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On Wijsman asymptotically lacunary $\mathcal{I}$-statistical equivalence of weight g of sequence of sets

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ABSTRACT. This paper presents the following definition which is a natural combination of the definitions of asymptotically equivalence, $\mathcal{I}$-convergence, statistical limit, lacunary sequence, and Wijsman convergence of weight g; where $g : \mathbb{N} \to [0, \infty)$ is a function satisfying $\lim_{n\to\infty} g(n) = \infty$ and $\frac{n}{g(n)} \nrightarrow 0$ as $n \to \infty$ for sequence of sets. Let (X, ρ) be a metric space, $\theta = \{k_r\}$ be a lacunary sequence and $\mathcal{I} \subseteq 2^{\mathbb{N}}$ be an admissible ideal. For any non-empty closed subsets $A_k, B_k \subseteq X$ such that $d(x, A_k) > 0$ and $d(x, B_k) > 0$ for each $x \in X$, we say that the sequences $\{A_k\}$ and $\{B_k\}$ are Wijsman $\mathcal{I}$-asymptotically lacunary statistical equivalent of multiple L of weight g if for every $\varepsilon > 0$, $\delta > 0$ and for each $x \in X$,

$$\left\{ r \in \mathbb{N} : \frac{1}{g(h_r)} \left| \left\{ k \in I_r : \left| \frac{d(x, A_k)}{d(x, B_k)} - L \right| \geq \varepsilon \right\} \right| \geq \delta \right\} \in \mathcal{I}$$

(denoted by $A_k \overset{S_\theta^L(\mathcal{I}_W)^g}{\sim} B_k$). We mainly investigate their relationship and also make some observations about these classes.

1. INTRODUCTION

Before continuing with this paper we present some definitions and preliminaries.

The concept of $\mathcal{I}$-convergence was introduced by Kostyrko et al. in a metric space [7]. Later it was further studied by ([2], [5], [6], [12], [13], [14], [15], [16], [17], [21]) and many others. $\mathcal{I}$-convergence is a generalization form of statistical convergence, which was introduced by Fast (see [3]) and that is based on the notion of an ideal of the subset of positive integers $\mathbb{N}$. The following definitions and notions will be needed.

Definition 1.1. ([7]) A family of sets $\mathcal{I} \subseteq 2^{\mathbb{N}}$ is called an ideal if and only if (i) $\emptyset \in \mathcal{I}$, (ii) For each $A, B \in \mathcal{I}$ we have $A \cup B \in \mathcal{I}$, (iii) For each $A \in \mathcal{I}$, each $B \subseteq A$ we have $B \in \mathcal{I}$.

An ideal is called non-trivial if $\mathbb{N} \notin \mathcal{I}$ and non-trivial ideal is called admissible if $\{n\} \in \mathcal{I}$ for each $n \in \mathbb{N}$. Throughout the paper, $\mathcal{I}$ will stand for a proper admissible ideal of $\mathbb{N}$.

Definition 1.2. ([7]) A family of sets $F \subseteq 2^{\mathbb{N}}$ is a filter in $\mathbb{N}$ if and only if (i) $\emptyset \notin \mathcal{F}$, (ii) For each $A, B \in \mathcal{F}$ we have $A \cap B \in \mathcal{F}$, (iii) For each $A \in \mathcal{F}$, each $B \supseteq A$ we have $B \in \mathcal{F}$.

Proposition 1.1. ([7]) *If $\mathcal{I}$ is a proper ideal of $\mathbb{N}$ (i.e., $\mathbb{N} \notin \mathcal{I}$), then the family of sets $\mathcal{F}(\mathcal{I}) = \{M \subset \mathbb{N} : \exists A \in \mathcal{I} : M = \mathbb{N} \backslash A\}$ is a filter of $\mathbb{N}$ it is called the filter associated with the ideal.*

Definition 1.3. ([7]) Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a proper admissible ideal in $\mathbb{N}$. The sequence (x_n) of elements of $\mathbb{R}$ is said to be $\mathcal{I}$-convergent to $L \in \mathbb{R}$ if for each $\varepsilon > 0$

$$A(\varepsilon) = \{n \in \mathbb{N} : |x_n - L| \geq \varepsilon\} \in \mathcal{I}.$$

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