breadth; Ovaloid.

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Intrinsic Equations for a Generalized Relaxed Elastic Line Due to the B-Darboux Frame of Space-Like Curve on a Surface in the Minkowski 3-Space

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Abstract

In this purpose, we worked a new frame on a surface which called as B-Darboux frame in Minkowski 3-space. The B-Darboux frame from the Darboux frame on a surface is derived in E_1^3. Then we have obtained the intrinsic equations for a generalized relaxed elastic line and their boundary conditions due to the B-Darboux farme for the space-like curve on an oriented surface in the Minkowski 3-space which has an important point on differential calculus.

Keywords: Generalized relaxed elastic line; Variational problem; Intrinsic equations; B-Darboux frame; Minkowski 3-space.

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Intrinsic Equations for a Generalized Relaxed Elastic Line Due to the B-Darboux Frame on an Oriented Surface

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Abstract

In this study, we introduce a new frame on a surface, called as B-Darboux frame. It is well known that we derive the parallel transport frame from the Frenet frame along a space curve. Analogously, we produce the B-Darboux frame from the Darboux frame on a surface. Then we have obtained the intrinsic equations for a generalized relaxed elastic line on an oriented surface in Euclidean 3-space. Finally, some applications of the result are given.

Keywords: Generalized relaxed elastic line; Variational problem; Intrinsic equations; B-Darboux frame.

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A New Approach on Dual Spherical Curves and Surfaces

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Abstract

In this paper, the vectorial moments of the Frenet vectors are reexpressed in terms of Frenet vectors. According to the new versions of these vectorial moments, the parametric equations of the ruled surfaces corresponding to the dual spherical curves are given. Further, this study gave a link between the classical surface theory and dual spherical curves on the dual unit spheres. Distribution parameters of the closed ruled surfaces corresponding to the dual spherical curves are given. The instantaneous pfaffian vector and the dual Steiner vector generated by the motion of the dual vectors are given. The integral invariants of the closed ruled surfaces are rederived and illustrated by presenting with examples.

Keywords: Dual spherical curves; Vectorial moment; Closed ruled surface; Dual angle of pitch; Distribution parameters; Gauss curvature.

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Relationships Between Symplectic Groupoids and Generalized Golden Manifolds

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Abstract

We introduce generalized Golden structures on manifolds and obtain integrability conditions in terms of classical tensor fields. We investigate big-isotropic subbundles and relate this new generalized manifolds with Lie groupoids. We observe that certain properties of generalized Golden manifolds are different from properties of generalized complex manifolds as well as generalized contact manifolds.

Keywords: Lie Groupoid; Lie Algebroid; Golden Manifold; Generalized Almost Golden Manifold.

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A Geometric Viewpoint on the Fixed-Circle Problem

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Abstract

In this talk, mainly we focus on the fixed-circle problem by a geometric viewpoint. For this purpose, we investigate some fixed-circle theorems using a new family of functions on metric spaces. We obtain a uniqueness condition of a fixed circle of a self-mapping and a condition which excludes the identity map. Also, the obtained results generalize the known fixed-circle theorems.

Keywords: Fixed circle; Metric space; Uniqueness condition.

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The Circling-Point Curve of Inverse Motion in Minkowski Plane

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Abstract

The locus of the points having trajectory with constant curvature in the moving plane is called circling-point curve in a planar motion. In this study, we investigate this curve for the inverse motion of Minkowski planes and also we deal with its degenerate cases individually. For this purpose firstly we state the trajectory of origin with respect to the instantaneous invariants of Bottema. Afterwards, we define the locus of the Ball points that are the intersection of the circling-point curve and imaginary inflection circle in Minkowski plane. Finally, we give the geometric interpretation of the circling-point curve by comparing the original and inverse motion in Minkowski planes.

Keywords: Circling-Point Curve; Ball Points; Minkowski Plane; Inverse Motion.

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Fractals of Infinite Area

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Abstract

In this presentation, we consider the dynamics of a special class of rational functions that are obtained from relaxed Newton's method,

$$N_{F,h}(z)= z - h\,(F(z)/F`(z)),\ \text{where } F(z)=z(1-z^3)e^z\ ,\ h\in(0,1]\ .$$

We pay special attention that fractal image of the rational iteration

$$N_{F,h}(z)= ((1-h)z+z^2-(4-h)z^4-z^5)/(1+z-4z^{3-}z^4).$$

In fact, these fractal images show that the basins of attraction of roots are unbounded and the boundary of the basins are Julia sets of the function $N_{F,h.}$

Keywords: Rational iteration; Relaxed Newton's method; Fractals; Julia set.

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A Neutral Relation Between Polynomial Structure and Almost Quadratic φ-Structure

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Abstract

In this paper, metallic structure and almost quadratic φ-structure are studied. Based on metallic (polynomial) Riemannian manifold, Kenmotsu quadratic, cosymplectic (Sasakian) quadratic manifold are defined and constructed some examples. Finally, we construct an almost quadratic φ-structure on the hypersurface M^n of a metallic Riemannian manifold M^{n+1}.

Keywords: Polynomial structure; Golden structure; Metallic structure; Almost quadratic φ-structure.

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On Vectorial Moments According to Bishop Frame in Minkowski -Space

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Abstract

In this paper, we study the vectorial moments in Minkowski 3-space. Firstly, we obtain vectorial moments of some special curves. Then, we give fundamental properties of these vectorial moments. Finally, we examine some examples according to Bishop frame in Minkowski 3-space.

Keywords: Vectorial moment; Bishop Frame; Minkowski space.

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New Fixed-Circle Theorems

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Abstract

In this talk, we obtain new fixed-circle theorems using some classical techniques which are used to obtain some fixed-point results on metric spaces. For this purpose, we introduce some different types of contractive conditions. Also, we give some illustrative examples to show the validity of our obtained results.

Keywords: Fixed circle; Metric space; Contractive condition.

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A Study on The Deformed Second Lift Metric on The Second Order Tangent Bundle

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Abstract

In the present paper, we study the geometry of deformed second lift metric on the second order tangent bundle $T^2(M)$ over a Riemannian manifold. The deformed second lift metric is obtained by adding a symmetric (0,2)-tensor field to the horizontal part of the second lift metric. The Levi-Civita connection and the Riemann curvature tensor of this metric are computed. As applications, the semi-symmetry property of $T^2(M)$ with respect to the deformed second lift metric and conditions for vector fields on $T^2(M)$ to be conformal and projective are established.

Keywords: Second-order tangent bundle; Deformed second lift metric; Semi-symmetric manifold.

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On the Principal Normal and Binormal Spherical Indicatrices of a Time-like W-Curve on Pseudohyperbolic Space $\mathbf{H}_0^3$

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Abstract

In this study, we investigate the principal normal, binormal spherical indicatrices of a timelike W-curve on pseudohyperbolic space H_0^3 in Minkowski space time E_1^4. The principal normal indicatrix of a timelike W-curve is determined as a spacelike curve lying on pseudohyperbolic space H_0^3 by taking the condition $\tau < \kappa < 0$ or $0 < \kappa < \tau$, then the Frenet-Serret invariants of the mentioned indicatrix curve is obtained in terms of the invariants of timelike W-curve. The binormal indicatrix is a spacelike curve which doesn't need any condition. Also, the Frenet-Serret invariants of the binormal indicatrix curve are obtained as similar to the principal normal indicatrix.

Keywords: Minkowski Space Time; Spherical indicatrix; Pseudo Null Curves.

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On Timelike Surfaces of Constant Breadth

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Abstract

This study focuses on time-like surfaces of constant breadth in Minkowski 3-space E_1^3. An ovaloid surface is determined as the convex closed time-like surface of constant breadth with non-vanishing, continuous principal curvatures. Since such a surface has two parallel tangent planes for each of their normal planes, we find the relations between these two opposite tangent planes. Additionally, we obtain some geometrical properties such as the radius of curvature, diameter of the surface for the ovaloid surface as the time-like surface of constant breadth.

Keywords: Minkowski space; Surfaces of constant breadth; Ovaloid.

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Various Types of Fixed-Circle Results on S-Metric Spaces

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Abstract

In this talk, we present new fixed-circle results using the modified C-Khan type contractive condition and the Suzuki-Berinde type F_C^S - contractive condition on an S - metric space. Some obtained results can be considered as fixed-disc theorems. Also, some illustrative examples of our results are given on S - metric spaces.

Keywords: Fixed circle; S - metric space; Modified C-Khan type contraction; Suzuki-Berinde type F_C^S - contraction.

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A Note on Neutral Slant Submersions

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Abstract

We investigate some geometric properties of three types of slant submersions whose total space is an almost para-Hermitian manifold.

Keywords: Para-Hermitian manifold; Pseudo-Riemannian submersion; Proper slant submersion.

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On Clairaut Anti-Invariant Semi-Riemannian Submersions

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Abstract

We investigate geometric properties of anti-invariant pseudo-Riemannian submersions whose total space is a paracosymplectic manifold. Then, we study new conditions for anti-invariant pseudo-Riemannian submersions to be Clairaut submersions. Also, examples are given.

Keywords: Paracosymplectic manifold; Semi-Riemannian submersion; Anti-invariant semi-Riemannian submersion, Clairaut submersion.

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On the Geometry of Conformal Slant Submersions

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Abstract

We introduce conformal slant submersions from almost contact metric manifolds onto Riemannian manifolds, mention a lot of examples and investigate the geometry of leaves of the vertical distribution and the horizontal distribution and find necessary and sufficient conditions for a conformal slant submersion to be totally geodesic and harmonic, respectively.

Keywords: Almost contact metric manifold; Conformal submersion; Slant submersion; Conformal slant submersion.

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Some Results for Generalized Null Mannheim Curves in 4-dimensional Semi-Euclidean Space with Index 2

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Abstract

In this talk, we give the necessary and sufficient conditions for null curves in 4-dimensional semi-Euclidean space with index 2 to be generalized null Mannheim curves in terms of their curvature functions and Frenet vectors by taking consideration of the plane spanned by the first binormal and the second binormal vectors, which in the case of a spacelike plane or a timelike plane, separately. Also, the related examples are given.

Keywords: Generalized Mannheim curve; Semi-Euclidean Space; Cartan null curve.

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Smarandache Curves According to q-Frame in Euclidean 3-Space

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Abstract

In this study, we investigate special Smarandache curves according to q-frame in Euclidean 3-space and we give some differential geometric properties of Smarandache curves.

Keywords: Frenet frame; Smarandache curves; q-frame; Natural curvatures.

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Translation Hypersurfaces in Isotropic Spaces

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Abstract

In this talk, we are interested in the translation hypersurfaces in 4-dimensional isotropic space. These surfaces are generated by three curves lying in 2-planes or hyperplanes. There are four different types of translation hypersurfaces in 4-dimensional isotropic space. We classify such hypersurfaces with constant Gauss-Kronocker and mean curvature.

Keywords: Translation hypersurface; Isotropic geometry; Gauss-Kronocker and mean curvature.

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On Affine Factorable Surfaces

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Abstract

An affine factorable surface is the graph of the form $z(x,y)= f(x)g(y+ax)$. In this talk, we are interested in the problem of obtaining such surfaces in isotropic geometry with constant Gaussian *K* and mean curvature *H*. The absolute of this geometry provides two different types of the affine factorable surfaces. We classify those surfaces of both type with $K, H = const$.

Keywords: Isotropic geometry; Factorable surface; Gaussian and mean curvature.

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Translation Surfaces in Galilean Spaces

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Abstract

The absolute of Galilean space provides five different types of translation surfaces generated by translating two curves. We obtain these surfaces having constant Gaussian and mean curvature, except the one in which both generating curves are non-planar.

Keywords: Translation hypersurface; Isotropic geometry; Gauss-Kronocker and mean curvature.

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Projective Vector Fields on the Tangent Bundle with respect to the Semi-symmetric Metric Connection

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Abstract

In the present paper, we firstly define a semi-symmetric metric connection on the tangent bundle with complete lift metric. Secondly, we characterize projective vector fields on the tangent bundle with respect to this connection. Finally, we present some results for special vector fields.

Keywords: Semi-symmetric metric connection; Tangent bundle; Projective vector field; Complete lift metric.

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Statistical Submersions in Cosymplectic-like Statistical Manifolds

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Abstract

In this study, we consider statistical submersions in cosymplectic-like statistical manifolds. We give examples of cosymplectic-like statistical manifolds and its statistical submersions. The properties of total and base spaces for the statistical submersions on cosymplectic-like statistical manifolds are studied under certain conditions.

Keywords: Statistical manifold; Statistical submersion; Cosymplectic-like statistical manifold; Kahler-like statistical manifold.

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Ricci Solitons on Lorentzian Hypersurfaces of Pseudo-Euclidean Spaces

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Abstract

The geometry of Ricci solitons has been working from different aspects by many mathematicians during the last two decades, especially after applied to solution of Poincare Conjecture by Grigori Perelman. Also, Ricci solitons on Euclidean submanifolds were studied by B.Y. Chen in [1-3].

In this work, we investigate the Ricci solitons on 3-dimensional Lorentzian hypersurfaces of a pseudo-Euclidean space. It is well known that a symmetric endomorphism of a vector space with Lorentzian inner product can be put into different forms. Thus, we make characterization of Ricci solitons on Lorentzian hypersurface of the pseudo-Euclidean space according to the form of shape operator.

Keywords: Ricci Solitons; Lorentzian Submanifolds; Position Vector Field.

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On the Isometries of the Generalized Taxicab Plane

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Abstract

In this talk, first we present the notion of gt-radian to measure angles in the generalized taxicab plane as natural generalized taxicab version of the radian notion of the Euclidean plane. Then, using this notion we define gt-reflection and gt-rotation as natural generalized taxicab versions of reflection and rotation notions of the Euclidean plane. Finally, we indicate the isometries of the generalized taxicab plane determining which gt-reflections and gt-rotations preserve the generalized taxicab distance.

Keywords: The generalized taxicab metric; Angle measure; Reflection; Rotation; Isometry.

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On the Pythagorean Theorem in the Generalized Taxicab Plane

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Abstract

In this talk, we give a generalized taxicab analogue of the Pythagorean Theorem, and show with two examples that the converse of the given analogue is not valid. Finally, we give a necessary and sufficient condition for a triangle in the generalized taxicab plane to have a right angle.

Keywords: Pythagorean theorem; Generalized taxicab metric.

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On gh-lifts of Some Tensor Fields

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Abstract

The main purpose of this paper is to transfer horizontal lifts of some tensor fields from tangent bundle to the cotangent bundle by using the musical isomorphism. In this study the $gh-$ lifts of tensor fields are described on the cotangent bundle newly. The $gh-$ lifts of tensor fields are obtained by transferring the horizontal lifts of tensor fields from tangent bundle to the cotangent bundle via musical isomorphism.

Keywords: $gh-$lift; Horizontal lift; Tensor fields; Musical isomorphism; Cotangent bundle; Tangent bundle.

