Weierstrass Representation, Degree and Classes of the Surfaces in the Four Dimensional Euclidean Space

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Abstract
We study two parameters families of Bour-type and Enneper-type minimal surfaces using the Weierstrass representation in the four dimensional Euclidean space. We obtain implicit algebraic equations, degree and classes of the surfaces.

Keywords — 4-space, surface, Weierstrass representation, degree, class.
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1 Surfaces in $\mathbb{R}^4$

In Moore [4], we find a general definition of rotation surfaces in $\mathbb{R}^4$:

$$\begin{aligned} X(u,t) = (&x_1(u)\cos(at) - x_2(u)\sin(at), \\ &x_1(u)\cos(at) + x_2(u)\sin(at), \\ &x_3(u)\cos(bt) - x_4(u)\sin(bt), \\ &x_3(u)\cos(bt) + x_4(u)\sin(bt)). \end{aligned}$$

We propose that we look at a restricted case of this, found in Ganchev-Milousheva [2] :

$$W(u,t) = (x_1(u), x_2(u), r(u)\cos(t), r(u)\sin(t)).$$

The first we think is a bit too general since the curve is not located in any subspace before rotation.

At any rate this has:

$$g(\partial u, \partial u) = r'^2 + (x_1)'^2 + (x_2)'^2 = 1$$

if we use arc length parametrization, $g(\partial u, \partial t) = 0$ and $g(\partial t, \partial t) = r^2$.

Using the Weierstrass representation in Section 2, we give two parameters familes of Bour's-type (in Section 3) and Enneper's-type (in Section 4) minimal surfaces in the four dimensional Euclidean space. We also calculate implicit algebraic equations of the surfaces, degrees and classes of the surfaces.

2 Weierstrass equations for a minimal surface in $\mathbb{R}^4$

In Hoffman and Osserman [3] , p.45, they give the Weierstrass equations for a minimal surface in $\mathbb{R}^4$: