FIBONACCI STATISTICAL CONVERGENCE OF DOUBLE SEQUENCES AND KOROVKIN TYPE APPROXIMATION THEOREMS

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Abstract: The purpose of this paper is twofold. First, the definition of new statistical convergence with Fibonacci double sequence is given and some important properties of statistical convergence are examined. Second, we provide various approximation results concerning the classical Korovkin theorem via Fibonacci type statistical convergence of double sequences.

Keywords: Korovkin type approximation theorems, statistical convergence, Fibonacci numbers, positive linear operators.

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1 Introduction and Background

Taking this definition, we can give the definition of statistical convergence which has been formally introduced by Fast [1]. The notion of convergence of real double sequences was first introduced by Pringsheim [13]. Hill [9] was the first who applied methods of functional analysis to double sequence. This notion of convergence of real double sequences has been extended to statistical convergence by Mursaleen et. al. [12].

Fibonacci sequence was initiated in the book Liber Abaci of Fibonacci. The sequence had been described earlier as Virahanka numbers in Indian mathematics [6]. In Liber Abaci, the sequence starts with 1, nowadays the sequence begins either with $f_0 = 0$ or with $f_1 = 1$. The numbers in the bottom row are called Fibonacci numbers, and the number sequence $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...$ is the Fibonacci sequence [5]. The Fibonacci numbers are a sequence of numbers (f_n) for $n = 1, 2, ...$ defined by the linear recurrence equation $f_n = f_{n+1} - f_{n-2}$, $n \geq 2$.

Korovkin type approximation theorems are practical tools to check whether a given sequence $(A_n)_{n\geq 1}$ of positive linear operators on $C[a, b]$ of all continuous functions on the real interval $[a, b]$ is an approximation process. That is, these theorems present a variety of test functions which provide that the approximation property holds on the whole space if it holds for them. Such a property was determined by Korovkin [8] in 1953 for the functions $1, x$ and x^2 in the space $C[a, b]$ as well as for the functions 1, cos and sin in the space of all continuous 2π-periodic functions on the real line.

The Fibonacci sequence was firstly used in the theory of sequence spaces by Kara and Başarır [11]. Afterward, Kara [7] defined the Fibonacci difference matrix $\widehat{F}$ by using the Fibonacci sequence (f_n) for $n \in \{1, 2, 3, ...\}$ and introduced the new sequence spaces related to the matrix domain of $\widehat{F}$. Until the study of Gadjiev and Orhan [10], there was no study related to statistical convergence and