Generalized Bour's theorem

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ABSTRACT

We give the classical isometric minimal helicoidal and rotational surfaces using generalized Bour's theorem in Euclidean 3-space. Furthermore, we investigate the minimality and have same Gauss map of the surfaces.

Keywords: Gauss map; gaussian curvature; helicoidal surface; mean curvature; rotational surface.

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INTRODUCTION

Theory of surfaces in three dimensional Euclidean and Minkowskian spaces have been studied for a long time and many examples of such surfaces have been discovered. Many nice books have been published, such as Do Carmo (1976), and Kühnel (2006).

In classical surface geometry in Euclidean space, it is well known that the right helicoid (resp. catenoid) is the only ruled (resp. Rotational or surface of revolution) surface which is minimal. If we focus on the ruled (helicoid) and rotational characters, we have Bour's theorem (Bour, 1862).

Ikawa (2000) determine pairs of surfaces by Bour's theorem with the additional condition that they have the same Gauss map in Euclidean 3-space. About helicoidal surfaces in Euclidean 3-space, Do Carmo & Dajczer (1982), prove that, by using a result of Bour (1862), there exists a two-parameter family of helicoidal surfaces isometric to a given helicoidal surface.

Some relations among the Laplace-Beltrami operator and curvatures of the helicoidal surfaces in Euclidean 3-space are shown by Güler *et al*. (2010). In addition, they give Bour's theorem on the Gauss map and some special examples.

Ji & Kim (2013) prove that, in Minkowski 3-space, a minimal helicoidal surface with Gauss curvature K has an isometric minimal rotational surface if and only if $K \le 0$. Moreover, they show that a timelike right helicoid does not have an isometric minimal rotational