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# The Gauss Map and the Third Laplace-Beltrami Operator of the Rotational Hypersurface in 4-Space

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**Abstract:** We study and examine the rotational hypersurface and its Gauss map in Euclidean four-space  $\mathbb{E}^4$ . We calculate the Gauss map, the mean curvature and the Gaussian curvature of the rotational hypersurface and obtain some results. Then, we introduce the third Laplace–Beltrami operator. Moreover, we calculate the third Laplace–Beltrami operator of the rotational hypersurface in  $\mathbb{E}^4$ . We also draw some figures of the rotational hypersurface.

**Keywords:** four-space; rotational hypersurface; Gauss map; Gaussian curvature; mean curvature; the third Laplace–Beltrami operator

## 1. Introduction

When we focus on the rotational characters in the literature, we meet Arslan et al. [1,2], Arvanitoyeorgos et al. [3], Chen [4,5], Dursun and Turgay [6], Kim and Turgay [7], Takahashi [8], and many others.

Magid, Scharlach and Vrancken [9] introduced the affine umbilical surfaces in four-space. Vlachos [10] considered hypersurfaces in  $\mathbb{E}^4$  with the harmonic mean curvature vector field. Scharlach [11] studied the affine geometry of surfaces and hypersurfaces in four-space. Cheng and Wan [12] considered complete hypersurfaces of four-space with constant mean curvature.

General rotational surfaces in  $\mathbb{E}^4$  were introduced by Moore [13,14]. Ganchev and Milousheva [15] considered these kinds of surfaces in the Minkowski four-space. They classified completely the minimal rotational surfaces and those consisting of parabolic points. Arslan et al. [2] studied generalized rotation surfaces in  $\mathbb{E}^4$ . Moreover, Dursun and Turgay [6] studied minimal and pseudo-umbilical rotational surfaces in  $\mathbb{E}^4$ .

In [16], Dillen, Fastenakels and Van der Veken studied the rotation hypersurfaces of  $\mathbb{S}^n \times \mathbb{R}$  and  $\mathbb{H}^n \times \mathbb{R}$  and proved a criterion for a hypersurface of one of these spaces to be a rotation hypersurface. They classified minimal, flat rotation hypersurfaces and normally flat rotation hypersurfaces in the Euclidean and Lorentzian space containing  $\mathbb{S}^n \times \mathbb{R}$  and  $\mathbb{H}^n \times \mathbb{R}$ , respectively. Senoussi and Bekkar [17] studied the Laplace operator using the fundamental forms  $I, II$  and  $III$  of the helicoidal surfaces in  $\mathbb{E}^3$ .

For the characters of ruled (helicoid) and rotational surfaces, please see Bour's theorem in [18]. Do Carmo and Dajczer [19] showed the existence of surfaces isometric to helicoidal ones by using Bour's theorem [18]. Güler [20] studied a helicoidal surface with a light-like profile curve using Bour's theorem in Minkowski geometry. Furthermore, Hieu and Thang [21] studied helicoidal surfaces by Bour's theorem in four-space. Choi et al. [22] studied helicoidal surfaces and their Gauss map in Minkowski three-space. See also [23–26]. Güler, Magid and Yaylı [27] studied the Laplace–Beltrami operator of a helicoidal hypersurface in  $\mathbb{E}^4$ .