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Spherical Caustic Curves Generated by Reflected Rays

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Abstract

In this study, a spherical curve on S^3 is considered as a mirror, then we obtain the mathematical formula of the curve generated by the reflected rays by this mirror curve and this curve is called orthotomic curve of the mirror curve. Spherical caustic curve is defined by using the spherical orthotomic curve. The contact points of the spherical orthotomic curve are examined in terms of the Sabban frame apparatus of the mirror curve. In addition, the contact points of the spherical caustic curve are characterized and some results are obtained by using the Sabban frame apparatus.

Keywords: Sabban frame; Spherical caustic curve; Spherical orthotomic curve.

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A Special Interpretation of the Concept "Constant Breadth" for a Space Curve

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Abstract

The definition of curve of constant breadth in the literature is made by using parallel and opposite direction tangent vectors at opposite points of the curve. In this study, we put parallel and opposite direction normal vectors of the curve to the output point of the concept of curve of constant breadth. And we work on the concept of curve of constant breadth according to normal vector. At the conclusion of the study, we obtain a system of linear differential equations with variable coefficients characterizing space curves of constant breadth according to normal vector. The coefficients of this system of equations are functions depend on the curvature and torsion of the curve. We then obtain an approximate solution of this system using the Taylor matrix collocation method. In summary, in this study, we first make a different interpretation for the concept of space curve of constant breadth. We then use this interpretation to obtain a characterization. And finally, we solve this characterization we've achieved.

Keywords: Curve of constant breadth; Special curves in space; Taylor matrix collocation method.

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Generalized Tessarine Numbers and Homothetic Motions

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Abstract

In this present paper, we first describe a matrix that is similar to Hamilton operators by using the generalized tessarine numbers product and additon and we give some algebraic properties of them. Then, by using one of the concepts of conjugates which is given according to the arbitrary units i_1, i_2 and i_3 for the generalized tessarines and this matrix in E_v^4 ($v=0$ and $v=2$), we obtain the hypersurfaces M, M_1 and M_2. By using these the hypersurfaces and this matrix we introduce two different types of homothetic motions in E^4 and E_2^4. Furthermore, for this one parameter homothetic motion, we give some theorems about velocities, pole points, and pole curves. Finally, it is found that these motions defined by the regular curve of order r curve lying curves on these hypersurfaces, at every $t-$instant, has only one acceleration centre of order $(r-1)$.

Due to the way in which the matter is given with the generalized tessarine numbers, the study gives some formulas, facts and properties about homothetic motion and variety of algebraic properties which are not generally known.

Keywords: Generalized tessarine numbers; Homothetic motion; Pole curves; Hypersurface.

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Partially Null Curves Lying Completely on the Subspace of $\mathbf{R}_2^4$

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Abstract

In this paper, we have investigated partially null curves in R_2^4 with curvatures $\kappa(s) \neq 0$, $\tau(s) \neq 0$ and $\sigma(s) = 0$ for each $s \in I \subset R$ using the Frenet formulas given in [5]. However, the differential equations of partially null curves were solved and it was investigated whether these curves were lying or not lying on the subspaces of this space.

Keywords: Semi-Euclidean 4- space with index 2; Partially null curves; Frenet frame.

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On the Complex Fibonacci 3-Vectors

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Abstract

The aim of this study, to investigate complex Fibonacci $3-$vectors. To achieve this, we give the definition of complex Fibonacci $3-$vectors, Hermitian inner product, vector product and the scalar triple product of complex Fibonacci $3-$vectors. Moreover, we present some properties of Hermitian vector product of complex Fibonacci $3-$vectors.

Keywords: Complex Fibonacci vectors; Anti-symmetric matrix; Hermitian Vector product.

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ACN on Geometric Graphs

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Abstract

A geometric graph is a graph in which the vertices or edges are associated with geometric objects. In graph theory, a partial cube is a graph that is an isometric subgraph of a hypercube. In other words, a partial cube is a subgraph of a hypercube that preserves distances, that is the distance between any two vertices in the subgraph is the same as the distance between those vertices in the hypercube. Every tree is a partial cube and every hypercube graph is itself a partial cube. In this paper, we study on average covering numbers of some hypercubes.

Keywords: Geometric graph theory; Covering; Vulnerability.

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Generalized Ricci Solitons on Lorentzian Twisted Product

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Abstract

In this talk, some relations between generalized Ricci solitons and twisted products, are established. Generalized Ricci solitons were introduced as a class of overdetermined systems of partial differential equations of finite type on pseudo-Riemannian manifolds, [4]. Twisted products have been widely used in differential geometry as well as in general theory of relativity to construct new examples of pseudo-Riemannian manifolds satisfying certain curvature conditions and to find exact solutions of Einstein field equations, [5]. Substantially, these two geometric notions are quite useful concepts that allow us to obtain some results for the existence of torqued vector field, which is recently introduced in [2]. For this purpose, we prove certain identities for the Ricci and the Weyl tensors on a Lorentzian twisted product admitting a timelike torqued vector field. Then, using these identities, we investigate some necessary conditions for such a Lorentzian twisted product to be a model of perfect fluids. The results of this presentation are based on our papers [1] and [3].

Keywords: Generalized Ricci Soliton; Twisted Product; Torqued Vector Field.

Acknowledgement: This work is supported by GAP project TGA-2018-41211 of Istanbul Technical University.

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Geometry of Statistical F-connections

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Abstract

Let M be an anti-Kähler manifold with an almost complex structure F, a pseudo-Riemannian metric g and a totally symmetric $(0,3)-$tensor field C. We first introduce statistical $F-$connections which are a special class of $\alpha-$connections on M and derive the conditions under which its curvature tensor field is holomorphic. Then, we present some local results concerning with curvature properties of the connection and the tensor C.

Keywords: Anti-Kähler structure; Einsten manifold; Holomorphic tensor; Statistical structure.

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On Lightlike W-Curves in 4-dimensional Semi-Euclidean Space with Index 2

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Abstract

In this study, we investigate the properties of lightlike W-curves with null normals in 4-dimensional semi-Euclidean space with index 2. We obtain parametric equations of W-curves with the help of curvatures functions and we give some examples.

Keywords: W- curve; Semi-Euclidean Space; Null curve; Curvatures functions.

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Codazzi Pairs on Almost Anti-Hermitian Manifolds

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Abstract

In this paper, we first introduce three types of conjugate connections of linear connections on an 2n-dimensional almost anti-Hermitian manifold M with an almost complex structure J, a pseudo-Riemannian metric g and the twin metric $G = g \circ J$. We obtain a simple relation among curvature tensors of these conjugate connections. To clarify relations of these conjugate connections, we prove a result stating that conjugations along with an identity operation together act as a Klein group. Secondly, we give some results exhibiting occurrences of Codazzi pairs which generalize parallelism relative to a linear connection ∇. Under the assumption that (∇, J) being a Codazzi pair, we derive a necessary and sufficient condition the almost anti-Hermitian manifold (M, J, g, G) is an anti-Kähler relative to a torsion-free linear connection ∇. Finally, we investigate statistical structures on M under ∇ (∇ is a J-invariant torsion-free connection).

Keywords: Anti-Kähler structure; Codazzi pair; Conjugate connection; Twin metric; Statistical structure.

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Constraint Manifolds for 2R Open Chain on Lorentz Plane

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Abstract

In a mechanism, there exist three kind of structure equations. These are planar, spherical and spatial. In this paper, using the structure equation of 2R planar open chain, we calculate constraint manifolds of it in Lorentz space. Then, in this space, we obtain geometric comments and conclusions by means of the constraint manifolds of the chain.

Keywords: 2R Planar Open Chain; Constraint Manifold; Planar Quaternions.

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Constraint Manifolds of 2R Spherical Open Chain in Lorentz Space

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Abstract

In Euclidean Space, the general form of structure equations is given for the cases of planar, spherical and spatial chains. In this paper, we present constraint manifolds of it via the structure equations of 2R Spherical Open Chain in Lorentz space. After then, If the constraint manifolds of the chain are taken into consideration, some geometric comments are studied.

Keywords: Structure Equation; Constraint Manifold; 2R Spherical Open Chain; Split Quaternion.

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On a Study of Lightlike Submanifolds of Metallic Semi-Riemannian Manifolds

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Abstract

The Metallic Ratio is fascinating topic that continually generated news ideas. A semi-Riemannian manifold endowed with a Metallic structure will be called a Metallic semi-Riemannian manifold. The main purpose of the present paper is to study the geometry of some types of lightlike submanifolds of Metallic Semi-Riemannian manifolds. We investigate the geometry of distributions and obtain necessary and sufficient conditions for the induced connection on these submanifolds to be a metric connection. We also obtain characterizations for transversal lightlike submanifolds of Metallic semi-Riemannian manifolds. Finally, we give an example.

Keywords: Metallic structure; Metallic semi-Riemannian Manifold; Lightlike submanifolds; Transversal Lightlike submanifolds; Radical transversal submanifolds.

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Rotation Minimizing Frame and its Applications in E^4

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Abstract

In this studied, in E^4, it is showed conditions that any frame is Rotation minimizing frame (RMF) using unit speed regular curves. It has also expressed how the Bishop frames can be obtained from frames of any curve on surface and in space. The necessary and sufficient conditions are given. Then, it is investigated whether obtained frames are Rotation minimizing frame (RMF) or not. Theorems, warnings and conclusions are expressed. The examined situations are shown over examples.

Keywords: Bishop Frame; Rotation Minimizing Frame; Rectifying Curve.

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Rectifying Slant Curves in Minkowski 3-Space

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Abstract

In this studied, in Minkowski 3-Space, we will study rectifying curves obtained from spherical curves. We will look for an answer to the question of how spherical curves will be selected for rectifying slant curves.

Keywords: Rectifying Curve; Slant Curve; Darboux Frame.

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An Alternative Approach to Tubular Surfaces

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Abstract

Let the envelope of one-parameter family of spheres in three-dimensional Euclidean space or briefly canal surfaces has the constant radius, then they can be renamed as tubular surfaces. The striking feature of tubular surfaces is that the radius vector of each sphere in the family and the center curve meet at a right angle. In this work, we change the condition on the angle and examine the characteristics of tubular surfaces.

Keywords: Circular surface; Tubular surface.

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Developable Surfaces with k-Order Frame

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Abstract

In this study, we investigate the three types of developable surfaces, introduced by Izumiya and Takeuchi, from the perspective of the singularity theory. By using k-order frame, we give a classification for the base curves of the surfaces in terms of the Nk-slant helices and analyze their efficiency on the singular sets diversity.

Keywords: Developable surface; Singularity; Slant helix.

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Generic Submersions from Kaehler Manifolds I

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Abstract

We introduce a new kind of Riemannian submersion such that the fibers of such submersion are generic submanifolds in the sense of Ronsse [3] that we call generic submersion. Some original examples are given. Necessary and sufficient conditions are found for the integrability and totally geodesicness of the distributions which are mentioned in the definition of these kinds of submersions.

Keywords: Riemannian submersion; Generic submersion; Kaehlerian manifold.

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Generic Submersions from Kaehler Manifolds II

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Abstract

We study a new kind of Riemannian submersion such that the fibers of such submersion are generic submanifolds in the sense of Ronsse [3] that we call generic submersion. We investigate the geometry of fibers. Also, we obtain some interesting results for generic submersion with parallel canonical structures.

Keywords: Parallel canonical structure; Generic submersion; Totally geodesic.

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Timelike Factorable Surfaces in Minkowski 4-Space IE_1^4

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Abstract

In the current work, we study timelike factorable surfaces in four-dimensional Minkowski space. We describe such surfaces in terms of their Gaussian and mean curvature functions. We classify flat and minimal timelike factorable surfaces in IE_1^4.

Keywords: Factorable surface; Minkowski 4-space; Timelike surface.

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Spacelike Aminov Surfaces of Hyperbolic Type in Four Dimensional Minkowski Space $\mathbb{E}_1^4$

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Abstract

In the present study, we discuss spacelike Aminov surfaces of hyperbolic type in four dimensional Minkowski space IE_1^4. We investigate the mean curvature and Gaussian curvature of this surface. We give flat and minimal spacelike Aminov surfaces in Minkowski 4-space IE_1^4.

Keywords: Aminov surface; Minimal surface; Minkowski 4-space.

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Spacelike Aminov Surfaces of Elliptic Type in Four Dimensional Minkowski Space IE_1^4

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Abstract

In the present study, we discuss spacelike Aminov surfaces of elliptic type in four dimensional Minkowski space IE_1^4. We investigate the mean curvature and Gaussian curvature of this surface. We give flat and minimal Aminov surfaces in Minkowski 4-space IE_1^4.

Keywords: Aminov surface; Minimal surface; Minkowski 4-space.

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Focal Surfaces of a Tubular Surface with Respect to Frenet Frame in $\mathbb{E}^3$

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Abstract

In this study, we focus on focal surfaces of a tubular surface in Euclidean 3-space E^3. We characterize these surfaces with respect to Frenet frame. Further, we get some results for these types of surfaces to become flat and minimal in E^3.

Keywords: Focal surface; Tubular surface; Bishop frame.

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A Characterization of Factorable Surfaces in Euclidean 4-Space IE^4

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Abstract

In this paper, we consider a factorable surface in Euclidean space IE^4 with its curvature ellipse. We classify the origin of the normal space of such a surface according to whether it is hyperbolic, parabolic, or elliptic. Further, we give the necessary and sufficient condition of the factorable surface to become Wintgen ideal surface.

Keywords: Curvature ellipse; Factorable surface; Wintgen ideal surface.

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Involute Curves of Order k of a Given Curve in Galilean 4-Space G_4

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Abstract

In the present study, we consider the curves in Galilean 4-space G_4. We find out the involute curves of order k (k=1,2,3) of a given curve. We get the relationships between the Frenet apparatus of a given curve and its involute curves of order k.

Keywords: Frenet frame; Involute curves; Galilean 4-space.

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Osculating Direction Curves and Applications

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Abstract

In this paper, we give definition a new type of direction curves in the Euclidean 3-space such as osculating-direction curve. And we give the characterizations for these curves. Moreover, we get the relationships between osculating direction curves and some special curves such as helix, slant helix or rectifying curves.

Keywords: Associated curves; Osculating-direction curves; Osculating-donor curves.

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Rectifying Direction Curves

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Abstract

In this study, we define a rectifying-direction and rectifying-donor curves in the Euclidean 3-space. And we give the characterizations for these curves. Also, we have the relationships between rectifying direction curves and some special curves such as helix, slant helix or rectifying curves.

Keywords: Rectifying-direction curves; Rectifying-donor curves.

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On T^*N^* Smarandache Curves of Involute-evolute Curve According to Frenet Frame in Minkowski 3-Space

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Abstract

In this study, let $\{\alpha^*, \alpha\}$ be involute evolute curve couple, when the Darboux vector of the spacelike involute curve α^* are taken as the position vectors, the curvature and the torsion of T^*N^* Smarandache curve are investigated. These values are expressed depending upon the timelike evolute curve α. Finally, we provide an illustrative example related to our results.

