UNIVERSITY

ISTANBUL MEDENIYET UNIVERSITY

# Ahyirac BOO 

# $\mathrm{a}^{r d}$ INTERNATIONAL CONFERENCE DN RECENT ADVANCEG IN PURE AND APPLIED MATHIEMATICE, [ICRAPAM rD16] 

Istan6u[ Commerce University, Istan6u[Medeniyet University Institute of Mathematics of $\mathcal{N}$ ational Academy of Science of Ukraine

$$
\text { 19-23 MAソ } 2016
$$

# $3^{r d}$ INTERNATIONAL CONFERENCE ON RECENT ADVANCES IN PURE AND APPLIED MATHEMATICS, <br> 19-23 MAY 2016 BODRUM-TURKEY 

## Abstract Book

ISBN: 978-975-00211-3-8


# $3^{r d}$ INTERNATIONAL CONFERENCE ON RECENT ADVANCES IN PURE AND APPLIED MATHEMATICS, 19-23 MAY 2016 BODRUM-TURKEY 

## Abstract Book

ISBN: 978-975-00211-3-8

## List of Major Sponsors

- Istanbul Commerce University
- İstanbul Medeniyet University
- Institute of Mathematics of National Academy of Science of Ukraine



## $3^{r d}$ INTERNATIONAL CONFERENCE ON RECENT ADVANCES IN PURE AND APPLIED MATHEMATICS, 19-23 MAY 2016, BODRUM- TURKEY

## Honorary Chairs of Scientific Committee

Prof. B. E. Rhoades, Indiana University, USA
Prof. G. Das, Utkal University, India
Prof. A. Ashyralyev, Fatih Uni., Turkey
Prof. M. Perestyuk, Kyiv Nat. Shevchenko Uni., Ukraine
Prof. O. Boichuk, Inst. of Math. NAS of Ukraine
Prof. I. Shevchuk, Kyiv Nat. Shevchenko Uni., Ukraine
Prof. F. Basar, Fatih University, Turkey
Prof. E. Misirli, Ege University, Turkey
Prof. V. Kalantarov, Koc Uni., Turkey
Prof. V. Guliyev, Ahi Evran Uni., Turkey
Prof. M. Mursaleen, Aligarh Muslim Uni., India
Prof. M. Abbas, Uni. of Pretoria, S. Africa
Prof. W. Sintunavarat, Thammasat Uni., Thailand
Prof. M. Bayramoglu, Inst. Math. \& Mech., Azerbaijan
Prof. M. Mardanov, Inst. Math. \& Mech., Azerbaijan
Prof. H. M. Srivastava, Uni. of Victoria, Canada
Prof. B. T. Bilalov, Inst. Math. \& Mech., Azerbaijan
Prof. F. Aliyev, Baku State Uni., Azerbaijan

## Organizing Committee

Prof.Dr. Ekrem Savas, Istanbul Commerce University, Turkey
Prof.Dr. Emine Can, Istanbul Medeniyet University, Turkey
Prof.Dr. Anatoliy M. Samoilenko, Inst. of Math. NAS of Ukraine, Ukraine
Prof.Dr. Martin Bohner, Missouri S\&T, USA
Prof.Dr. Ram Mohapatra, Uni. of Central Florida, USA
Prof.Dr. Richard Patterson, North Florida University, USA
Prof. Dr. Ants Aasma, Tallinn Uni. of Tech., Estonia
Prof.Dr. Mustapha Cheggag, Polytechnic Nat. School of Oran, Algeria
Prof.Dr. P. D. Srivastava, IIT Khargapur, India
Prof.Dr. Fairouz Tchier, King Saud University, Saudi Arabia
Prof.Dr. Mehmet Dik, Rockford University, USA
Prof.Dr. Lubomira Softova, Second University of Naples, Italy
Prof.Dr. Ali M. Akhmedov, Baku State University, Azerbaijan
Prof.Dr. Agron TatoPolytechnic, Uni. of Tirana, Albania
Prof.Dr. Debasis Giri, Haldia Institute of Technology, India
Prof.Dr. Naim Braha, Uni. of Prishtina, Republic of Kosova
Assoc. Prof.Dr. Fatma Aydin Akgun, Yildiz Technical University, Turkey
Assoc. Prof.Dr. Rahmet Savas Eren, Istanbul Medeniyet University, Turkey
Assoc. Prof.Dr. Bunyamin Aydin, Necmettin Erbakan University, Turkey
Assoc. Prof.Dr. Necip Simsek, Istanbul Commerce University, Turkey
Assoc. Prof.Dr. Erhan Deniz, Kafkas University, Turkey
Assoc. Prof.Dr. Mahpeyker Ozturk, Sakarya University, Turkey
Assist. Prof.Dr. Sukran Konca, Bitlis Eren University, Turkey


