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Family of Enneper Minimal Surfaces

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Abstract: We consider a family of higher degree Enneper minimal surface E_m for positive integers m in the three-dimensional Euclidean space $\mathbb{E}^3$. We compute algebraic equation, degree and integral free representation of Enneper minimal surface for $m = 1,2,3$. Finally, we give some results and relations for the family E_m.

Keywords: Enneper minimal surface family; Weierstrass representation; algebraic surface; degree; integral free representation

1. Introduction

Minimal surfaces have an important role in the mathematics, physics, biology, architecture, etc. These kinds of surfaces have been studied over the centuries by many mathematicians and also geometers. A *minimal surface* in $\mathbb{E}^3$ is a regular surface for which the mean curvature vanishes identically.

There are many important classical works on minimal surfaces in the literature such as [1–10]. However, we only see a few notable works about algebraic minimal surfaces, including general results and the properties. They were given by Enneper [11,12], Henneberg [13,14] and Weierstrass [9,15].

One of them is the classical Enneper minimal surface that was given by Enneper. See [11,12] for details. About Enneper minimal surface, many nice papers were done such as [16–24] in the last few decades.

In this paper, we introduce a family of higher degree Enneper minimal surface E_m for positive integers m in the three-dimensional Euclidean space $\mathbb{E}^3$. In Section 2, we give the family of Enneper minimal surfaces E_m. We obtain the algebraic equation and degree of surface E_1 (resp., E_2, E_3). Using the integral free form of Weierstrass, we find some algebraic functions for E_m $(m \geq 1, m \in \mathbb{Z})$ in Section 3. Finally, we give some general findings for a family of higher degree Enneper minimal surface E_m with a table in the last section.

2. The Family of Enneper Minimal Surfaces E_m

We will often identify $\overrightarrow{x}$ and $\overrightarrow{x^t}$ without further comment. Let $\mathbb{E}^3$ be a three-dimensional Euclidean space with natural metric $\langle . , . \rangle = dx^2 + dy^2 + dz^2$.

Let $\mathcal{U}$ be an open subset of $\mathbb{C}$. A *minimal* (or *isotropic*) *curve* is an analytic function $\Psi : \mathcal{U} \to \mathbb{C}^n$ such that $\Psi'(\zeta) \cdot \Psi'(\zeta) = 0$, where $\zeta \in \mathcal{U}$, and $\Psi' := \frac{\partial \Psi}{\partial \zeta}$. In addition, if $\Psi' \cdot \overline{\Psi'} = |\Psi'|^2 \neq 0$, then Ψ is a *regular minimal curve.*

Thus, let see the following lemma for complex minimal curves.

Lemma 1. *Let $\Psi : \mathcal{U} \to \mathbb{C}^3$ be a minimal curve and write $\Psi' = (\varphi_1, \varphi_2, \varphi_3)$. Then,*

$$\mathcal{F} = \frac{\varphi_1 - i\varphi_2}{2} \quad \text{and} \quad \mathcal{G} = \frac{\varphi_3}{\varphi_1 - i\varphi_2}$$