Keywords: Smarandache curves; Involute-evolute curve couple; Minkowski space.

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Reisnerr-Nordström Spacetime Geometry: Derivation of the Euler and Burgers Models

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Abstract

A relativistic generalization of the Euler and Burgers models have recently been introduced and analyzed both theoretically and numerically. In this work we extend these analysis to a particular type of the Lorentzian manifold, so called the Reissnerr-Nordström (R-S) spacetime geometry. We introduce basic properties of the R-S spacetime and its metric components containing electrical charge term which distinguish the R-S spacetime from the Schwarzshild geometry. Furthermore, we present a derivation of the Euler and Burgers models for a 1+1 dimensional R-S geometry with some numerical results.

Keywords: Reissnerr-Nordström Spacetime; Lorenzian geoemetry; Relativistic Equations; Finite Difference Method.

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Some Results for CA Surfaces with Higher Codimension

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Abstract

In this article, we give a mini survey about constant angle (CA) surfaces in different ambient surfaces obtained so far. Also, we obtained some results for CS-surfaces in Euclidean spaces.

Keywords: Constant angle surfaces; Fixed direction; Parallel normal vector.

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On CPD Surfaces in Euclidean Spaces

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Abstract

In this article, we give a mini survey about surfaces endowed with canonical principal direction (CPD) in different ambient surfaces obtained so far. Also, we obtained some results for CPD-surfaces in Euclidean spaces.

Keywords: Canonical principal direction; Parallel normal vector.

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Geometric Inextensible Timelike Curve Flows and mKDV Soliton Equation in SO(n,1)/SO(n-1,1)

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Abstract

In this study, we show that the integrability structure of multi-component systems of the defocusing mKdV equation (the modified Korteveg-de Vries) arise geometrically from inelastic timelike curve flows in Lorentzian symmetric Space.

Keywords: Curve flow; Lorentzian space.

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Properties of Berger Type Deformed Sasaki Metric

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Abstract

The paper deals with the tangent bundle with Berger type deformed Sasaki metric over an anti-paraKahlerian manifold. We define an almost paracomplex structure on the tangent bundle which is compatible with the Berger type deformed Sasaki metric and investigate under which the tangent bundle with these structures is an anti-paraKahlerian. Also, we examine the curvature properties of this metric.

Keywords: Tangent bundle; Paracomplex structure; Berger type deformed Sasaki metric; Geodesics.

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On Doubly Twisted Submanifolds

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Abstract

We define new kind of doubly twisted submanifold in Kaehler manifolds. We show that there exist a such submanifold by giving an illustrate example. We investigate the geometry of such submanifolds. In particular, we establish an inequality for the squared norm of the second fundamental form for this kind of submanifolds.

Keywords: Doubly Twisted product; Pointwise Hemi-Slant Submanifold; Kaehler manifold.

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Timelike Tubular Surfaces with Flc-Frame

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Abstract

This study aimed to introduce a new version of timelike tubular surfaces in E_1^3. Firstly, we define a new adapted frame along a spacelike(timelike) space curve, and this denote the Flc-frame (Frenet-like curve). We then derive a relationship between the Frenet frame and Flc-frame. Finally, we obtain parametric representation of timelike Flc-tubular surfaces. Moreover, we derived the differential geometric properties of these surfaces.

Keywords: Frenet frame; Tubular surface; Minkowski space; Flc-frame.

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Second Order Parallel Symmetric Tensor on a S-Manifold

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Abstract

In this talk we study a second order parallel symmetric tensor in a s-manifold and we deduce some results on semi-parallel hypersurface in s-space forms M^2n+s (c) with c≠ s.

Keywords; Second order symmetric tensor; S manifold; Semi parallel hypersurface.

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Bertrand Offsets of Ruled Surfaces with B-Darboux Frame

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Abstract

In this study, Bertrand offset of ruled surface with B-Darboux frame is introduced. Then using B-Darboux frame of ruled surface, some characteristic properties of ruled surface such as developability, striction point, and distribution parameter are given.

Keywords: Ruled Surface; Darboux Frame; Bertrand; B-Darboux Frame.

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A Study on Rectifying Non-Null Curves in Minkowski 3-space

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Abstract

In this study, we study rectifying curves via the dilation of unit speed non-null curves on S_1^2 or H_0^2 in the Minkowski 3-space E_1^3. Also, we characterize the Frenet-Serret apparatus of the centrode in the Minkowski 3-space. Then we obtain a necessary and sufficient condition for which the centrode $D(s)$ of a unit speed non-null curve $\alpha(s)$ in E_1^3 is a rectifying curve to improve a main result of [2].

Keywords: Rectifying non-null curve; Centrode; Minkowski 3-space.

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An Alternative Method for Finding n-th Roots of a 2x2 Real Matrix

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Abstract

In this study, we give a new method for finding n-th roots of a 2×2 real matrix with the help of hybrid numbers. First, we define the argument and polar forms of a real 2×2 matrix and express the De Moivre's formulas according to the type and character of the matrix.

Keywords: Hybrid Numbers; Roots of Matrices; De Moivre Formulas.

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The Fermi-Walker Derivative on the Binormal Indicatrix of Spacelike Curve

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Abstract

In this study, we have investigated the Fermi-Walker derivative along the binormal indicatrix of any spacelike curve with a spacelike or timelike principal normal in Minkowski 3-space. Fermi-Walker parallelism and non-rotating frame concepts are defined throughout the binormal indicatrix of any spacelike curve. It is shown that while any vector field is Fermi-Walker parallel along the binormal indicatrix of the spacelike curve the vector field is not Fermi-Walker parallel along the spacelike curve.

We have examined the Frenet frame whether it is a non-rotating frame or not. We have proved that Frenet frame is a non-rotating frame along the binormal indicatrix of the curve. Fermi-Walker parallel.

Keywords: Fermi-Walker derivative; Fermi-Walker parallelism; Non-rotating frame; Spacelike curve; Binormal indicatrix.

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The Fermi-Walker Derivative on Spacelike Surfaces

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Abstract

In this study, we have analyzed the Fermi-Walker derivative along any curve lying on the spacelike surface in Minkowski 3-space. Fermi-Walker parallelism, non-rotating frame and Fermi-Walker termed Darboux vector notions are given throughout the curve that lying on the spacelike surface.

It is shown that the Darboux frame is a non-rotating frame along the spacelike principal line. Furthermore, we have proved that when the geodesic torsion is constant Fermi-Walker termed Darboux vector is Fermi-Walker parallel.

Keywords: Fermi-Walker derivative, Fermi-Walker parallelism, Non-rotating frame, Fermi-Walker termed Darboux vector, Spacelike surface, Darboux frame

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Hamiltonian Mechanical Energy on Super Hyperbolic Spiral Curve

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Abstract

The aim of this article is to improve Hamiltonian energy equations for hyperbolic spiral on supermanifold with super jet bundle. The super logarithmic spiral's super coordinates on the super bundle structure of supermanifolds have been given for body and soul part and also even and odd dimensions. This study showed a physical application and interpretation of super velocity and super time dimensions in super Hamiltonian energy equations for this curve which is de.ned by super coordinates.

Keywords: Supermanifold; Supersymmetry; Jet bundle; Hamiltonian mechanical system; Hamiltonian energy equation.

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Lagrangian Mechanical Energy on Super Logarithmic Spiral Curve

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Abstract

The aim of this article is to improve Lagrangian energy equations for equiangular spiral (logarithmic spiral) on supermanifold with super jet bundle. The super logarithmic spiral's super coordinates on the super bundle structure of supermanifolds have been given for body and soul part and also even and odd dimensions. This study showed a physical application and interpretation of super velocity and super time dimensions in super Lagrangian energy equations for this curve which is defined by super coordinates.

Keywords: Supermanifold; Supersymmetry; Jet bundle; Lagrangian mechanical system; Lagrangian energy equation.

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Half Derivative Formulation for Fuzzy Space with Caputo Method

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Abstract

The concept of derivative may mean motion in terms of physical meaning and as a result some studies have made by this concept of derivative on the geometry of point, line and plane motion. Many application of fractional calculus amount to replacing the time derivative in an evolution equation with a derivative of fractional order. On the other hand, a fuzzy space is a space for the representation of information. It is described by on n-dimensional vector where the components are in the range $[0,1]$. The aim of this paper is to improve the fractional derivative calculus especially half derivative on fuzzy space with Caputo method. And we will present a geometric application of this fractional derivation.

Keywords: Fuzzy Space; Caputo method; Fractional derivative; Fuzzy manifold.

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New Fixed-Circle Results via Some Families of Functions

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Abstract

In this talk, we mention the concept of fixed-circle on S-metric spaces. Modifying some known families of functions to the fixed-circle problem, we introduce new fixed-circle results. Some comparisons are made by presenting new illustrative examples.

Keywords: Fixed circle; *S*-metric space; Uniqueness condition.

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D-homothetic Deformation on Almost Contact B-Metric Manifolds

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Abstract

In this paper, the notion of D−homothetic deformation of an almost contact B−metric structure is introduced. We give an example of D−homothetic deformation of a 3−dimensional Sasaki-like accR manifold. Finally, the notion of D−homothetic warping is defined.

Keywords: Almost contact B-metric manifolds; D−homothetic deformation; D−homothetic warping, Sasaki-like accR manifold.

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On Fibonacci Spinors

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Abstract

In this paper, we study on spinors that are vectors with two complex components. Cartan firstly introduced spinors in [1] geometrically. Moreover, Vivarelli expressed some relations about spinors and real quaternions in [4]. In this study, we give a relationship between spinors and Fibonacci quaternions. Then, we express spinor representation of Fibonacci quaternions. Finally, we prove some theorems using spinors for Fibonacci quaternions.

Keywords: Spinors; Fibonacci quaternions.

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A Note on Bicomplex Matrices

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Abstract

In this paper, we consider bicomplex numbers and bicomplex matrices. Firstly, we give some properties of bicomplex numbers. After that we investigate bicomplex matrices using properties of complex matrices. Then, we define the complex adjoint matrix of bicomplex matrices and we describe some of their properties. Furthermore, we give the definition of q-determinant of bicomplex matrices.

2000 Mathematics Subject Classification: 15B33, 11E88, 11R52.

Keywords and Phrases: Complex number; Bicomplex Number; Complex matrix; Bicomplex matrix.

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The Beam Models Depending on Geometry of Deformed Beams

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Abstract

The beams, one of the structural elements, is widely used in many engineering area from aircraft and aerospace engineering to offshore structures and tall buildings. Several different mathematical model of this structural elements is improved. The difference between these models depends on geometry of deformation of the beam under load. Namely, the various beam theories are generated depending on the form of the beam section at the time of deformation. Substituting the displacements obtained from the beam kinematics to the Green-Lagrange strain, the unit displacements expressions of various beam theories are obtained. The strains are attained for the constitutive laws used in the beam theories with the assumptions performed here. In this study, the finite and infinitesimal strain statements are found from differential geometry for many beam theories presented in the literature.

Keywords: Deformed beam; Strain; Green-Lagrange strain.

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Conformal Semi-slant Submersions with Total Space a Kahler Manifold

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Abstract

Park and Prasad [10] defined and studied semi-slant submersions as a generalization of slant submersions, semi-invariant submersions, anti-invariant submersions. As a generalization of semi-slant submersions, we introduce conformal semi-slant submersions and study the new submersions from almost Hermitian manifolds onto Riemannian manifolds. We study the integrability of distributions and the geometry of leaves of a conformal submersion. Moreover, we show that there are certain product structures on the base manifold of a conformal semi-slant submersion. We also obtain totally geodesic conditions for such maps. Finally, we give lots of examples.

Keywords: Second fundamental form of a map; Riemannian submersion; Semi-slant submersion; Conformal semi-slant submersion.

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Biharmonic Riemannian Submersions

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Abstract

In this paper, we study biharmonic Riemannian submersions. We first derive bitension field of a general Riemannian submersion, we then use it to obtain biharmonic equations for Riemannian submersions with 1-dimensional fibers and Riemannian submersions with basic mean curvature vector fields of fibers. These are used to construct examples of proper biharmonic Riemannian submersions with 1-dimensional fibers and to characterize warped products whose projections onto the first factor are biharmonic Riemannian submersions.

Keywords: Biharmonic maps; Riemannian submersions; biharmonic Riemannian submersions; warped product; twisted product.

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On Distance Formulae in Two Convex Dual Spaces

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Abstract

As we can consider polygons in the plane, we can consider polyhedra in space. A polyhedron is a closed, three-dimensional figure which faces are polygons. A polyhedron is called convex if the line segment joining any two points in the interior of the figure lies completely within the figure. A polyhedron called dual if either of a pair of polyhedra in which the faces of one are equivalent to the vertices of the other. For every polyhedron there exists a dual polyhedron. Starting with any convex polyhedron, the dual can be constructed in the following manner: place a point in the center of each face of the original polyhedron and connect each new point with the new points of its neighboring faces.

Some mathematicians studied on distance formulae in some spaces ([1], [3], [4]). Dual convex spaces have an important place in mathematics. In this study we give some distance formula in two spaces which metrics are induced by two dual convex polyhedra called tetrakis hexahedron [5] and truncated octahedron [2]. Tetrakis hexahedron is a Catalan solid and truncated octahedron is an Archimed solid.

Keywords: Tetrakis hexahedron space; Truncated octahedron space; Distance of a point to a plane; Distance of a point to a line; Distance between two lines.

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A Study on Constant Angle Surfaces Constructed on Curves in Minkowski 3-Space

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Abstract

In this study, we deal with the normal, binormal, rectifying developable, Darboux developable and conical surfaces from the point of view the constant angle property in $\mathbb{R}^3_1$. Moreover, we give some related examples with their figures by using the Maple Program.

Keywords: Constant angle surfaces; Ruled surface; Helix; Slant helix; Minkowski Space.

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Pseudo Cyclic Z-Symmetric Manifold

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Abstract

The object of the present paper is to introduce a type of non-flat Riemannian manifolds called pseudo cyclic Z-symmetric manifold and study its geometric properties. Among others it is shown that a pseudo cyclic Z-symmetric manifold is a quasi-Einstein manifold. We also study conformally flat pseudo cyclic Z-symmetric manifolds. The existence of such notion is ensured by a non-trivial example.

Keywords: Pseudo cyclic Ricci symmetric manifold; Z-symmetric tensor; Conformally flat.

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3-Dimensional Quasi-Sasakian Manifolds with The Schouten-Van Kampen Connection and D_a-Homotetic Deformation

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Abstract

In this paper we study the Schouten-van Kampen connection on D_a-homotetic deformed 3-dimensional quasi-Sasakian manifolds. Also, we study locally ϕ-symmetry, semisymmetry and η- parallelism conditions on D_a-homotetic deformed 3-dimensional quasi-Sasakian manifolds with the Schouten-van Kampen connection.