# $3^{r d}$ INTERNATIONAL CONFERENCE ON RECENT ADVANCES IN PURE AND APPLIED MATHEMATICS, 19-23 MAY 2016, BODRUM- TURKEY 

## Local Organizing Committee

## Armagan Elibol

## Ekber Girgin

Neslihan Kaplan
Rabia Savas

## Betul Hicdurmaz

Abdurrahman Buyukkaya
Sefa Anil Sezer

## Scientific Committee

Prof.Dr. Huseyin Cakalli, Turkey
Prof.Dr. Mikail Et, Turkey
Prof.Dr. Metin Basarir, Turkey
Prof.Dr. S. A. Mohiuddine, S. Arabia
Prof.Dr. T. A. Chishti, India
Prof.Dr. Cihan Orhan, Turkey
Prof.Dr. Ayhan Serbetci, Turkey
Prof.Dr. Bilal Altay, Turkey
Prof.Dr. Ljubisa Kocinac, Serbia
Prof.Dr. Ismail Ekincioglu, Turkey
Prof.Dr. A. Sinan Cevik, Turkey
Prof.Dr. Leiki Loone, Estonia
Prof.Dr. Akbar B. Aliyev, Azerbaijan
Prof.Dr. Vali M. Gurbanov, Azerbaijan
Prof.Dr. Faqir M. Bhatti, Pakistan
Prof.Dr. Said Melliani, Morocco
Prof.Dr. Seyit Temir, Turkey
Prof.Dr. Halit Orhan, Turkey
Prof.Dr. Vatan Karakaya, Turkey
Prof.Dr. Amir Khosravi, Iran
Prof.Dr. Seifedine Kadry, Kuwait
Prof.Dr. Ali M. Akhmedov, Azerbaijan
Prof.Dr. Ziyatkan Aliyev, Azerbaijan
Prof.Dr. Poom Kumam, Thailand
Prof.Dr. Agacik Zafer, Kuwait
Prof.Dr. Tunay Bilgin, Turkey
Prof.Dr. Gangaram S. Ladde, USA

Prof.Dr. Claudio Cuevas, Brazil
Prof.Dr. Reza Saadati, Iran
Prof.Dr. Salih Aytar, Turkey
Prof.Dr. Charles Swartz, USA
Prof.Dr. Yagub A. Sharifov, Azerbaijan
Prof.Dr. Niyazi A. Ilyasov, Azerbaijan
Prof.Dr. Mujahid Abbas, S. Africa
Prof.Dr. Nazim Kerimov, Turkey
Prof.Dr. Aref Jeribi, Tunisia
Prof.Dr. Yusuf Yayli, Turkey
Prof.Dr. Husamettin Coskun, Turkey
Prof.Dr. Abdullah Aziz Ergin, Turkey
Prof.Dr. Cemil Tunc, Turkey
Prof.Dr. Maria Zeltser, Estonia
Prof.Dr. Salih Celebioglu, Turkey
Prof.Dr. Kamalmani Baral, Nepal
Prof.Dr. Ants Aasma, Estonia
Prof.Dr. Ismail N. Cangul, Turkey
Prof.Dr. Sadek Bouroubi, Algeria
Dr.Lejla Miller Van-Wieren, Bosnia
Prof.Dr. Murat Tosun, Turkey
Prof.Dr. Yilmaz Simsek, Turkey
Prof.Dr. Harry Miller, Bosnia
Prof.Dr. Ali Fares, France
Prof.Dr. Ibrahim Canak, Turkey
Prof.Dr. Mehmet Gurdal, Turkey

## Dear Collogues;

First of all I wish to offer you a warm welcome to the third International Conference on Recent Advances in Pure and Applied Mathematics (ICRAPAM 2016).

The last conference of this series was organized in Istanbul, Turkey, during 03-06 June 2015 and it was attended by 400 scientists from 48 different countries, contributing 360 oral presentations and 39 posters.

As the past conference, the aim of this conference is to provide a platform for mathematicians to present their recent Works, exchange ideas and new methods in several important areas of Mathematics and to provide an opportunity to improve collaboration between local and international participants in the wonderful historic city of Istanbul. Further we believe that, the development in various fields of Mathematics lead to new research areas in Mathematics and the richness of the new results can also provide basis for interdisciplinary collaborations. That is why; we have planned to provide a common forum for scientists to communicate their original results in various fields of analysis and applied mathematics.

The conference is supported by Istanbul Commerce University and Istanbul Medeniyet University from Turkey, and Institute of Mathematics of National Academy of Science of Ukraine.

We also like to thank all the invited speakers who have kindly accepted our invitation and have come to spend their precious time by sharing their ideas during the conference. Finally, we would also like to thank all of the members of the Scientific Advisory Committee and the Organizing Committee of this conference.

Again we would like to convey our heartiest welcome to each of you who have come to attend this conference and we wish for an enjoyable high scientific level conference and hope to meet you again in the future.
With our best wishes and warm regards,

Prof. Dr. Ekrem SAVAS
Chair of the Conference


## $3^{r d}$ INTERNATIONAL CONFERENCE ON RECENT ADVANCES IN PURE AND APPLIED MATHEMATICS, 19-23 MAY 2016 BODRUM- TURKEY

## INVITED TALKS

Soft Contraction Theorem ..... 1Mujahid Abbas
Uniform and Pointwise Inequalities for Algebraic Polynomials on a Regions in the Complex Plane ..... 2
Fahreddin G. Abdullayev
Some Problems of the Theory of Bifurcations of Non-Linear Problems of Fourth Order ..... 3
Ziyatkhan S. Aliyev
Matrix Maps Between Statistical Sequence Spaces ..... 4
Jeff Connor
Applications of Newton Interpolating Series in Archimedean and Non-Archimedean Analysis ..... 5
Ghiocel Groza
Some Classes of Divergent Sequences ..... 6
Ljubiša D.R. Kočinac
Morrey Regularity of the Weak Solutions of Some Quasilinear Elliptic Systems ..... 7Softova Lubomira
The Navier-Stokes Equations - New Trends on Weak and Strong Solutions ..... 8
Werner Varnhorn

