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I_2-Convergence of Double Sequences in Topological Groups

Topolojik Gruplarda Çift Dizilerin I_2-Yakınsaklığı

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Abstract

Let $2^{N\times N}$ be a family of all subsets of N×N. Following the definition of ideal convergence in a metric space by Kostyrko et al. in 2000, ideal convergence for double sequences in a metric space was introduced by Das et al. (2008). In this paper, I investigate I_2-convergence and I_2-convergence of double sequences in a topological space and establish some basic teorems. Furthermore we introduce of I_2-Cauchy and I_2-Cauchy notions for double sequences in topological groups.

Keywords: I_2-convergence, I_2-convergence, topological group, double sequence, ideal

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Öz

$2^{N\times N,}$ N×N kümesinin tüm alt kümelerinin ailesi olsun. Kostyrko ve arkadaşlarının 2000'de bir metrik uzayda ideal yakınsaklığı tanımlamalarının ardından, çift diziler için ideal yakınsaklık Das ve arkadaşları tarafından tanımlandı (2008). Bu makalede bir topolojik uzayda çift dizilerin I_2-yakınsaklığı ve I_2-yakınsaklığı incelenmiş ve bazı önemli teoremler inşa edilmiştir. Ayrıca topolojik uzaylarda çift diziler için I_2-Cauchy ve I_2-Cauchy kavramları tanımlanmıştır.

Anahtar Kelimeler: I_2-yakınsaklık, I_2-yakınsaklık, topolojik grup, çift diziler, ideal

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1. Introduction

By an ideal on a set X we mean a nonempty family of subsets of X closed under taking finite unions and subsets of its elements. In other words, a non-empty set $I \subseteq 2^N$ is called an ideal on N if;

i $B \in I$ whenever $B \subseteq A$ for some $A \in I$. (closed unders subsets)

ii $A \cup B \in I$ whenever $A, B \in I$. (closed under unions)

If $N \notin I$ then we say that this ideal is a proper ideal. Similarly an ideal is proper and also contains all finite subsets then we say that this ideal is admissible. Filter is a dual notion of ideal and generally we will use ideals in our proofs but if the notion is more familiar for filters, we will use the notion of filter.

Similarly, a non-empty set $F \subseteq 2^N$ is called a filter on N if;

i $B \in F$ whenever $B \supseteq A$ for some $A \in F$. (closed unders supersets)

ii $A \cap B \in F$ whenever $A, B \in F$. (closed under intersections)

Proposition 1.1. If I is a non-trivial ideal in N, then the family of sets $F = F(I) = \{M = N \backslash A : A \in I\}$

is a filter in N and it is called the filter associated with the ideal.

Definition 1.1. Let $x = (x_k)$ be a real sequence. This sequence is said to be *I*- convergent to $L \in R$ if for each $\varepsilon > 0$ the set

$A_\varepsilon = \{k \in N : |x_k - L| \geq \varepsilon\}$

belongs to *I*. In this definition the number *L* is *I*-limit of the x.

I-convergence generalizes ordinary convergence and statistical convergence. This means that if we choose two

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