Keywords: D_a-homotetic deformation; The Schouten-van Kampen connection; Locally ϕ-symmetry; η- parallelism; Semisymmetric manifolds.

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Some Conditions on 3-Dimensional Quasi-Sasakian Manifolds with The Schouten-Van Kampen Connection

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Abstract

In this paper we study the Schouten-van Kampen connection on 3-dimensional quasi-Sasakian manifolds. Also, we study locally ϕ-symmetry, semisymmetry and η- parallelism conditions on 3-dimensional quasi-Sasakian manifolds with the Schouten-van Kampen connection.

Keywords: D_a-homotetic deformation; The Schouten-van Kampen connection; Locally ϕ-symmetry; η- parallelism; Semisymmetric manifolds.

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The Geometry of Complex Golden Conjugate Connections

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Abstract

In this study, we give some properties of the conjugate connection on a complex Golden structure. We express the complex Golden conjugate connections in terms of structural and virtual tensors from the almost complex structure. In addition, the existence of duality between the complex Golden and almost complex conjugate connection is investigated.

Keywords: Complex Golden structure; (Conjugate) Linear connection; Almost complex manifold; Structural and virtual tensor field.

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Some Geometric Properties of Rhombic Dodecahedron Space

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Abstract

Polyhedra have interesting symmetries. Therefore, they have attracted the attention of scientists and artists from past to present. Thus, polyhedra are discussed in a lot of scientific and artistic works. There are only five regular convex polyhedra known as the platonic solids. Semi-regular convex polyhedron which are composed of two or more types of regular polygons meeting in identical vertices are called Archimedean solids. The duals of the Archimedean solids are known as the Catalan solids.

Minkowski geometry is a non-Euclidean geometry in a finite number of dimensions. Linear structure of Minkowski geometry is the same as the Euclidean one. There is only one difference which distance is not uniform in all directions. This difference cause chancing concepts with respect to distance. Unit ball of Minkowski geometries is a general symmetric convex set [6]. Therefore, this show that one can find a relation between symmetries convex set and metrics [1,2,3,4]. In [1], we introduce a new metric, and show that the spheres of the 3-dimensional analytical space furnished by this metric is rhombic dodecahedron.

One of the fundamental problem in geometry for a space with a metric is to determine the group of isometries. In this work, we show that the group of isometries of the 3-dimesional space covered rhombic dodecahedron metric is the semi-direct product of octahedral group O_h and T(3), where T(3) is the group of all translations of the 3-dimensional space.

Keywords: Polyhedra; Rhombic dodecahedron; Isometry group.

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Some Distance Formulae in 3-Dimensional Truncated Octahedron Space

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Abstract

Some mathematicians studied on metrics and improved metric geometry (some of these are [1], [3], [4], [5], [7], [8]). Each of geometries induced by metrics in mentioned studies is a Minkowski geometry. Minkowski geometry is a non-euclidean geometry in a finite number of dimensions that is different from elliptic and hyperbolic geometry (and from the Minkowskian geometry of space-time). In a Minkowski geometry, the linear structure is just like the Euclidean one but distance is not uniform in all directions. That is, the points, lines and planes are the same, and the angles are measured in the same way, but the distance function is different. Instead of the usual sphere in Euclidean space, the unit ball is a certain symmetric closed convex set [2].

Truncated Octahedron space is a Minkowski Geometry with truncated octahedron metric which unit sphere is a truncated octahedron; an Archimedean solid [6].

In this study, we give some distance formulae in truncated octahedron space.

Keywords: Truncated octahedron metric; Distance of a point to a plane; Distance of a point to a line; Distance between two lines.

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Surfaces with Constant Slope According to Darboux Frame

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Abstract

A ruled surface in R^3 is a surface which contains at least one 1-parameter family of straight lines. We define a ruled surface whose generating lines are given by points on the curve, while in all points they have the constant slope with respect to the tangent planes to the given surface. These surfaces will be called generalized surfaces with constant slope with respect to the given surface. In this study, we define generalized surfaces with a constant slope with respect to the given planes according to Darboux Frame. We give necessary and sufficient conditions for these types of surfaces to be developable. Also, the investigation is observed under some special cases in terms of the director vector of surface be a asymptotic curvature, geodesic curvature and line curvature, and the angle be a constant. Especially we obtain that the striction lines and Gaussian curvature of the surfaces and as a result of singular locus of the surface that is coincide with the striction line of the surface.

Keywords: Surfaces with Constant Slope; Ruled Surface; Darboux Frame.

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The Circle Inversion Fractals in Terms of Alpha Metric

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Abstract

In this representation, we introduce the α - circle inversion by using α - distance function instead of Euclidean distance in definition of classical inversion. We give some properties of α - circle inversion. Analogous to the fractals generated by iterated function systems (IFS) are the limit sets of circle inversions. Typically, these are generated by circles that are mutually external, that is, each lies outside all the others. Then new fractal patterns are obtained. Moreover, we generalize the method called circle inversion fractal be means of the α -circle inversion. In alpha plane, $\mathbb{R}^2_\alpha$, we give a generalization of α - circle inversion fractal by using the concept of star-shaped set inversion which is a generalization of circle inversion fractal.

Keywords: Alpha plane geometry; Inversion; Fractal.

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On Some Arcs in the Smallest Cartesian Group Plane

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Abstract

In this work, an algorithm for the classification of the 3-arcs and some examples of the 3-arcs in the projective plane of order 25 over the smallest Cartesian Group are given.

Keywords: Projective plane; Cartesian group; Arcs.

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On Coordinatization and Fibered Projective Plane

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Abstract

In this work, we consider a mono-point-generated fibered projective plane with base projective plane. A coordinatization fibered points and lines of a fibered projective plane and some observations regarding the triangular norm are given.

Keywords: Fibered projective plane; Triangular norm.

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Parallel Frame of Nonlighlike Curves in Minkowski Space-time by Means of Lorentzian Rotations

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Abstract

This study is concern with nonlightlike curves in Minkowski space-time with the use of parallel frame. Firstly, the parallel frame of nonlightlike curves in Minkowski space-time is defined in four different cases depending on the character of the Frenet-Serret frame of the curve. For each case, the relations between the Frenet-Serret and parallel frame of the curve are stated with the angles θ, α and β. Moreover, principal curvature functions, $k_1(s)$, $k_2(s)$ and $k_3(s)$, are determined in terms of curvature function $\kappa(s)$ of the curve and the angles θ, α and β. Finally, the curvature, first and second torsion functions of the curve are expressed by the principal curvatures $k_1(s)$, $k_2(s)$ and $k_3(s)$ and the angles θ, α and β.

Keywords: Minkowski Space-time; Nonlightlike Curve; Frame Field.

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Inversion About a Circle in PT Metric Space

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Abstract

Considering distance of air travel or travel over water in terms of Euclidean distance, these travels are made through the interior of spherical Earth which is impossible [5]. So, researchers give alternative distance functions (d_T, d_{CC}, d_α , d_m and d_{PT}) of which paths are different from path of Euclidean metric in the distance geometry. The common property of these metrics d_T, d_{CC} and d_α is that at least one of the line segments forming their paths is parallel to the coordinates axes. Also, their paths compose only union of line segments. But path of metric d_{PT} consist of arc and line segment.

The purpose of this work is to define an inversion with respect to the circle in the plane equipped with d_{PT}.

Keywords: Metric Geometry; Distance Functions; Inversion.

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On Graphs Obtained from Projective Planes

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Abstract

In this work, we introduce a new method for obtaining a graph from projective planes. Also combinatorial properties of the graphs which are obtained from finite projective planes by using this method are investigated. The relations between these combinatorial properties and the order of the projective plane are examined. Finally, the properties of the degree sequences of the corresponding graphs of projective planes are studied.

Keywords: Projective Plane; Graph; Regular Graph; Degree Sequence.

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On the Construction of Bertrand Curves

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Abstract

Before Salkowski published his paper in 1909, circle had only known as constant curvature curve. Salkowski studied a curve whose curvature is equal to 1. Now, this curve is said Salkowski curve. While Salkowski studied this curve, he gave a method for Salkowski curve. In this paper, we modify Salkowski method and we give some example of the Bertrand curve.

Keywords: Salkowski curve; Bertrand curve; Spherical curve.

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A Curve Theory in Sliced Almost Contact Metric Manifolds

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Abstract

In the classical theory of contact structures Legendre curves play an important role. Because contact distributions carry the Legendre curves to Legendre curves. With the new definition of sliced Sasaki manifolds, we defined new type of Legendre curves. In this work we defined π-Legendre curves by using the classical definition of Legendre curves. Also, we constructed the frame vector fields of a π-Legendre curve in the 3-dimensional subdistribution H^3 of TM where $(M, \phi_\pi, \omega_\pi, \pi, g, \xi)$ is a $2n+1$-dimensional sliced almost contact metric manifold.

Keywords: Almost contact manifolds; Legendre curve; π-Legendre curve.

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On Rectifying Spherical Curves in Euclidan Space

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Abstract

Let α be a curve on the sphere. The osculating sphere at every point of the spherical curves is unique and it is equal to the sphere where the curves lie on. The characterization of the spherical curves was done with the use of contact theory. The solution of the differential equation which is found in the characterization was done by Breuer in 1971. The curves which lie on the rectifying plane are called rectifying curves.

In this study te rectifying spherical curves defined by using the definition of spherical curves. Also, the characterization of the rectifying curves is given and the features of these curves were investigated. Finally, we gave examples of these curves.

Keywords: Spherical curve; Rectifying spherical curves; Osculating sphere.

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Isometry Group in TD and TI Metric Spaces

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Abstract

Platonic solids (tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron) have been studied by mathematicians, physicists, chemists, biologists and artists from past to resent, because their symmetries are very interesting. Nowadays, modern technology and multidisciplinary studies have confirmed that many elementary particles in the sub-atomic world take on some Platonic solids. Therefore, Platonic solids and their metric structures could provide new applications in various studies based on the physical properties of elementary particles. Similarly, these applications can be considered for polyhedra with different symmetries.

In this work, we study on truncated dodecahedron and truncated icosahedron, which their symetry groups are the icosahedral group I_h . We first introduce that two metrics of which unit spheres are truncated dodecahedron and truncated icosahedron, then show that group of isometries of the three dimensional space equipped with these metric is the semi-direct product I_h and T(3), where T(3) is the group of all translations of the three dimensional space.

Keywords: Metric Geometry; Distance Functions; Polyhedra; Isometry Group.

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The Slant Helices According to N-Bishop Frame of The Timelike Curve in Minkowski 3-Space

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Abstract

N-Bishop frame is a convenient alternative frame to the Serret-Frenet frame constructed by {N,C,W}. We present some characterizations of the slant helices according to N-Bishop frame of the timelike curve in Minkowski 3-space.

Keywords: Slant helix; N-Bishop; Timelike curve.

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Introduction to Dual Covariant Derivative on Time Scales

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A time scale is an arbitrary nonempty closed subset of the real numbers. The method of time scale achieves to unify discrete and continuous forms. In our study, we introduce the concept of the dual covariant derivative on time scales at first time. Then some properties are given.

Keywords: Screw Theory; Dual Space; Time Scales; Dual Covariant Derivative.

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A Partial Solution to an Open Problem of Frenet Frame of a Curve Parametrized by Time Scales

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Abstract

The Frenet Frame of a curve on time scale was introduced an open problem in [1]. We investigate a partial solution for this open problem. First, we give some basics for a curve parametrized by time scales. Then we present some theoretical computation for Frenet frame using the delta derivative on time scales. Finally, we give a numeric example at the end of the paper.

Keywords: Frenet Frame; Time Scale; Delta Derivative; Curve.

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Nearly Metallic Kähler Manifolds

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Abstract

In this paper, we construct nearly metallic Kähler structures on Riemannian manifolds. For such a manifold with these structures, we study curvature properties. Also, we define special connections on the manifold, which preserve the associated the fundamental 2-form and satisfy some special conditions and present some results concerning them.

Keywords: Metallic Kähler structures; Connection; Curvature tensor.

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The Parallel Equidistant Ruled Surfaces on the Dual Space

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Abstract

In this work, we have defined the correspondence on the dual space of two equidistant ruled surfaces which are obtained assuming generator vectors (tangent vectors) are parallel and the distance between asymptotic planes at corresponding points is constant along their striction curves in Euclidean space. We have given the relationship between the curvatures of directrix curves of these dual parallel equidistant ruled surfaces and the relationship between Blaschke vectors belong to their spherical indicatrix curves. Also, we have showed the relationships between of these invariants and the integral invariants of these closed ruled surfaces in case of the striction curves of these dual parallel equidistant ruled surfaces are close.

Keywords: Dual parallel equidistant ruled surfaces; E. Study mapping; Integral invariants.

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The Gauss Curvatures of the Dual Parallel Equidistant Ruled Surfaces

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Abstract

In this work, we have calculated the Gauss curvatures of the dual parallel equidistant ruled surfaces which are obtained assuming generator vectors (dual tangent vectors) are parallel and the dual distance between asymptotic planes at corresponding points is constant along their striction curves (on the dual space) and we have given the relationships between of these curvatures.

Keywords: Dual parallel equidistant ruled surfaces; E. Study mapping; Gauss curvatures.

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On Semi Biharmonic Legendre Curves in Sasakian Space Forms

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Abstract

In present study, we consider semi-biharmonic Legendre curves in Sasakian space forms. We obtain the necessary and sufficient conditions for Legendre curves in Sasakian space forms to be semi-biharmonic.

Keywords: Semi-biharmonic; Legendre curve; Sasakian space form.

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f-Biminimal Maps in Generalized Space Forms

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Abstract

In this talk, we consider *f*-biminimal maps in generalized complex space forms and generalized Sasakian space forms. We also obtain the necessary and sufficient conditions for submanifolds in generalized space forms to be *f*-biminimal.

Keywords: *f*-biminimal maps; Generalized complex space form; Generalized Sasakian space form.

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Notes on Geodesics of SO(3)

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Abstract

In this study, we consider the studies related to the rotation of a rigid body. These studies contain some subjects such as the representations of the rotation matrices depended on Euler angles, Euler-Rodrigues formula and the unit quaternions. Then we examine the special orthogonal group in Euclidean 3-space, SO(3) and its tangent vector space, so(3), a diffeomorphic map from tangent sphere bundle of 2-sphere T_1S^2 to SO(3). In addition, we study Riemannian metric structure on SO(3), h and Sasaki Riemann metric g^S on T_1S^2 and then we examine the isometric map from (T_1S^2, g^S) to (SO(3), h). Moreover, we study the Euler motions of any point on a rigid body, which determine geodesics of SO(3).

In this paper we show that a system of differential equations which gives geodesics of SO(3) by using rotation matrices is equal to a system of differential equations which gives geodesics of (T_1S^2, g^S).

Keywords: Tangent sphere bundle; Special Orthogonal Group; Geodesic.