## $3^{r d}$ INTERNATIONAL CONFERENCE ON RECENT ADVANCES IN PURE AND APPLIED MATHEMATICS, 19-23 MAY 2016, BODRUM- TURKEY

## LIST OF TALKS

Neo-Equilibrium Point in the Network Game ..... 10
Ahmad Termimi Ab-Ghani
An Extension of Derivation on Prime Near-Rings ..... 11
Madugu Abdu and Moharram A. Khan
Polynomial Inequalities on Unbounded Region ..... 12F. G. Abdullayev and G. A. AbdullayevUsing Wavelet Denoising Methodology for Enhancing the Performance of Forecasting of ARIMA13MethodologyQais Mustafa Abdulqader
The Use of Conformable Variational Iteration Method, Conformable Reduced Differential Transform ..... 14 Method and Conformable Homotopy Analaysis Method for Solving Different Types of Nonlinear Partial Differential Equations
Omer Acan, Omer Firat and Yildiray Keskin
Rough Semicontinuous Set Valued Maps ..... 15Fatma Gecit Akcay and Salih Aytar
On a New Subclass of Meromorphic Functions with Positive Coeffitients Defined by a Certain İntegral ..... 16 Operator
Arzu Akgul
Some Properties of Generalized Hausdorff Matrices ..... 17
F.Aydin Akgun and B.E.Rhoades
Matrices Which Preserve Decreasing Sequences ..... 18
F.Aydin Akgun, S. Mahmood and B.E.Rhoades
Some Mixed Means of Fourier Series in Lebesgue Spaces Lp, $\omega(\mathrm{Td})$ with Muckenhoupt Weights $\omega$ ..... 19
Ramazan Akgun
On Iterative Processes and Spectral Problems of Generalized Difference Operator-Matrices ..... 20
Ali M. Akhmedov
On Compactness of Bigeneralized Topological Spaces ..... 21
Samer Al-Ghour
Hermitian Part, and Skew Hermitian Part of Normal Matrices ..... 22
M.Al-Hawari
On $(\mu, \lambda)-c$-Continuous Functions in Generalized Topological Spaces ..... 23
Ahmad Al-Omari
Fuzzy Hypergraphs: Medical Application ..... 24
Hanan A. Alolyian
Numerical Solution of Fractional Second-Order System by Using Reproducing Kernel Algorithm ..... 25
Zuhier Altawallbeh, Mohammed Al-Smadi and Ali Ateiwi

Majorization by Starlike Functions26Osman Altıntaş and Öznur Özzan Kılıç
A New Approximate Solution for Quadratic Riccati Differential Equation ..... 27
Nidal Ankira
An Algorithm for Solving Bi-Matrix Games ..... 28
Aicha Anzi, Ramzi Kasri, Hicham Lenouar and Radjef Mohammed Said
Boundary Stabilization of a Flexible Beam with a Tip Rigid Body Without Dissipativity ..... 29
Moulay Driss Aouragh
Migrativity Property for Uninorms and Nullnorms ..... 30
Emel Asici
Totally Free Square Complexes ..... 31Hasan Atik and Murat Atik
Trigonometric Approximation in Weighted Lorentz Spaces ..... 32
Ahmet Hamdi Avsar
On Multiplicative Complex Integral ..... 33
Agamirza E. Bashirov and Sajedeh Norozpour
The Spectrum and Trace of a Discontinuous Value Problem with Retarded Argument ..... 34
M. Bayramoglu and A. Bayramov
Numerical Solution of Semilinear Elliptic Equation via Difference Schemes ..... 35
Elif Özturk Beıgmohammadı and Esra Demirel
The Mathematical Models for Pavement Design ..... 36
Mouloud Belachia and Nadjette Bouacha
On Quadratic Integral Equations of Volterra Type in Fréchet Spaces ..... 37
Latifa Benhamouche
On A-Statistical Convergence with Density of Moduli ..... 38
Stuti Borgohain and Ekrem Savas
The Boundary Integral Method for the Laplace Equation with Mixed and Oblique Conditions ..... 39
Hamza Bouguerne
Efficiency Conditions for Multiobjective Bilevel Optimization Problem Under Generalized Invexity ..... 40
Karima Bouibed, Hachem Slimani and Mohammed Said RadjefConvergence Result in the Approximation of Some Matrix Functions by Krylov Method41(Application on an Inverse Problem)
A. Bouzitouna, N. Boussetila and F. Rebbani
Stability Result of a Fractional Differential Equation42
S. Bouzitouna and R. Atmania
Analysis of Centrodes in Planar and Spherical Motion ..... 43
Serife Nur Bozdag and Ali Caliskan
Empirical Likelihood for Copula Moment Based Estimation Method ..... 44
Brahim BrahimiApproximation of Entropic and Admissible Solution of the Hyperbolic Problems of Conservation Laws by45the New Scheme Ziti's $\delta$-Scheme in Several Dimensions. Application to Burgers Equation, AdvectionConvection Equation, Gaz Dynamics and Biological ProblemsLarbi Bsiss and Chérif Ziti

## Abdurrahman Buyukkaya and Mahpeyker Ozturk

A Characterization of Curves According to Parallel Transport Frame in Euclidean $n$-Space ..... 49
Sezgin Buyukkutuk, İlim Kisi and Günay OzturkInitial Coefficients for a Subclass of Bi-Univalent Functions Defined by Salagean Differential Operator50
Murat Caglar
Beyond the $\lambda$-Statistical Ward Continuity ..... 51
Huseyin Cakalli
On Abel Statistical Continuity ..... 52
Huseyin Cakalli and E.Iffet Taylan
A Variation on $\mathrm{N}_{\theta}$ Ward Continuity ..... 53
Huseyin Cakalli, Mikail Et and Hacer Sengul
A Variation on Strongly Lacunary Ward Continuity ..... 54
Huseyin Cakalli and Huseyin Kaplan
On the Smallest Bounding Disks of Graph-Directed Fractals ..... 55
Gokee Cakmak and Ali Deniz
A Tauberian Theorem for $(\mathrm{C}, 1,1)$ Summable Double Sequences of Fuzzy Numbers ..... 56
Ibrahim Canak, Umit Totur and Zerrin Onder
Chemical Applications of Graph Indices ..... 57Ismail Naci Cangul, Aysun Yurttas, Muge Togan and Ahmet Sinan Cevik
Permuting $n$-Derivations From Semilattice to Lattice ..... 58Sahin Ceran and Utku Pehlivan
Existence of Solutions for Third Order Boundary Value Problems on an Infinite Interval ..... 59
Erbil Cetin
In DES Cryptologic System Using Graph Adjacency Matrice for Random Variables ..... 60Ahmet Sinan Cevik, Mustafa Yonet
Inclusion Regions for Matrix Eigenvalues ..... 61
Mao-Ting Chien
Estimation Procedure for Multi-Parameter Archimedean Copulas Based on the Trimmed L-Moments ..... 62 Method
A.Chine and F. BenatiaHomology Groups of Certain Khalimsky Digital Spaces63Ismet Cinar and Ismet Karaca
A Quintic B-Spline Based Differential Quadrature Method for Numerical Solution of Extended Fisher- ..... 64 Kolmogorov Equation
Sumita Dahiya and R. C. Mittal
Certain Types of Open Covers and Selection Perinciples Using İdeals ..... 65
Pratulananda Das
Imprimitive Action with Continued Fractions for the Suborbital Graphs ..... 66
Ali Hikmet Deger and Ummugulsun Akbaba