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Line Intersections on Some Projective Klingenberg Planes

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Abstract

Projective Klingenberg planes (PK-planes) are introduced as a generalization of projective planes. A PK-plane can be coordinated with any given local ring. In this work, we investigate the situations of two lines respect to each other in a PK-plane coordinated with the dual local ring $Z_4 + Z_4 \varepsilon$.

Keywords: Projective Plane; Projective Klingenberg Plane; Local Ring; Dual Local Ring.

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Plane Mechanism and Dual Spatial Motions

Aydın Altun

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Abstract

The spherical representations of the roselike curves being generated by hy(t;n,m,r,α) and ep(t;n,mr,α), the dual developable ruled roselike and hyperbolic surfaces and their graphs are given. It is well-known that the roses are generated by natural mechanism on the plane. Translation operations of curves and surfaces in generally from any Euclidean space E^n to the real sphere are given originally in this paper. The dual ruled or developable ruled surfaces are obtained by the unit spherical representations of the planer or any dimensional Euclidean space curves and surfaces.

Keywords: Plane Mechanism; Dual Spatial Motions.

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Some Properties of Parabolas Whose Vertices are on Sides of Orthic Triangle and Foci are Orthocenter

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Abstract

The parabolas which are tangent to two sides of triangle where the tangents points are two vertices are known as Artzt parabolas since they were Örst described by Artzt in 1884, [1]. Some of properties of these parabolas studied in [3] and [4] with name "Properties of parabolas inscribed in a triangle". The other parabola which is tangent to two sides of a triangle is known as parabola related to orthic triangle in [8]. Also, some properties of parabolas whose vertex and foci are on bisector line and tangent to two sides of a given triangle are studied and associated with Euler line for the triangles with particular angles in [2].

In this work, we give the properties of tangency, ideal points, intersection points, parallelism, cross ratio and harmonic conjugates points related to parabolas with the orthocenter as the focus and the pedal triangleís vertices of the orthic triangle as the vertex in a given triangle, by using barycentric coordinate points.

Keywords: Parabolas; Cross ratio; Barycentric coordinate.

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The Inverse Kinematics of Rolling Contact Motion of Timelike Surfaces in the Direction of Spacelike Unit Tangent Vector with Point Contact

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Abstract

Rolling contact between two surfaces plays an important role in robotics and engineering such as spherical robots, single wheel robots, and multifingered robotic hands to drive a moving surface on a fixed surface. The rolling contact pairs have one, two, or three degrees of freedom (DOFs) consisting of angular velocity components. Rolling contact motion can be divided into two categories: spin-rolling motion and pure-rolling motion. Spin-rolling motion has three (DOFs), and pure-rolling motion has two (DOFs). Further, it is well known that the contact kinematics can be divided into two categories: forward kinematics and inverse kinematics. In this paper, we investigate the inverse kinematics of rolling contact motion without sliding of one timelike surface on another timelike surface in the direction of its spacelike unit tangent vector by determining the desired motion and the coordinates of the contact point on each timelike surface. We get three nonlinear algebraic equations by using curvature theory in Lorentzian geometry. These equations can be reduced as a univariate polynomial of degree six by applying the Darboux frame method. This polynomial enables us to obtain rapid and accurate numerical root approximations. Moreover, we engender two fundamental parts of the spin velocity in Lorentzian 3-space: the induced spin velocity and the compensatory spin velocity.

Keywords: Darboux Frame; Forward Kinematics; Inverse Kinematics; Lorentzian 3-Space; Pure-Rolling; Rolling Contact; Spin-Rolling.

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The Inverse Kinematics of Rolling Contact Motion of Timelike Surfaces in the Direction of Timelike Unit Tangent Vector with Point Contact

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Abstract

Rolling contact between two surfaces plays an important role in robotics and engineering such as spherical robots, single wheel robots, and multifingered robotic hands to drive a moving surface on a fixed surface. The rolling contact pairs have one, two, or three degrees of freedom (DOFs) consisting of angular velocity components. Rolling contact motion can be divided into two categories: spin-rolling motion and pure-rolling motion. Spin-rolling motion has three (DOFs), and pure-rolling motion has two (DOFs). Further, it is well known that the contact kinematics can be divided into two categories: forward kinematics and inverse kinematics. In this paper, we investigate the inverse kinematics of rolling contact motion without sliding of one timelike surface on another timelike surface in the direction of its timelike unit tangent vector by determining the desired motion and the coordinates of the contact point on each timelike surface. We get three nonlinear algebraic equations by using curvature theory in Lorentzian geometry. These equations can be reduced as a univariate polynomial of degree six by applying the Darboux frame method. This polynomial enables us to obtain rapid and accurate numerical root approximations. Moreover, we engender two fundamental parts of the spin velocity in Lorentzian 3-space: the induced spin velocity and the compensatory spin velocity.

Keywords: Darboux Frame; Forward Kinematics; Inverse Kinematics; Lorentzian 3-Space; Pure-Rolling; Rolling Contact; Spin-Rolling.

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A New Approach for Inextensible Flows with Modified KdV Flow

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Abstract

In this paper, we study inextensible flows and give present a new approach for computing the geometry properties of curves by integrable geometric curve flows. We give some new solutions by using the Modified KdV flow. Finally, we obtain figures of this solutions.

Key Words: Modified KdV flow; Bäcklund transformations; curve flows.

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On Fermi-Walker Derivative with Modified Frame

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Abstract

In this paper, we obtain a new construction of curves by Fermi-Walker parallelism and derivative with modified frame. Finally, we give some characterizations according to Modified frame.

Keywords: Modified frame; Fermi Walker derivative-parallelism; Frenet frame.

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Computation of the Lines of Curvature of Parametric Hypersurfaces in $\mathbb{E}^4$

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Abstract

A line of curvature is a curve whose velocity vectors at its every point correspond to the principal directions of the surface. Maekawa et al. [3] studied the differential geometry properties of a line of curvature of parametric surfaces in 3-dimensional Euclidean space. In this talk, we give an analogue algorithm for the computation of the lines of curvature of parametric hypersurfaces in Euclidean 4-space.

Keywords: Line of curvatures; Darboux frame; Curvature.

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Translation Surfaces According to a New Frame

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Abstract

In this study, we investigated translation surfaces according to a new frame which is called Flc-frame in three-dimensional Euclidean space. We obtained the curvatures of the translation surface in terms of new curvatures. Also, some characterizations were found for these surfaces.

Keywords: Translation surface; Flc-frame; Curvatures.

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Timelike Conchoid Curves in Minkowski Plane

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Abstract

A conchoid d(t) of a planar curve c(t) is a curve at a constant length d measured with respect to a fixed-point O. For every line through O that intersects the given curve at a focus A, the two points on the line which are distance d from the focus A are on the conchoid. In this paper, we studied timelike conchoid curves in Minkowski plane.

Keywords: Conchoid of Nicomedes; Minkowski plane; Lorentz polar representation.

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A New Class of Nearly Kenmotsu Manifolds

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Abstract

The aim of this work is to show that, in η Einstein nearly Kenmotsu manifolds with projective curvature tensor P, and conharmonic curvature tensor N, satisfy the conditions $R(X,Y).P = 0$ and $R(X,Y).N = 0$ respectively. And so, to obtain a new class of η Einstein nearly Kenmotsu manifolds.

Keywords: Nearly Kenmotsu manifold; η Einstein manifold; Projective curvature tensor; Conharmonic curvature tensor.

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Householder Transformation with Hyperbolic Numbers

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Abstract

In this study, we examine some properties of Householder transformation using the set of hyperbolic numbers D={a+bh : h²=1} and give some applications as reflection in the Lorentzian plane. Moreover, we express the householder transformation in the modul of n-dimensional hyperbolic number vectors.

Keywords: Hyperbolic (Double) Numbers; Householder Transformation, Lorentz Plane, Reflections.

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On Contact CR-Submanifolds of a Sasakian Manifold

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Abstract

In this paper, we study the differential geometry of contact CR-submanifolds of a Sasakian manifold. Necessary and sufficient conditions are given for a submanifold to be a contact CR-submanifold in Sasakian manifolds and Sasakian space forms. Finally, the induced structures on submanifolds are investigated, these structures are categorized and we discuss these results.

Keywords: Sasakian manifold; Sasakian space form; Contact CR-submanifold.

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Pseudoparallel Invariant Submanifolds of (LCS)n-Manifolds

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Abstract

The aim of this paper is to study the invariant submanifolds of (LCS)n-manifolds. We study pseudoparallel, generalized Ricci-pseudoparallel and 2-pseudoparallel invariant submanifolds of a (LCS)n-manifold and get the necessary and sufficient conditions for an invariant submanifold to be totally geodesic and give some new results contribute to differential geometry.

Keywords: Normal paracontact metric manifold; Ricci-pseudosymmetric manifold; Ricci soliton.

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On Contact Pseudo-Slant Submanifolds in a LP-Cosymplectic Manifold

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Abstract

New results are shown for the totally geodesic situations of contact pseudo-slant submanifolds in a LP-cosymplectic manifold. Necessary and sufficient conditions for a submanifold to be contact pseudo-slant are given. The contact pseudo-slant product is characterized and necessary and sufficient conditions for a contact pseudo-slant submanifold to be the contact pseudo-slant product is given.

Keywords: LP-Cosymplectic manifold; Contact pseudo-slant submanifold; Totally geodesic submanifold**.**

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Bernoulli Polynomial Solutions of System of Frenet-Like Linear Differential Equations in Normal Form Arising from Differential Geometry

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Abstract

In many scientific problems, systems of differential equations have been encountered. Usually, the system of linear differential equations in the normal form appears in differential geometry, physics, and kinematics. Most of these type systems have not analytic solutions; so numerical methods are required. In this study, by using a new matrix method based on Bernoulli polynomials we obtain the solution sets of the Frenet-Like system of differential equations with variable coefficients in the normal form under the initial conditions. The presented method converts the problem into a system of algebraic equations based on the matrix operations and collocation points; thereby, the main results associated with the solution and applicability of the method is performed.

Keywords: Bernoulli polynomials; Frenet-Like system of differential equations; Matrix method; Collocation points.

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An Examination on Curves with Common Principal Normal and Darboux Vectors in E^3

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Abstract

Deriving a curve based on the other curve is a useful method in differential geometry. Evolute and involute curve, Manheim curves are given as the famous examples. Also, another example; Bertrand curves have common principal normal lines. In this paper we examined curves with common principal normal and Darboux vectors in E^3. If the principal normal vector of first curve and Darboux vector of second curve are linearly dependent, then first curve is called ND* curve, and the second curve is called ND* partner curve. Also, we give Frenet apparatus of the second curve based on the Frenet apparatus of first curve.

Keywords: Darboux vector; Deriving curve; Frenet frame.

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On Fuzzy Line Spreads

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Abstract

In this study, the fuzzy line spreads are studied in fuzzy projective plane with base plane that is a projective plane over GF(2) and GF(3).

Keywords: Projective plane; Fuzzy projective plane; Spread.

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Fiber Diagonal triangle and Fiber Quadrangular Set

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Abstract

In this study, the fibered versions of the diagonal triangle and the quadrangular set of a fiber complete quadrangle in fibered projective planes are determined. And then some related results with them are given.

Keywords: Fibered projective plane; Complete quadrangle; Quadrangular set.

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A New Method to Obtain the Position Vector of Slant Helices in Lorentz-Minkowski 3-Space

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Abstract

We work the problem of finding parametric representation of the position vector of slant helices in the Lorentz-Minkowski 3-space. To do this, we constitute a new coordinate system by means of the natural coordinate system of $\mathbb{R}_1^3$. Also, we give a proposition which presents the condition of being slant helix. Using them, we obtain a theorem which presents the parametric representation of the position vector of slant helices. As an application of this theorem, we also obtain that of the Salkowski curves and the anti-Salkowski curves.

Keywords: Local coordinate system; Lorentz-Minkowski 3-space; Slant helix.

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The Frenet Frames of Lorentzian Spherical Timelike Helices and Their Invariants

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Abstract

We gave the general parametrizations of the Lorentzian spherical timelike helices at 3-dimensional Lorentz-Minkowski space R_1^3 with sign $(+;+;-)$ and, obtained Darboux derivative formulas and geodesic curvature functions [1]. In this paper we find the curvature functions of the Frenet frames of the Lorentzian spherical timelike curves and give the Lorentzian geometric interpretations and some examples of these curves for special values of the hyperbolic angle ρ.

Keywords: Lorentzian sphere; Timelike curves; Frenet frame.

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Timelike Helices on The Lorentzian Sphere $S_1^2(r)$

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Abstract

In this paper, we define the parametric equations of timelike helices on a Lorentzian sphere with radius r making a hyperbolic angle ϴ with the timelike axis z. The Darboux frame, the derivative formulae, instantaneous rotation vector and geodesic curvature function are obtained. Furthermore, we give some examples of the timelike curves for special values of the hyperbolic angle ϴ.

Keywords: Lorentzian sphere; Timelike helices; Darboux frame.

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Quaternionic And Split Quatenionic Principal Curvatures and Principal Directions

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Abstract

In this study, firstly we study about geometrical applications of quaternions and split quaternions. Because the vector part of a quaternion is a vector in R^3, the geometry of R^3 is reflected in the algebraic structure of the quaternions. Many operations on vectors can be defined in terms of quaternions. The maximum and minimum of the normal curvature at a given point on a surface are called the principal curvatures. The principal curvatures measure the maximum and minimum bending of a regular surface at each point. The principal directions corresponding to the principal curvature are perpendicular to one another. Also, we built the shape operator for quaternions and split quaternions. We get the matrix corresponding to this shape operator. This is the matrix whose eigenvalues and eigenvectors we want to find. Finally, the quaternionic principal curvatures and quaternionic principal directions were created with these characteristic values.

Keywords: Quaternion; Split Quaternion; Principal Curvature; Quaternionic Principal Curvature; Principal Directions; Quaternionic Principal Directions.

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Some Classes of Invariant Submanifolds of (k,μ)-Contact Manifold

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Abstract

The aim of this work is to study the invariant pseudoparallel, 2-pseudoparallel, Ricci-generalized pseudoparallel and 2-Ricci generalized pseudoparallel submanifolds of (k,μ)-contact manifolds. Also, the conditions $Z(X,Y)h = 0$ and $Z(X,Y)\tilde{\nabla}h = 0$ are searched on invariant submanifolds of (k,μ)-contact manifolds and manifold is classified, where Z is the concircular curvature tensor.

Keywords: Contact Metric manifolds; (k,μ)-contact manifolds; Invariant Submanifold, Pseudoparallel and Ricci generalized pseudoparallel submanifolds.

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On Intuitionistic Fuzzy Menelaus and Ceva Theorems

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Abstract

In this study, the intuitionistic fuzzy projective plane with base plane that is a projective plane is considered. The conditions to intuitionistic fuzzy versions of the 6-figures of Menelaus and Ceva are determined in this plane.