Bifurcation of Flow Patterns and Eddy Structure in an L-Shaped Cavity with Lids Moving in the Same Directions

## Ali Deliceoglu

A Nonself-Adjoint Dirac Operators with a Spectral Parameter in the Boundary Condition and with
Transmission Condition
Isil Acik Demirci and Bilender P.Allahverdiev

A Nonself-Adjoint Dirac Operators with a Spectral Parameter in the Boundary Condition and with
Isil Acik Demirci and Bilender P.Allahverdiev
Close-to-convexity of a cross-product of Bessel functions
Erhan Deniz
Existence Criteria of Three Positive Solutions to Boundary Value Problems for p-Laplacian Dynamic
Equations on Time Scales
Abdulkadir Dogan and Bünyamin Aydin
A Study on Some New Results Arising from (p,q)-Calculus
Ugur Duran, Mehmet Acikgoz and Serkan Araci
$\begin{array}{ll}\text { Upper and Lower Solutions for Fourth Order Boundary Value Problems } & 73 \\ \text { Serife Müge Ege }\end{array}$
Invariant Subspace Analysis of the Fractional Modifed Kuramoto-Sivashinsky Equation
E. H. El Kinani

On the Study of the Holditch Theorem for the Non-Linear Three Points in $\boldsymbol{C}_{p}$
75
Tulay Erisir and Mehmet Ali Gungor
The Stability Analysis of a Delay Differential Equation with State Dependent Delay
Sertac Erman and Ali Demir
Generation of the Trigonometric Cubic B-Spline Collocation Solutions for Generalized Burgers-Fisher Equation
Ozlem Ersoy and Idris Dag
An Exponential Cubic B-Spline Finite Element Method for Solving the Nonlinear Coupled Burgers’
Equation
Ozlem Ersoy and Idris Dag
$\begin{array}{ll}\text { On Weakly Sections in Sequence Spaces } & 79 \\ \text { Merve Temizer Ersoy, Hasan Furkan and Bilal Altay } & \end{array}$

## Dispersive Properties of Conservative Schemes for Three Coupled Nonlinear Schrödinger Equation <br> Sevim Ertug and Ayhan Aydın

A Model of Nickel-Iron Alloy Electrodeposition on Rotating Disk Electrode: The Global Existence in the Quadratic Case
Nadia Idrissi Fatmi
Mathematical Analysis for a Class of Quasilinear Elliptic Equations with Nonlinearity in the Gradient and L1-Data
Nadia Idrissi Fatmi
Homoclinical Structure of Hybrid Systems with Impacts
Mehmet Onur Fen and Fatma Tokmak Fen
$\begin{array}{ll}\text { Multiple and Nodal Solutions for Nonlinear Problems } & 84 \\ \text { Michail E. Filippakis }\end{array}$
Estimation of the Scatter Matrix of an Elliptically Symmetric Distributions. Orthogonally Invariant


## Dominique Fourdrinier, Fatiha Mezoued and Martin T. Wells

New Summability Method and Its Applications ..... 86
Mubariz T. Garayev, Mehmet Gurdal and Ulas Yamanci
Some New Ideal Convergent Double Sequence Spaces and Weighted Lacunary I-Statistical Convergence ..... 87 for Double Sequences
Ergin Genc and Sukran Konca
Linear Combinations of L-Functions Satisfying the Same Riemann-Type of Functional Equation ..... 88
Dorin Ghisa
Common Fixed Points of $(\alpha, \beta)$-Implicit Graph Contraction via Cyclic Admissible Pair in Modular Spaces ..... 89
Ekber Girgin and Mahpeyker Ozturk
Improvement of a Secure Authentication Scheme for Session Initiation Protocol Based on ECC ..... 90
Debasis Giri, Tanmoy Maitra and P. D. Srivastava
Lower Envelopes in Vector Spaces ..... 91
Nihat Gokhan Gogus
Matrix Operators on the Series Space $\left|\overline{\boldsymbol{N}}_{\boldsymbol{p}}^{\boldsymbol{\theta}}\right|(\boldsymbol{\mu})$ and Applications ..... 92
Fadime Gökce and Mehmet Ali Sarıgol
Numerical Investigation on the Effect of the Rotation Intensity of a Tornadic Wind Xixiong Guo and Jun Cao
Compact Finite Difference Solutions of Soybean Hydration Model as Stefan Problem ..... 94
Seda Gulen and Turgut Ozis
Enneper-Type Surfaces in Three Dimensional Minkowski Space ..... 95
Erhan Guler, Semra Saraçoğlu Celik and Omer Kisi
A Banach-Stone Type Theorem for Isometries on $L^{p}$ Spaces ..... 96
Banu Gunturk and Bahaettin Cengiz
On Some Codes Over $\mathrm{R}_{2}$ ..... 97
Murat Guzeltepe and Alev Altinel
Closure Algebras of Metric Spaces ..... 99
Ahmet Hamal
Positive Solutions for Semipositone Fractional Boundary Value Problems ..... 100
Nuket Aykut Hamal, Fulya Yoruk Deren and Tugba Senlik Cerdik
On $k$-Quasi Class Q* Operators ..... 101
Valdete Rexhëbeqaj Hamiti, Shqipe Lohaj and Qefsere Gjonbalaj
General Robin Boundary Value Problems for Elliptic Operational Differential Equations with Variable ..... 102 Operators
Rabah Haoua and Ahmed Medeghri
Solutions of Fuzzy Fractional Heat-Like and Wave-Like Equations by Variational Iteration Method ..... 103 Atimad Harir, Said Melliani, L. Saadia Chadl
Absolute Cesàro Series Spaces and Matrix Operators ..... 104
G. Canan Hazar and M. Ali Sarıg̈̈l
Some New Numerical Approximations for Time Fractional Schrödinger Equations ..... 105
Betul Hicdurmaz
New Numerical Approximations for the Nonlinear Population Model ..... 106Betul Hicdurmaz and Emine Can