Keywords: Projective plane; Fuzzy projective plane; Intuitionistic fuzzy projective plane; Menelaus and Ceva 6-figures.

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On Some Classical Theorems in Fibered Projective Plane

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Abstract

In this work, we considered fibered projective planes and looked at the fibered versions of some classical theorems in projective planes.

Keywords: Projective plane; Fibered projective plane.

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Some Applications of Generalized Bicomplex Numbers on Motions in Four Dimensional Spaces

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Abstract

In this paper, we give some applications of generalized bicomplex numbers on motion in four dimensional generalized spaces $R^4_{\alpha\beta}$ and define homothetic motions and rotational motions on some hypersurfaces in R^4_2 and R^4.

Keywords: Bicomplex Numbers; Generalized Bicomplex Numbers; Kinematics.

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On the Affine Planes Embedded in NFPG(2,9)

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Abstract

In this study, we determine projective and affine planes embedded in Near Field Plane of order 9 by using C# program.

Keywords: Near Field; Projective Plane; Affine Plane.

References

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On Some Properties of the (6,2)-arc in NFPG(2,9)

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Abstract

In this work, we examine the non-Veronesian arc and Pascal's Theorem by using the points of (6,2)-arc in the left nearfield projective plane of order 9.

Keywords: Projective plane; Veronesian arc; Pascal's Theorem.

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Right Conoid in Euclidean 3-Space

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Abstract

In this study, the conditions for the Right Conoid Surface to be flat and minimal are determined. Then, by using Gauss and mean curvature with second Gauss and mean curvature of surface, the conditions for Right Conoid Surface being Weingarten, linear Weingarten and quadratic are researched. Furthermore, we classify Right Conoid in the three-dimensional Euclidean space E^3 satisfying some algebraic equations in terms of the coordinate functions and the Laplacian operators with respect to the first, the second and the third fundamental forms of the Right Conoid. We also give explicit forms of this surfaces.

Keywords: Right Conoid; Second Gaussian curvature; Second mean curvature; Weingarten equation; Linear Weingarten equation; Laplacian operators; Quadric surface.

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On the Differential Geometry of $GL_{p,q}(1|1)$

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Abstract

In the last decade, the theory of quantum super groups [1] has arised as a natural generalization of the notion of Lie supergroups. They have found application in several areas of mathematics and mathematical physics [2]. Quantum supergroups can be realized on a quantum superspace in which coordinates are noncommuting [3]. In the recent development of differential calculus on the quantum groups two main concepts are readily seen. First of them, formulated by Woronowicz [4] is known as bicovariant differential calculus on the quantum groups. We shall consider this concept.

The differential calculus on the quantum supergroups involves functions on the supergroup, differentials, and differential forms. A differential calculus on the quantum supergroup $GL_{p,q}(1|1)$ was introduced by Celik [5], using consistency of calculus. In this work, we construct a two-parameter differential calculus on the quantum supergroup $GL_{p,q}(1|1)$ using covariance.

Keywords: Quantum supergroup; Super-Hopf algebra; $\mathbb{Z}_2$-graded Differential calculus.

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The Symmetry Group of the Differential Calculus on $F(\mathbb{R}_q(1|1))$

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Abstract

A differential calculus on the quantum groups formulated by Woronowicz [5] is known as bicovariant differential calculus on the quantum groups. Differential calculus can be applied to a super-Hopf algebra considered as a left (right) quantum superspace with respect to the coproduct.

The function algebra on the extended quantum superplane, denoted by $F(\mathbb{R}_q(1|1))$, is a super-Hopf algebra [1]. Two bicovariant differential calculi on the super-Hopf algebra $F(\mathbb{R}_q(1|1))$ is given in [4].

In this work, we have introduced quantum supergroups which are the symmetry groups of the differential calculi and show that both of them are the super-Hopf algebra. For some specific choices of deformation parameters, these supergroups coincide with the groups given in [3] and [2].

Keywords: Quantum superplane; Super-Hopf algebra; Differential calculus; Quantum supergroup.

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On Characterization of Inextensible Flows with Modified Orthogonal Frame

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Abstract

In this paper, we study inextensible flows of curves according to modified orthogonal frame in space. We research inextensible flows of curves according to modified orthogonal frame with necessary and sufficient conditions for an inelastic curve flow.

Key Words: Inextensible flows; Modified Orthogonal frame.

References

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On Some Surfaces by Ribbon Frame

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Abstract

In this paper, we study ruled surfaces by Ribbon frame. Finally, we give some characterizations for developable surfaces according to Ribbon frame.

Key Words: Ribbon frame, Ruled Frame, Frenet frame.

References

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On the Classification Of Generalized m-Quasi Einstein Manifolds

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Abstract

The study of Einstein manifolds and their several generalizations is always an attractive topic in modern Riemannian geometry. There has been increasing interest especially on so-called quasi-Einstein manifolds. As known, Riemannian manifold $((M)^n, g)$ with a potential function f on M, is called generalized m-quasi Einstein, if the associated m-Bakry-Emery Ricci tensor $Ric_m^f = Ric + \nabla^2 f - \frac{1}{m} df \otimes df$ is a constant multiple of the metric g [2].

In this talk, it will be mentioned that in which conditions the Ricci soliton structure can be observed on the generalized m-quasi Einstein manifolds. Then, some examples for this kind of manifolds will be given in the following part of the talk. Additionally, the conditions of rigidity and being warped product are researched on these manifolds.

The study came into being with the motivation of the following papers in the reference list.

Keywords: Generalized m-quasi Einstein manifolds; Ricci Solitons; Rigidity.

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On the Square Roots of 2x2 Real Matrices

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Abstract

In this study, first, we give some known methods to find square roots of a real 2x2 matrix. After, we give a new method for finding the square root of a 2×2 real matrix. For this, we use polar forms and De Moivre's formulas for 2x2 matrices.

Keywords: Hybrid Number, Split Quaternion, Roots of Matrix, De Moivre's Formula.

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A Geometric Modeling of Tracheal Elements of Chard (*Beta vulgaris* var. *cicla* L.) Leaf

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Abstract

In this study, we give a geometric description of the tracheal elements of chard (*Beta vulgaris* var. *cicla* L.) which is a widespread cultivated plant in Turkey. It is used as an edible plant and antidiabetic in traditional medicine plant for its leaves. We have shown that the tracheal elements which are taxonomical value of the plant can be considered as a surface of revolution or a tubular shape along a special curve.

Keywords: Chard; Tracheal elements; Geometric model.

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Geometric Modeling and Statistical Comparison of Some Sage (*Salvia* L.) Glandular Hairs

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Abstract

In this study, we give a geometric description of glandular hairs of some Sage (*Salvia* L.) which are aromatic and often used as herbs, spices, folk medicines and fragrances thanks to their glandular hairs. *Salvia* has the richest glandular hair with in Family Lamiaceae. Glandular trichomes that develop from epidermal cells are generally considered as the site of biosynthesis or accumulation of essential oils. The compounds produced by glandular hairs with antiseptic characteristics decrease DNA synthesis in the cell. This feature is important in the diagnosis and treatment of cancer. Glandular hair is divided into different types according to the shape of the head cells. These differences correspond to the production of the different materials. In the study, the head part of glandular hairs of 18 *Salvia* taxa tried to be defined geometrically and numerical data obtained from anatomical studies were evaluated statistically to compare the examined taxa with each other. It has been also found that the results from numerical analysis of the glandular hairs characters can provide additional evidences that correspond to the anatomy for the recognition of the taxa.

Keywords: Anatomy; Geometric model; Glandular hair; Salvia; Statistically.

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New Type Direction Curves in E_1^3

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Abstract

In the present paper, the notion of osculating direction curve and osculating donor curve of the non-lightlike Frenet curve in the Minkowski 3-space E_1^3 are introduced. In addition, some new characterizations and results for these curves are given. Furthermore, the relationships between these curves and some special curve pairs is examined.

Keywords: Frenet curve; direction curve; Mannheim curve.

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A Study on Null Quaternionic Curves in Minkowski spaces

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Abstract

In this study, we have some results on null quaternionic rectifying curves and null quaternionic similar curves in Minkowski space E_1^3. Besides, we give definition of null quaternionic (1,3)-Bertrand partner curves in E_1^4. Thus, we obtain relations between curvatures of (1,3)-Bertrand partner curves in Minkowski spaces.

Keywords: Null Quaternionic Curve; Rectifying curve; Similar partner curve; Bertrand partner curve.

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A Study on Differential Equations of Null Quaternionic curves

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Abstract

In this presentation, we have some results about differential equations of null quaternionic curves according to components of Frenet frames in Minkowski 3-space E_1^3 and Minkowski space-time E_1^4.

Keywords: Null Quaternionic curve; Differential equations; Serret-Frenet formulae.

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On the L-Hyperbola and L-Parabola in the Lorentz-Minkowski Plane

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Abstract

We present the L-hyperbolas with two foci and focus-directrix in Lorentz- Minkowski plane. We show a parabola as the locus of points equidistant from a focus and a line (the directrix). Also, we define the locus of points equidistant from two distinct points and lines in this plane.

Keywords: Lorentz- Minkowski plane; L-hyperbola; L-parabola.

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Some Properties Generic Submanifolds of LP-Cosymplectic Manifold

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Abstract

The aim of the present paper is to define and study generic submanifolds of LP-Cosymplectic manifold. We investigate the geometry of leaves which arise the definition of a generic submanifold and proving a necessary and sufficient condition for a generic submanifold to be totally geodesic. We also consider parallel distributions of generic submanifolds.

Keywords: Generic submanifold; Lorentz almost para contact manifold; LP Cosymplectic manifold.

Acknowledgment: This paper is supported by Amasya University Research Project (FMB-BAP 18-0335)

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Some Curvature Properties of CR-Submanifolds of a Para Sasakian Manifold

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Abstract

In this paper different curvature tensors on para Sasakian manifold have been studied. We investigate constant φ-holomorphic sectional curvature and L-sectional curvature of para Sasakian manifolds, obtaining conditions for them to be constant of para Sasakian manifolds in such condition. We calculate the Ricci tensor and scalar curvature in all the cases. Moreover, we investigate some properties of CR-submanifolds of a para Sasakian space form whose φ-sectional curvature is constant. We consider sectional curvature of CR-product of a para Sasakian manifolds.

Keywords: Para Sasakian manifold; CR submanifold; CR product.

Acknowledgment: This paper is supported by Amasya University Research Project (FMB-BAP 18-0335)

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A Hamilton-Jacobi Theory for Implicit Differential Systems

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Abstract

We propose a geometric Hamilton-Jacobi theory for systems of implicit differential equations. In particular, we are interested in implicit Hamiltonian systems, described in terms of Lagrangian submanifolds of TT*Q generated by Morse families. The implicit character implies the nonexistence of a Hamiltonian function describing the dynamics. This fact is here amended by a generating family of Morse functions which plays the role of a Hamiltonian. A Hamilton–Jacobi equation is obtained with the aid of this generating family of functions. To conclude, we apply our results to singular Lagrangians by employing the construction of special symplectic structures.

Keywords: Lagrangian submanifolds; Hamilton-Jacobi equations; Implicit differential equations; Morse families.

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Conformal Generalization of Nambu-Poisson Geometry in 3D

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Abstract

Non-canonical Hamiltonian realizations of dynamical systems will be discussed in the framework of Nambu-Poisson geometry. To this end, we shall first address the cosymplectic geometry, the conformal theory of Hamiltonian dynamics, and the Nambu-Poisson manifolds. After presenting basics of these theories, a conformal generalization of Nambu-Poisson geometry will be introduced in order to arrive at Hamiltonian formulations of non-autonomous dynamical systems. Here, while deriving integrals of the dynamical systems, the method of Jacobi's last multiplier will be employed. Accordingly, the proposed theoretical constructions will be studied in some biological models by exhibiting their non-canonical Hamiltonian characterizations.

Keywords: Cosymplectic manifolds; Nambu-Poisson manifolds; Conformal Hamiltonian Systems.

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Indicatrices of the Curves in Affine 3-Space

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Abstract

In this study, we investigated the tangent, normal and binormal indicatrix curves of space curves in affine 3-space in both general case and in special case of space curve is constant curvature curve.

Keywords: Affine sphere; Indicatrices Curves; Affine Frenet Vectors.

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Special Curves in Euclidean 3-Space

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Abstract

In this study, we investigated the special curves in Euclidean 3-space which are the curves whose tangents, principal normals, binormals and Darboux lines intersect a constant proper line at each points of the curve and we obtained some characterizations.

Keywords: Helix; Euler Spiral; Frenet vectors.

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Characterizations of Dual Curves in Dual Space D^3 According to Dual Bishop Frame

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Abstract

In this study, some characterizations of some special dual curves such as dual general helices, dual slant helices, dual Darboux helices and dual similar partner curves are given according to dual Bishop frame.

Keywords: Dual Bishop Frame; Dual general helices; Dual slant helices; Dual Darboux helices.

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On Normal Complex Contact Metric Manifolds Admitting a Semi-symmetric Non-Metric Connection

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Abstract

In this work, the object is to study a semi-symmetric non-metric connection on a normal complex contact metric manifolds. Curvature properties and fundamental facts are given. Also, flatness of some curvature tensors is studied.

Keywords: Complex contact manifolds; Semi-symmetric non-metric connection; Conformal curvature tensor; Concircular curvature tensor.

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Invariant Submanifolds of Normal Complex Contact Metric Manifolds

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Abstract

The aim of this paper is to study invariant submanifolds of normal complex contact metric manifolds. The integrability conditions for invariant distribution are given. Also, necessary and sufficient conditions are given for these types of submanifolds to be totally geodesic.

Keywords: Normal complex contact metric manifolds; Invariant submanifolds; semi-parallel invariant submanifolds.

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Approaching Generalized Quaternions from Matrix Algebra

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Abstract

In recent years, generalized quaternions, a natural generalization of quaternion and split quaternions, have increasingly become the focus of attention. Essentially generalized quaternions are derived from quadratic form theory studies. Many authors are studying the generalized quaternions with different aspects.

In this study, the existence of ordered triple matrices isomorphic to a base of the generalized quaternion algebra is shown. Then the properties of the fundamental matrices obtained from these matrix triplets are examined.

Keywords: Generalized quaternion; Fundamental matrices; Eigenvectors.

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Generalized Quaternions in Spatial Kinematics in an Algebraic Sense

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Abstract

Generalized quaternion algebra has arised in recent years as a means of quadratic form theory studies. This structure is essentially a natural generalization of quaternion algebra. Quaternions and split quaternions, as well as in generalized quaternions, are also being studied by many researchers.

In these studies, generalized quaternions and Hamilton operators are investigated for finite spatial displacements. There are 4x4 type matrices in the screenings. Relative movement for a generalized 3-dual sphere is taken by Hamilton operators for a generalized quaternion. In addition, the relation between Hamiltonian operators and transformation matrices is given differently.

Well-known results for quaternion and split quaternions are obtained as special cases from the results which are obtained for the generalized quaternions.

Keywords: Generalized quaternion; Spatial kinematics; Hamilton operators

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Generalized Complex Contact Space Forms

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Abstract

The normal complex metric contact manifold which has constant $\mathcal{GH}$ $-$sectional is called complex contact space form. This type of manifold has a curvature form. In this study we generalized the curvature form of complex contact space form and we call this generalized complex contact space forms. This notion includes both complex space forms and real contact space forms classes. We obtain some curvature properties of generalized complex contact space forms and examine flatness conditions.

Keywords: Complex contact manifolds; Complex contact space forms; Complex space forms.

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On the Curvatures Properties of Tangent Bundle of Hypersurfaces in a Euclidean Space

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Abstract

Let's consider an immersed orientable hypersurface $f: M \to \mathbb{R}^{n+1}$ of the Euclidean space (f an immersion) and observe that the tangent bundle TM of the hypersurface M is an immersed submanifold of the Euclidean space $\mathbb{R}^{2n+2}$. First, we introduce an induced metric on tangent bundle, which we are calling as rescaled induced metric. Then we investigate some curvature properties of such a tangent bundle.

Keywords: Tangent bundle; Hypersurface; Rescaled induced metric; Curvature tensor.

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Geometry of Lightlike Submanifolds of Golden Semi Riemannian Manifolds

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Abstract

The Golden Ratio is fascinating topic that continually generated news ideas. A Riemannian manifold endowed with a Golden Structure will be called a Golden Riemannian manifold. The main purpose of the present paper is to study the geometry of radical transversal lightlike submanifolds and transversal lightlike submanifolds of Golden Semi-Riemannian manifolds. We investigate the geometry of distributions and obtain necessary and sufficient conditions for the induced connection on these manifolds to be metric connection.

Keywords: Lightlike manifold; Golden semi Riemannian manifold; Radical transversal lightlike submanifold; Radical screen transversal submanifold.

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A Study on the Rotated Surfaces in Galilean Space

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Abstract

The scalar product of the vectors $u = (u_1, u_2, u_3, u_4), v = (v_1, v_2, v_3, v_4)$ in G_4 is defined as

$$\langle u, v\rangle_{G_4} = \begin{Bmatrix} u_1 v_1, & if\ u_1 \neq 0\ or\ v_1 \neq 0 \\ u_2 v_2 + u_3 v_3 + u_4 v_4, & if\ u_1 = 0,\ v_1 = 0 \end{Bmatrix}$$

Let $\alpha: I \subset R \to G_4$, $\alpha(s) = \big(s, y(s), z(s), w(s)\big)$ be a curve parametrized by arclength parameter s in G_4. The vectors of the Frenet-Serret frame, that is, respectively, are defined as

$$t(s) = \alpha' = (1, y'(s), z'(s), w'(s)),\ n(s) = \frac{t'(s)}{\kappa(s)},\ b(s) = \frac{n'(s)}{\tau(s)},\ e(s) = \mu t(s) \times n(s) \times b(s),$$

where the real valued functions $\kappa(s) = \|t'(s)\|$ is called the first curvature of the curve $\alpha,$ the second curvature function is defined as $\tau(s) = \|n'(s)\|,$ the third curvature function is defined as $\sigma = \langle b', e\rangle$. For the curve in G_4, we have the following Frenet-Serret equations:

$$t' = \kappa n, n' = \tau b, b' = -\tau n + \sigma e, e' = -\sigma b. \qquad (1.1)$$

In this study, we give a brief a description of surfaces of rotation of four-dimensional Galilean space using a curve in G_4. Firstly, we obtain the matrices of rotation corresponding to the appropriate Galilean space and then we generate surfaces of rotated and the first fundamental form of these π^1(u,v,s), π^2(u,v,s), π^3(u,v,s).

Keywords: Rotated surfaces; Galilean space; First fundamental form.

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The Fermi-Walker Derivative and Dual Frenet Frame

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Abstract

In this study, we have analyzed the Fermi-Walker derivative along any dual curve in dual space. Fermi-Walker transport, non-rotating frame and Fermi-Walker termed Darboux vector concepts are redefined along the dual curve according to the dual Fenet frame.

We proved the dual Frenet frame is a non-rotating frame along the dual planar curves. Moreover, we show that Fermi-Walker termed Darboux vector is Fermi-Walker transported along the dual anti-Salkowski curves.

Keywords: Fermi-Walker derivative; Fermi-Walker transport; Non-rotating frame; Fermi-Walker termed Darboux vector; Dual curve; Dual Frenet frame.

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The Fermi-Walker Derivative and Non-Rotating Frame in Dual Space

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Abstract

In this study, we have explained that the Fermi-Walker derivative and Fermi-Walker transport along any dual curve in dual space. Non-rotating frame and Fermi-Walker termed Darboux vector notions are analyzed according to the different dual frames.

We proved that unlike the dual Frenet frame, the dual Darboux Frame and the dual Bishop frame are non-rotating frame along the all dual curves. In addition, these concepts are applied to an example. In the example, it is shown that the dual Bishop frame is a non-rotating frame for a dual helix curve.

Keywords: Fermi-Walker derivative; Fermi-Walker transport; Non-rotating frame; Fermi-Walker termed Darboux vector; Dual curve; Dual Darboux frame; Dual Bishop frame; Dual helix.

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On 3-dimensional Almost Golden Riemannian Manifold

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Abstract

The differential geometry of the Golden on Riemannian manifolds is a popular subject for mathematicians. In 2007, Hretcanu [1] introduced the golden structure on manifolds. In [2], Z. Olszak derive certain necessary and sufficient conditions for an almost contact structure to be normal.

Our goal in this talk, is to introduce a new class of three dimensional almost Golden Riemannian manifolds that has a relationship with the almost contact structure, and we construct a concrete example. Then, we are particularly interested in three more special types (Golden Sasaki, Golden Kenmotsu and Golden cosymplectic). We present many examples.

Keywords: Almost Golden structure; Almost contact metric structure.

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Sasakian Structure on The Product of Manifolds

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Abstract

When studying the product of two almost contact metric manifolds, Caprusi [2] established that this product is an almost Hermitian manifold. He characterized it for some classes of manifolds in the topic of cosymplectic geometry. He showed that this product is Hermitian, Kählerian, almost Kählerian or nearly Kählerian if and only if the two factors are normal, cosymplectic, almost cosymplectic or nearly cosymplectic, respectively.

Blair and Oubiña [1] studied conformal and related changes of the product metric on the two almost contact manifold. They proved that if one factor is Sasakian, and the other is not, but is locally of the type studied by Kenmotsu. The results are more general and given in terms of trans-Sasakian, α-Sasakian and β-Kenmotsu structures. Finally, they asked the open question: What kind of change of the product metric will make both factors Sasakian?

Regarding this result, one can ask if it is valid only in the case of cosymplectic geometry. In other words, what remarkable classes of structures can be induced on the product of two manifolds in Riemannian geometry?

Here we introduce a new complex structure on the product of an almost contact metric manifold and an almost Hermitian manifold with exact Kähler form. We prove that this product is Sasakian if and only if the first factor is Sasakian and the other is Kählerian. Next, we construct an example.

Keywords: Product manifolds; Sasakian structures; Kählerian structures.

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A Generalization of Surfaces Family with Common Smarandache Asymptotic Curves in Galilean Space

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Abstract

In this study, we examine how to construct surfaces family with special Smarandache asymptotic curves in G^3. We give the family of surfaces as a linear combination of the components of the Galilean frame and derive the conditions for coefficients to hold both the asymptotic and the isoparametric requirements. Finally, by using generalized marching-scale functions, we illustrated some surfaces about our method.

Keywords: Galilean space; Asymptotic curve; Smarandache curve.

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Some Characterizations of Curves in Spaces with Density

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Abstract

In this study, we investigate curves in the plane with density e^{ax+by} and we calculate the φ-curvature and φ-torsion of a curve in the plane with density e^{ax+by}. We also examine some special cases of these curves according to the constants a and b. In these special cases, we find the curvature and torsion of the curve in the plane with density. By using the computed φ-curvature and φ-torsion of a curve in the plane with density we obtain the necessary conditions for being the straight line or planar of curve in Galilean space with density. In addition, we calculate the torsion and the curvature of a curve in Galilean space with densities $e^{ax+by+cz}$ and $e^{ax^2+by^2+cz^2}$, respectively, where a, b, and c are arbitrary real constants. Finally, we give some characterizations and some examples.

Keywords: Curves; Curves with density; Curvature; Galilean metric and Galilean space.

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Some Characterizations of Rotational Surfaces Generated by Cubic Hermitian Curves

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Abstract

In order to provide flexible approaches for designers, we construct the rotational surfaces generated by cubic Hermitian curves. Possessing two local shape control parameters exhibit better performance when adjusting its local shapes through two local shape control parameters. Particularly, to adjust and control the shapes of rotational surfaces more elegantly, we present the rotational surfaces generated by cubic Hermitian curves with two local shape parameters. Further, we explore the properties of rotational surfaces, as well as its applications in rotational surface designs. Moreover, we supply the modeling examples to illustrate the proposed method in admitting the easy control of the shape of a surface. Finally, we give some characterizations for these surfaces obtaining the Gaussian and mean curvatures.

Keywords: Hermitian curves; Rotational surfaces; Shape parameter.

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Shape Operator Along a Surface Curve and Its Applications

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Abstract

In this study, we consider a surface curve in Euclidean 3-space and compute the matrix of the shape operator of the surface along the curve depending on the normal curvature and geodesic torsion of the curve. We obtain the Gaussian and mean curvatures of the surface via these curvatures and give an easy proof of the Beltrami-Enneper theorem. Also, the geodesic curves of a plane, a sphere and a circular cylinder are obtained by a new method.

Keywords: Shape operator; Darboux frame; Geodesic curve.

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Screen Conformal Lightlike Hypersurfaces of a Golden Semi-Riemannian Manifold

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Abstract

We study screen conformal screen semi-invariant lightlike hypersurfaces of a golden semi-Riemannian manifold. We obtain necessary and sufficient conditions for screen conformal screen semi-invariant lightlike hypersurfaces to be locally lightlike product manifolds. We give a condition for its Ricci tensor to be symmetric.

Keywords: Golden semi-Riemannian manifolds; Screen semi-invariant lightlike hypersurfaces; Screen conformal.

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Half Lightlike Submanifolds of a Golden Semi-Riemannian Manifold

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Abstract

We present half lightlike submanifolds of a golden semi-Riemannian manifold. We prove that there is no radical anti-invariant half lightlike submanifold of a golden semi-Riemannian manifold. We obtain results for screen semi-invariant half lightlike submanifolds of a semi-Riemannian golden manifold. We prove the conditions of integrability of distributions screen semi-invariant half lightlike submanifolds and we study the geometry of leaves of distributions.

Keywords: Golden semi-Riemannian manifolds; Golden Structure; Half lightlike submanifolds; Screen Semi-invariant.

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Basic Concepts of Lorentz Space

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Abstract

In the study, Pedoe inequality are examined for spacelike pure triangles, timelike pure triangles, and non pure triangles in Lorentz plane. Also, some fundamental concepts of linear algebra and analytical geometry are examined in Lorentz space.

Keywords: Lorentz space; Lorentz plane; Trigonometry

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Inequalities on Screen Homothetic Lightlike Hypersurfaces of Lorentzian Product Manifolds with Quarter-Symmetric Non-metric Connection

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Abstract

In this paper, we introduce some inequalities involving k-Ricci curvature and k-scalar curvature on a screen homothetic lightlike hypersurface of a Lorentzian product manifold admitting with quarter-symmetric non-metric connection. Using these curvatures, we compute Chen-Ricci inequality and Chen inequality on a screen homothetic lightlike hypersurface of a Lorentzian product manifold with quarter-symmetric non-metric connection. Finally, we give some characterizations for lightlike hypersurfaces.

Keywords: Lightlike hypersurface; Lorentzian manifold; Quarter-symmetric non-metric connection.

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Screen Transversal Cauchy Riemann Lightlike Submanifolds of Indefinite Kaehler Manifolds

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Abstract

In the present paper, we introduce a new class of lightlike submanifolds, namely, Screen Transversal Cauchy Riemann (STCR)-lightlike submanifolds, of indefinite Kaehler manifolds. We show that this new class is an umbrella of screen transversal lightlike, screen transversal totally real lightlike and CR-lightlike submanifolds. We give an example of a STCR lightlike submanifold, investigate the integrability of various distributions, obtain a characterization of such lightlike submanifolds in a complex space form and find new conditions for the induced connection to be a metric connection.

Keywords: Lightlike hypersurface; Lorentzian manifold; Quarter-symmetric non-metric connection.

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Some Associated Curves of Binormal Indicatrix of a Curve in Euclidean 3-Space

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Abstract

In this study, some associated curves of binormal indicatrix of a curve in Euclidean 3-space are defined as integral curves of vector fields generated by Frenet vectors of the binormal indicatrix. Some intrinsic properties between these curves such as Frenet vectors and curvatures are obtained. By the aid of these associated curves, efficient methods to construct helices and slant helices from special spherical curves such as circles on unit sphere, spherical helices, and spherical slant helices are given.

Keywords: Associated curves; Binormal indicatrix; Direction curves; Helix; Slant helix; Spherical helix; Spherical slant helix.

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Stability Measures of Sierpinski Fractal

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Abstract

Almost everything around us belongs to a fractal family. Fractal is defined as nested objects extending to infinity. In its simplest form, a fractal is a geometric structure that resembles itself on different scales. Graph theory owes many powerful ideas and structures to geometry. Fractals are also the structures that come to mind in graph theory.

The Sierpinski graphs are strongly associated with the famous fractals called Sierpinski triangle or Sierpinski gasket. Sierpinski graphs have applications in different areas of graph theory, topology, probability, dynamic systems, and psychology.

In our daily life many problems can be solved by using graphs. A graph is a way of modelling relations between objects. In graph theory, many parameters are defined to measure the reliability of a graph to deformation after the collapse of certain vertices (points) or edges (sides). If a vertex or an edge is damaged, then the graph loses its effectiveness. In this study, some of these vulnerability parameters are applied to Sierpinski graph to measure its stability and general results are obtained.

Keywords: Sierpinski Triangles; Fractals; Graph Theory.

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Hyperelastic Curves in SO(3)

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Abstract

We study hyperelastic curves in Lie groups G equipped with bi-invariant Riemannian metrics. We first give a theorem that states a hyperelastic curve in a Riemannian manifold can be characterized by a differential equation given by special initial conditions. When the manifold is a Lie group G equipped with bi-invariant Riemannian metrics, we derive the Euler-Lagrange equation which characterizes the hyperelastic curve with regard to the Lie reduction of a curve γ in G. For G=SO(3), we investigate solutions of the Euler-Lagrange equation.

Keywords: Hyperelastic curve; Hyperelastic Lie quadratic; Lie groups.