Conbinatoric Bijection ..... 107Fadila Hocini
To Compute Topological Complexity Numbers Using Steenrod Squares ..... 108
Melih Is and Ismet Karaca
On Darboux Vector in Lorentzian 5-Space ..... 109
Esen Iyigun
Weak Module $(\sigma, \tau)$-Amenability of Triangular Banach Algebras of Order Three ..... 110
Hulya Inceboz and Berna Arslan
Theoretical and Numerical Methods for Two-Phase Flow Modeling ..... 111 Hyeonseong Jin
What is a Multiset? ..... 112
Helmut Jürgensen
On the Exact Distribution of the Product of Two Independent Hypoexponential Random Variables ..... 114
Therrar Kadri, Khaled Smaili, Seifedine Kadry and Ali El-Joubbeh
Finite Dimensional Chebyshev Subspaces of Classical Banach Spaces
Aref K. Kamal
On the Moments of Semi-Markovian Inventory Model When the Demand Distribution Belongs to the General Class of Regularly Varying Distributions with Infinite Variance
A.B.Kamıslık, T. Kesemen and E.Şenol
Inverse Problem of Elliptic Equation with Nonlocal Boundary Conditions ..... 119
Fatma Kanca
Groebner Shirshov Basis of Aut(Fn) for the Word Problem ..... 120
Esma Kangal, Nurten Urlu and Ahmet Sinan Cevik
The Existence of Positive Solutions for Fractional-Order Boundary Value Problems on Finite Interval ..... 121
Ilkay Yaslan Karaca and Serenay AbaliPositive Solutions of Impulsive Time-Scale Boundary Value Problems with p-Laplacian on the Half-Line122Ilkay Yaslan Karaca and Aycan Sinanoglu
The Existence of Positive Solutions for Fractional-Order Nonlinear Boundary Value Problems on Infinite123IntervalIlkay Yaslan Karaca and Dondu Oz
Some Properties of Digital Persistent Homology Groups ..... 124
Ismet Karaca and Ozgur Ege
Calibration Problem with Unknown Operator ..... 125
Karima Belaide
Euler-Lagrange Dynamical Equations on 3-Dimensional Normal Almost Contact Geometry ..... 126
Zeki KasapWeyl-Euler-Lagrange Movement Equations on Almost Paracontact Metric Manifold127Zeki Kasap
Generalized Berinde-Type Contractions in Partially Ordered $\mathrm{G}_{\mathrm{p}}$-Metric Spaces ..... 128
Meltem Kaya and Hasan FurkanOn the Uniform Convergence of Spectral Expansions for a Spectral Problem Rationally Dependent on the129EigenparameterNazim B. Kerimov, Sertac Goktas and Emir A. Maris