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On Complex Sasakian Manifolds Satisfying Certain Curvature Conditions

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Abstract

In this work, we study on complex Sasakian manifolds. We give fundamental facts and definitions. In addition, we obtain some curvature properties. Also, we investigate on some special curvature tensors with certain conditions.

Keywords: Complex contact manifolds; Complex Sasakian manifolds; Conformal curvature tensor; Concircular curvature tensor; Projective curvature tensor.

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Semi-slant Submanifolds with Schouten-van Kampen Connection

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Abstract

In this paper, we study semi-slant submanifolds with the Schouten-van Kampen connection on Kenmotsu manifolds.

Keywords: Semi-slant submanifolds; Schouten-van Kampen connection; Kenmotsu manifolds.

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Generalized Normalized δ-Casorati Curvature for Statistical Submanifolds in Quaternion Kaehler-like Statistical Space Form

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Abstract

In 2017, C.W. Lee derived optimal Casorati inequalities with normalized scalar curvature for statistical submanifolds of statistical manifolds of constant curvature. In this talk, I wish to give our recent results on bounds for generalized normalized δ-Casorati curvatures for statistical submanifolds in quaternionic Kaehler-like statistical space forms published in Journal of Geometry.

Keywords: Casorati curvature; Conjugate connection; Statistical manifold; Quaternion Kaehler-like statistical space form.

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On the Invariants of Finite Blaschke Products

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Abstract

Let $B(z)$ be a finite Blaschke product of order $n, n \geq 2$ for the unit disc. In this study, we investigate the problem when such Blaschke products has the property $B \circ M = B$, where M is a Möbius transformation different from the identity and B can be written as a composition $B = B_2 \circ B_1$ of two finite Blaschke products of lower degree.

Keywords: Finite Blaschke products; Composition of Blaschke products.

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Lucas Polynomial Approach to Determine Lorentzian Spherical Timelike Curves in Minkowski 3-Space

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Abstract

In this paper, we give necassary condition for an arbitrary-speed regular timelike curve to lie on a Lorentzian sphere. Then, we obtain the position vector of timelike curve lying on a Lorentzian sphere satisfies a third-order linear differential equation. Also, Lucas collocation method is applied for the approximate solutions of this differential equation. Furthermore, by using this method we obtain the parametric equation of the Lorentzian spherical timelike curve, approximately. Moreover, in order to show efficiency of the method, an example is given and the results are compared with tables and figures.

Keywords: Lorentzian spherical curves; Lucas polynomial and series; Differential equation; Collocation points; Frenet frame.

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D-H Representation in Lorentzian Space and Mechanical Applications

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Abstract

D-H representation is introduced by Richard S. Hartenberg and Jacques Denavit for kinematic and synthesis of linkages. D-H parameters in mechanism and kinematics are very handy and strong algorithm. Especially in computer design using the D-H parameters are indispensable. This paper presents D-H parameters in Lorentzian space and gives mechanical applications.

Keywords: D-H representation; Kinematics; Lorentzian space; Mechanism.

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Differential Equations of Space-Like Loxodromes on Canal Surfaces in Minkowski 3-Space

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Abstract

In this talk, we investigate the differential equations of space-like loxodromes on non-degenerate canal surfaces depending on both causal characters of these canal surfaces and their meridians in Minkowski 3-space.

Keywords: Loxodrome; Canal Surface; Euclidean 3-Space.

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Loxodromes on Helicoidal and Canal Surfaces in Euclidean 4-Space

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Abstract

In this talk, we generalize the equations of loxodromes on helicoidal surfaces as well as canal surfaces in Euclidean 3-space to Euclidean 4-space. Also, we give some examples by using Mathematica computer program.

Keywords: Loxodrome; Helicoidal Surface; Canal Surface; Euclidean Space.

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Non-Euclidean form of Minkowski Space and 4-Dimensional Geometry

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Abstract

The relativity theory is structured on two discoveries according to Albert Einstein. They are the special theory and the general theory of relativity. Einstein had need of a third theory and technique. This is the spacetime model and 4 dimensions geometry which is constructed by Hermann Minkowski. This study presents the necessity of the 4 dimensions geometry and Lorentz structure for relativity theory.

Keywords: Lorentz space; Minkowski space; Spacetime geometry.

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On the Geometry of Trans-Sasakian Manifolds with The Schouten-Van Kampen Connection

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Abstract

The object of the present paper is to the study 3-dimensional trans-Sasakian manifolds with $\alpha = \beta =$constant with respect to the Schouten-van Kampen connection.

Keywords: Schouten-van Kampen connection; Trans-Sasakian manifold; Projective curvature tensor; Conharmonical curvature tensor.

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A Work on Homology Groups of Simple Closed H-Curves

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Abstract

Topological invariants are very useful tools in several areas related to digital images and geometric modelling. The present work deals with the singular homology groups which is a way for distinguishing or identifying the digital images by means of combining the algebraic topological notions (the homology groups) with the digital topology (the H-topology). The aim of this work is to introduce the singular homology groups of digital images, especially we investigate simple closed H-curves in 3-dimensional digital images, and the homology functor between HAC and Ab categories.

Keywords: (generalized) Marcus-Wyse topology; Digital topology; HA-map; H-adjacency; Simple closed H-curves; Singular homology.

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Abstracts of Poster Presentations

On A Class of Finite Projective Klingenberg Planes

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Abstract

In this presentation, we study on a class of projective Klingenberg planes coordinatized by a plural algebra of order *m*. So, the incidence matrices for the special cases of the class are obtained. Also, the number of collineations of the class is found. Finally, we give an example of a collineation for the class.

Keywords: Plural algebra; Local ring; Projective Klingenberg plane.

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On Ruled Surface Pair Generated by Darboux Vectors of a Curve and its Natural Lift in Dual Space

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Abstract

In this study, firstly, the Darboux vector $\overline{W}$ of the natural lift $\bar{\alpha}$ of a curve α is calculated in dual space. Secondly, we defined ruled surfaces which are given by the curve α and its natural lift $\bar{\alpha}$. Finally, we obtained striction lines and distribution parameters of the ruled surface pair generated by the natural lift $\bar{\alpha}$.

Keywords: Darboux Vector; Natural Lift; Striction Line; Distribution Parameter.

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$\bar{M}$- Geodesic Spray and $\bar{M}$- Integral Curve in the Dual Space

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Abstract

In this study, firstly, $\bar{M}$- vector field Z on M, $\bar{M}$- integral curve of Z, and $\bar{M}$- geodesic spray concepts are given. $\bar{M}$ is a Riemann manifold and M is a hypersurface of $\bar{M}$ in Dual space.

Then, "The natural lift $\bar{\alpha}$ of the curve α is an $\bar{M}$- integral curve of $\bar{M}$- geodesic spray Z if and only if α is an $\bar{M}$- geodesic on M." is proved in Dual space.

Keywords: $\bar{M}$- vector field; $\bar{M}$- integral curve; $\bar{M}$- geodesic spray.

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Ruled Surfaces with Striction Scroll in Dual Space

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Abstract

In this study, ruled surfaces with striction scroll are described in dual space, and then some characterizations of their surfaces are given.

Keywords: Ruled surfaces with striction scroll; Dual Space.

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Some Characterizations for Konoidal Ruled Surfaces

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Abstract

In this study, firstly, some basic properties of ruled surface are given in E^n. Secondly, konoidal, strongkonoidal and orthokonoidal ruled surfaces are defined and some characterizations of these surfaces are given. Finally, those notions are compared with each other in E^n.

Keywords: Ruled Surface; Konoidal Ruled Surface; Strongkonoidal Ruled Surface; Orthokonoidal Ruled Surface.

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On Almost alpha-Cosymplectic Pseudo-Metric Manifolds

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Abstract

In this presentation, we introduce a study of almost alpha-cosymplectic manifolds with pseudo-Riemannian metrics, emphasizing the analogies and differences with respect to Riemannain case. After obtaining some fundamental formulas about curvature properties, some results are investigated.

Keywords: Almost Kenmotsu manifold; Almost alpha-cosymplectic manifold; Pseudo-metric; Curvature.

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On Almost alpha-Cosymplectic Manifolds

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Abstract

In this presentation, we study almost alpha-cosymplectic manifolds with some tensor fields. In particular, we consider some certain semi-symmetric conditions related to locally symmetry and eta parallelism. Finally, we give some illustrating examples on almost alpha-cosymplectic manifolds depending on alpha.

Keywords: Almost Kenmotsu manifold; Almost alpha-cosymplectic manifold; Semi-symmetry; Eta parallelity; Flat conditions.

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On Smarandache Curves of Involute-evolute Curve According to Frenet Frame in Minkowski 3-space

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Abstract

In this presentation, let $\{\alpha^*, \alpha\}$ be involute evolute curve couple, when the Darboux vector of the spacelike involute curve α^* are taken as the position vectors, the curvature and the torsion of Smarandache curve are calculated. These values are expressed depending upon the timelike evolute curve α. Finally, we give an example of our main results.

Keywords: Smarandache curves; Involute-evolute curves; Minkowski space.

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Ruled Surface Pair Generated by a Curve and Its Natural Lift in IR^3

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Abstract

In this study, firstly, the Frenet vector fields $\overline{T}, \overline{N}, \overline{B}$ of the natural lift $\overline{\alpha}$ of a curve α are calculated in terms of those of α in IR^3. Secondly, we obtained striction lines and distribution parameters of ruled surface pair generated by the curve α and its natural lift $\overline{\alpha}$. Finally, for α and $\overline{\alpha}$ those notions are compared with each other.

Keywords: Natural Lift; Ruled Surface; Striction Line; Distribution Parameter.

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Mannheim Offsets of Ruled Surfaces Under the Symmetrical Helical Motions in E^3

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Abstract

In this study, situation of Mannheim offsets of ruled surfaces under the symmetric helical motions in 3-dimensional Euclidean space will be investigated. Firstly, relationships between geodesic Frenet trihedrons of Mannheim offsets of ruled surfaces will be obtained and the relationship between the curvatures of the surface pairs will be examined. Also, change of integral invariants the surface pairs under the 1-parameter motions will be studied. Finally, the relevant example will be given.

Keywords: Mannheim curve; Motion; Symmetrical Helical Motion.

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On Fuzzy Hyperplanes of Fuzzy 5-Dimensional Projective Space

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Abstract

In this work, we construct fuzzy subgeometries in a fuzzy projective space as defined in [5]. We give a classification of fuzzy projective hyperplanes of fuzzy 5-dimensional projective space from fuzzy 6-dimensional vector space.

Keywords: Fuzzy Vector Space; Fuzzy Projective Space.

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An Application for Fibered Projective Plane of Order 2

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Abstract

In this work, a computer program is designed to determine all fibered projective planes with base projective plane which is Projective Plane of Order 2. By applying this program, the fiber points and lines of all these fibered projective planes are obtained using membership degrees of the points.

Keywords: Fibered Projective Plane; Fiber Point; Fiber Line.

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On Isometries of $R^2_{\pi n}$

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Abstract

In this work, we introduce a family of distance functions and show that the group of isometries of the plane associated with the induced metrics is the semi-direct product of the Dihedral group D_{2n} and the translation group T(2).

Keywords: Group; Isometry; Distance function; Metric.

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Notes on Quaternionic Frame in R^4

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Abstract

In this paper, we introduce a new version of quaternionic frame for quaternionic curve in R^4 and give an application of this new quaternionic frame by an example.

Keywords: Quaternions; Quaternionic frame; Serret-Frenet Formulae.

Acknowledgment: The author is supported by Ahi Evran University Scientific Research Projects Coordination Unit. Project Number: EGT.A4.18.031.

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On the Complete Arcs in NFPG(2,9)

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Abstract

In this study, the complete (k, 2)- arcs with $6 \leq k < 10$ containing the quadrangles constructing the Fano planes in NFPG(2,9) were determined and classified by using C# program.

Keywords: Projective plane; Near Field; Complete Arcs.

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Translation Surfaces Generated by Spherical Indicatrices of Space Curves in Euclidean 3-Space

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Abstract

In this paper, we study translation surfaces generated by spherical indicatrices of space curves in 3-dimensional Euclidean space and obtain some characterizations for such surfaces.

Keywords: Translation surfaces; Euclidean 3-space; Spherical indicatrix.

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Surfaces of Revolution in G_3

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Abstract

In this study, a complete classification of surfaces of revolution in the three dimensional Galilean space G_3 in terms of the position vector field, Gauss map, pointwise 1-type Gauss map equation and Laplacian operators of the first, the second and the third fundamental forms on the surface is made.

Keywords: Surface of revolution; Galilean space; Gauss map.

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Clairaut's Theorem on The Some Special Surfaces in Galilean Space

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Abstract

The scalar product of the vectors $u = (u_1, u_2, u_3), v = (v_1, v_2, v_3)$ in G_3 is defined as

$$\langle u, v \rangle_{G_3} = \begin{cases} u_1 v_1, & if\ u_1 \neq 0\ or\ v_1 \neq 0 \\ u_2 v_2 + u_3 v_3, & if\ u_1 = 0,\ v_1 = 0 \end{cases}$$

Let $\alpha: I \subset R \to G_3$, $\alpha(s) = \big(s, y(s), z(s)\big)$ be a curve parametrized by arclength parameter s in G_3. The vectors of the Frenet-Serret frame, that is, respectively, are defined as

$$t(s) = \alpha' = (1, y'(s), z'(s)), n(s) = \frac{t'(s)}{\kappa(s)}, b(s) = \frac{n'(s)}{\tau(s)},$$

where the real valued functions $\kappa(s) = \|t'(s)\|$ is called the first curvature of the curve $\alpha,$ the second curvature function is defined as $\tau(s) = \|n'(s)\|$. For the curve in G_3, we have the following Frenet-Serret equations:

$$t' = \kappa n,\ n' = \tau b,\ \ b' = -\tau n. \qquad (1.1)$$

In this study, we explored these different types of tubular surfaces (with normal curves, osculating curve, rectifying curve), and we generalized Clairaut's theorem to these surfaces.

Keywords: Galilean space; Tubular surfaces; Clairaut's theorem.

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Some Characterizations of Timelike Clad Helices in Minkowski 3-space

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Abstract

In this study we introduce timelike clad (C-slant curves or 2-type slant curves) and g-clad helices (3-type slant curves) which are generalizations of the concept of timelike helices. We obtain some characterizations of these curves via new alternative frame in Minkowski 3-space. Then, we give the axis of the clad and g-clad helices. Finally, we present some characterizations via the properties of slant, clad and g-clad helices.

Keywords: Timelike clad helices; Timelike g-clad helices; Minkowski 3-space.

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Special Class of Curves in Affine 3-Space

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Abstract

In this study, we investigated the special curves in affine 3-space which are the curves whose affine binormal lines intersect a constant proper line at each points of the curve and we obtained some characterizations.

Keywords: Affine space; Equi-affine Frame; Affine curvatures.

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