Sink Mobility Under Extreme Conditions in Wireless Sensor Networks ..... 130
M. Emre Keskin
Car Insurance Customer Segmentation by Data Mining ..... 131
Sadi Khadidja, Kherchi Hanya and Lounici Nadjib
The Close Relationship Between Architecture and Mathematics ..... 132
Murat Kilic
An Inverse Result for the Periodic Boundary Conditions ..... 133
Alp Arslan Kirac
New Definitons About $A^{\prime}$ - Statistical Convergence with Respect to a Sequence of Modulus Functions and Lacunary Sequences
Omer Kisi, Hafize Gumus and Ekrem Savas
A Combinatorial Method for Characterizing the Linear Combinations of Finitely Many Diagonalizable Matrices That Mutually Commute
Emre Kisi and Halim Ozdemir
A New Approach to Canal Surface with Parallel Transport Frame ..... 136
İlim Kisi and Gunay Ozturk
A Pseudo-Spectral Approach to the Multi-Pantograph Equation Systems ..... 137
Ayse Betul Koc
Some Remarks Related to Orthogonality in the Space of p-Summable Sequences ..... 138
Sukran Konca and Mehmet AslanOn Some Congruences140Sibel Koparal and Nese Omur
On Solutions to the Sylvester s-Conjugate Equations $A\left({ }^{s} \bar{X}\right)-X B=C, s=1,2,3$ Over Elliptic Quaternion141
Matrices
Hidayet Huda Kosal and Murat Tosun
Scale Laws of Prime Number Distributions by the Modified Chi-square Function ..... 142
Daniele Lattanzi
Some Properties of k-Quasi Class Q Operators ..... 143
Shqipe Lohaj and Valdete Rexhëbeqaj Hamiti
Hele-Shaw Flow with a Time-Dependent Gap: The Schwarz Function Approach to the Interior Problem ..... 144
K. Malaikah, T.V. Savina and A.A. Nepomnyashchy
Numerical Study of Heat Transfer and Entropy Generation in a Nanofluid Filled Two-Sided Lid-Driven ..... 146 Cavity
Sumit Malik and A.K.Nayak
Modeling of Tomatoes Solar Drying Systm in Arid Areas in Algeria ..... 147
Saadeddine Manaa, A. Beatriz, D. Karlo and N. Moummi
An Optimal Control Problem with Control in Coefficients ..... 148Gabriela Marinoschi
Asymptotic Representations for Heavy-Tailed Distributions Under Random Censoring ..... 149
Djamel Meraghni150Behavior of Cellular Beams with Various Opening ShapesMostefa Mimoune, Saad Siouane and Fatima Z. MimouneMagnetic Fields of Asymmetric Magnetic Recording Heads Using the Superposition of the Head Filed151Ammar Edress Mohamed
$\square$
On Fractional Differential Inclusions in Banach Space ..... 152
Fatima Zohra Mostefai
Some Inequalities for the Multiplicative Zagreb Indices of Graph Operations ..... 153
Yasar Nacaroglu and A.Dilek Maden
Statistics of the Extreme Values Under Random Truncation ..... 154
Abdelhakim Necir
Vaccination for Preventing Future Rubella Epidemic in Japan ..... 155
Hiroshi Nishiura and Ryo Kinoshita
Application of Data Mining to Insurance Multi-Risk Habitation and Professional Branch ..... 156
Lounici Nora, Sadi Khadidja and Djemaa Hassiba
Exact Solutions of Some Nonlinear Evolution Equations ..... 157
Meryem Odabasi and Emine Misirli
Existence of Solutions for Dynamic Systems on an Infinite Time Scale ..... 158
Arzu Denk Oguz and S. Gulsan Topal
An Extension of a Valuation $v$ on a Field $K$ with rankv $=2$ to $K(x)$159Figen Oke
On Tauberian Conditions for the $(\bar{N}, p)$ Summability of Integrals ..... 160
Muhammet Ali Okur, Ibrahim Canak and Umit Totur
Eigenvalues and Spectral Singularities of Non-Selfadjoint Matrix Sturm-Liouville Operators with ..... 161
Eigenvalue-Dependent Boundary Conditions
Murat Olgun
A New Type of Convergence for a Sequence of Rays ..... 162
Oznur Olmez and Salih Aytar
Some Commutativity Theorems for Rings with Involution ..... 163
Lahcen Oukhtite
A Nonstandard Numerical Scheme for a Predator-Prey Model with Allee Effect ..... 164
Nihal Ozdogan and Mevlude Yaitt Ongun
On the Stability of a Neural Field Model ..... 165
Berrak Ozgur and Ali Demir
Value Groups and Residue Fields Writing via Distinguished Chains ..... 166
Burcu Ozturk and Figen Oke
Fixed Point Results for General Type Contractions in Modular Spaces ..... 167
Mahpeyker Ozturk
Common Best Proximity Points of Generalized Almost $(\alpha, \beta)-(\psi, \phi)$-Geraghty Contractive Mappings in ..... 168
Metric Spaces
Mahpeyker Ozturk and Neslihan Kaplan
Relations Between Darboux and Bishop Frames on a Regular Curve in Minkowski Space ..... 169Emin Ozylmaz and Amine Yilmaz
Results on Hadamard Codes and Codes over Rings ..... 170
Mustafa Ozkan and Figen Oke

Perturbation-Iteration Method for Singular Problems ..... 171
Mehmet Pakdemirli
Transportation Situations and Related Games with Interval Uncertainty ..... 173
Osman Palanci, Sirma Zeynep Alparslan Gok, Hamid Alzaki and Sule Nizamoglu
Solutions of Some Diophantine Equations ..... 174
Bilge Peker
Numerical Solution for a Free Convection Flow ..... 175
Haldun Alpaslan Peker
A Method for Decision Making Problems by Using Graph Representation of Soft Set Relations ..... 176
Nazan Cakmak Polat and Bekir Tanay
The Best Quadratic Approximation of Hyperbola with Order Four ..... 177
Abedallah RababahExistence Results for Weak Solution of Multi-Term Fractional Differential Equations in Non-Reflexive178
Banach Spaces via Riemann Pettis İntegral
Ghaus Ur Rahman
An M/M/2 Retrial Queue with Breakdowns and Repairs ..... 179
Lila Raiah and Nadia Oukid
Isothermic Hopf Cylinders and Slant Helices ..... 181
Cagla Ramis and Yusuf Yayli
On the M-Power Class (N) ..... 182
M. H. M. Rashid
Generalized Topological Vector Spaces ..... 183
Salti Samarah
On $\mathrm{S}_{\lambda}{ }^{\mathrm{L}}(\mathrm{I})^{\alpha}$ Asymptotically Statistical Equivalent of Functions ..... 185
Rabia Savas and Mahpeyker Ozturk
On Asymptotically I-Lacunary Statistical Equivalent Functions of Order $\alpha$ ..... 186Rahmet Savas
A New Numerical Method Based on Hybrid Taylor and Lucas Polynomials for Solving a Class of Linear ..... 187 Volterra-Type Functional Integto-Differential Equations with Proportional and Variable Delays Nurcan Baykus Savasaneril and Mehmet Sezer
Existence Result for Nonlinear Fractional Differential Equations with Nonlocal Fractional Integro- ..... 188
Differential Boundary Conditions in Banach Spaces
Djamila Seba
On Statistical Convergence of Order $(\alpha, \beta)$ ..... 189
Hacer Sengul
Reduced Differential Transform Method with Fixed Grid Size for Solving Klein Gordon Equations ..... 190
Sema Servi, Yildiray Keskin and Galip OturancTauberian Conditions Under Which Ordinary Convergence Follows From Logarithmic Type Summability191
MethodsSefa Anıl Sezer and Ibrahim Canak
BEM Solution of MHD Flow in a Semi-Infinite Channel with Variable Wall Conductivity ..... 192
M. Tezer-Sezgin and Canan Bozkaya
A General Solution to a Dynamical Problem of Continuous Media Having Cubic Nonlinearities ..... 193
B.Gültekin Sınır and Sümeyye Sınır

Kinematic Surfaces with Constant Scalar Curvature in Euclidean 5-Space ..... 194
Emad Solouma
Fuzzy Complex System Reliability Analysis Using Depth-First Search ..... 195
Halil İbrahim Sahin and Melek Eris
Interpretation of Syllogisms on Boolean Algebras ..... 196
Ibrahim Senturk, Tahsin Oner and Urfat NuriyevSelf-Organizing Topological Maps for Classification of the Hercynian Granitoids From Their Geochemical197Characteristics: Case of the Aouli Pluton (High Moulouya, Morocco)Abdelghani Talhaoui, Imad Manssouri, Abdellah El Hmaidi, Mohamed Berrada and TajeddineManssouri
Demonic Operational and Denotational Semantics Commute ..... 198
Fairouz Tchier
On a Fractional Model Arising in Spintronics ..... 199
Mouhcine Tilioua
Existence and Multiplicity of Positive Solutions for Fractional Boundary Value Problems ..... 200
Fatma Serap Topal
Applying Graph Coloring to Schedule Doctors' Work in Hospital ..... 201
Fardous Toufic and Khouloud Al-Gahtani
A-Compactly Uniform Integrability of Sequences of Random Elements ..... 202
Mehmet Unver
Solution to General Weakly Non-Linear Dynamic Problem ..... 203
Leyla Usta and B.Gültekin Sinir
Self-Dual Codes and the Steenrod Algebra ..... 204
Tane Vergili and Ismet Karaca
Integral Equation Methods for the Planar Exterior Robin Boundary Value Problem of the Laplacian ..... 205 Olha Ivanyshyn Yaman and Gazi Ozdemir
On Numerical Radius and Berezin Number Inequalities for Reproducing Kernel Hilbert Space ..... 206
Ulas Yamanci and Mehmet Gurdal
A Note on Power Central Values of Generalized Skew Derivations with Annihilating Conditions ..... 207
Nihan Baydar Yarbil
New Analytic Solutions of the Space-Time Fractional Cahn-Hilliard Equations ..... 208
Handan Cerdik Yaslan
Multi-Point Boundary Value Problems on an Unbounded Time Scale ..... 209
Ismail Yaslan and Mustafa Gunendi
Results on Soft Continuous Functions in the Soft Topological Spaces Equipped with Soft Scott Topology ..... 210Gozde Yaylalı and Bekir Tanay
Idempotent and Nilpotent Subsemimodules of Semimodules ..... 211
Gulsah YesilkurtConvolution and Approximation in Weighted Lorentz Spaces212Yunus Emre Yildirirk-Chinese Postman Problem Approach for Snow Plowing Operations: A Case Study213Mustafa Yilmaz, Merve Kayaci Codur

A Bound for the Number of Symmetric Colorings of a Finite Group ..... 216
Yuliya ZelenyukOne by One Embedding the Crossed Hypercube into Pancake Graph217
Mohamed Faouzi Zerarka and Smain Femmam


# Soft Contraction Theorem 

Mujahid Abbas<br>Department of Mathematics and Applied Mathematics, University of Pretoria, Hatfield, Pretoria, South Africa<br>abbas.mujahid@gmail.com


#### Abstract

Soft set theory has become a full-fledged research area and a vast amount of mathematical activity has been carried out to obtain many remarkable results showing the applicability of soft set theory in decision making, demand analysis, forecasting, information science, mathematics and other disciplines. On the other hand, fixed point theory deals with the conditions which guarantee that a mapping T of a set X into itself admits one or more fixed points. Fixed point theory serves as an essential tool for solving problems arising in various branches of mathematical analysis. Intersection of metric fixed point and soft set theories is a very recent development. The aim of this talk is to discuss the concept of soft contraction mapping on soft metric spaces and then, a theorem of Banach contraction principle type called soft contraction theorem in the setup of soft complete metric spaces. An example which illustrate some restrictions of soft metric fixed point theory is discussed.


# Uniform and Pointwise Inequalities for Algebraic Polynomials on a Regions in the Complex Plane 

Fahreddin G. Abdullayev<br>Department of Mathematics, Mersin University, Mersin, Turkey<br>fabdul@mersin.edu.tr


#### Abstract

In many areas of research in mathematics (in approximation theory, embedding theory and etc.), one can be faced with two important problems: determining how to undergo a change of (semi)norm of the holomorphic function when the given region expands; determining the relationships between different (semi)norms of analytic functions in a given finite Jordan region on the complex plane in the various (semi)normed space. We will consider this problems for algebraic polynomials of complex variables in the well known Bergman and Lebesgue spaces, and investigate the following problems: evaluating the increase of the modulus of polynomials in the exterior of the given region with respect to the (semi)norm of the polynomial in the this region [1,2,3]; determining a change of (semi)norm of polynomials for the given region [4,5] and, finally, combining obtained estimations for the modulus of polynomials, we will get the estimation modulus of polynomials in whole complex plane [5].


Keywords: Algebraic Polynomials, Conformal Mapping, Smooth Curve, Quasicircle.

## References:

[1] Hille E., Szegö G., Tamarkin J.D., On some generalization of a theorem of A.Markoff , Duke Math., 3(1937), 729-739.
[2] Stylianopoulos N., Strong asymptotics for Bergman polynomials over domains with corners and applications, Const. Approx., 38(2013), 59-100.
[3] Abdullayev F.G., The properties of the orthogonal polynomials with weight having singularity on the boundary contour. Journal of Com. Anal. and its Appl.,Vol. 6(1)(2004), 43-60.
[4] Jackson D., Certain problems on closest approximations, Bull. Amer. Math. Soc., 39, (1933), 889-906.
[5] Abdullayev F.G., Özkartepe P., On the Behavior of the Algebraic Polynomial in Unbounded Regions with Piecewise Dinı-Smooth Boundary, Ukr. Math. J. , Vol. 66(5) (2014), 645-665.


# Some Problems of the Theory of Bifurcations of Non-Linear Problems of Fourth Order 

Ziyatkhan S. Aliyev<br>Baku State University, Institute of Mathematics and Mechanics of NAS of Azerbaijan,<br>z_aliyev@mail.ru


#### Abstract

For nonlinear Sturm-Liouville problems are obtained sufficiently general results on the local and global bifurcations, which are reflected in Rabinowitz and Berestycki well-known global bifurcation theorems and in works many other authors. But the global bifurcation of solutions of nonlinear fourthorder problems are studied in some papers, only in cases of specific differentiable perturbations of linear fourth-order problem with constant coefficients for specially chosen boundary conditions. In the present report, will be provided the construction of meaningful theory local and global bifurcations of a broad class of nonlinear eigenvalue problems for ordinary differential equations of fourth order.


# Matrix Maps Between Statistical Sequence Spaces 

Jeff Connor<br>Department of Mathematics Ohio University Athens, Ohio 45701<br>connorj@ohio.edu


#### Abstract

This talk is intended for a general audience, and will present an approach to describing maps between "statistical" sequences spaces: sequence spaces where the role of finite sets has been replaced with members from an ideal of subsets of natural numbers. The best known of these ideals is the collection of subsets of natural numbers that have arithmetic mean density zero. Although this talk will focus more on broad principles than particular results, there will be some discussion of results related to matrices that map statistically convergent sequences into themselves, maps that preserve the statistical core, and maps that preserve statistical asymptotic limits.


# Applications of Newton Interpolating Series in Archimedean and Non-Archimedean Analysis 

Ghiocel Groza<br>Department of Mathematics and Computer Science, Technical University of Civil Engineering Bucharest, Bucharest, Romania<br>grozag@utcb.ro


#### Abstract

Newton interpolating series, which is a natural generalization of Taylor series, have useful applications in different areas of mathematics. Since the partial sums of a Newton interpolating series are Newton interpolation polynomials they are a useful tool in the interpolation theory and they can be used to approximate solutions of boundary value problems for differential equations (see, for example, [1], [2], [3]). Also the expansion of the exponential function into a suitable Newton interpolation series is the key of proof of an important result in number theory (see [4]). In the case of complex functions (see [5], [6]) important results on entire functions follow easily by using Newton interpolation series. Useful results are obtained also for non-archimedean fields (see [7]), where they generalize Mahler series.


Keywords: Newton Interpolating Series, Boundary Value Problems, Analytic Function, Non-Archimedean Field, Mahler Series.

## References:

[1] G. Groza and N. Pop, "Approximate solution of multipoint boundary value problems for linear differential equations by polynomial functions", J. Difference Equ. Appl., 14 (2008), No. 12, 1289-1309.
[2] G. Groza and N. Pop, "A numerical method for solving of the boundary value problems for ordinary differential equations", Result. Math., 53(2009), No. 3-4, 295-302.
[3] G. Groza, M. Jianu and N. Pop, "Infinitely differentiable functions represented into Newton interpolating series", Carpathian J. Math., 30(2014), No. 3, 309-316.
[4] A. Shidlowski, "Transcendental Numbers", de Gruyter, 1989.
[5]. G. Groza, A. Haider and S. M. Ali Khan, "Interpolation of entire functions", Bol. Soc. Mat. Mex., III. 17 (2011), No. 2, 127-133.
[6] Y. Martin, "Sur les séries d'interpolation", Ann. Sci. ' Ecole Norm. Sup., 66 (1949), sér. 3, 311-366.
[7] G. Groza and A. Haider, "A generalization of strictly convergent power series and applications", Bull. Transilv. Univ. Brasov, Ser. III, Math. Inform. Phys. (Proceedings of the Seventh Congress of Romanian Math.), 5(54) (2012), 155-166.


# Some Classes of Divergent Sequences 

Ljubiša D.R. Kočinac<br>University of Niš, Faculty of Sciences and Mathematics, 18000 Niš, Serbia lkocinac@gmail.com


#### Abstract

We present some known and some new results concerning certain classes of divergent real sequences related to the Karamata theory in asymptotic analysis. Relations of these classes of sequences with selection principles theory will be discussed. We also consider statistical versions of some of the mentioned notions. Keywords: Regular Variation, Rapid Variation, Selection Principles, Statistical Convergence.


# Morrey Regularity of the Weak Solutions of Some Quasilinear Elliptic Systems 

Lubomira Softova<br>Department of Civil Engineering, Design, Construction and Environment, Second University of Naples, Italy<br>luba.softova@unina2.it


#### Abstract

We consider quasilinear divergence form elliptic systems defined via Carathéodory maps satisfying controlled growth conditions with data belonging to some Morrey spaces. In addition we suppose that the principal operator satisfies componentwise coercivity condition for large values of the solution. We show essential boundedness, higher integrability and Morrey regularity of the weak solution in bounded domain, satisfying the Reifenberg flatness condition. Keywords: Quasilinear Elliptic Systems, Controlled Growth Conditions, Coercivity Condition, Boundedness, Morrey Spaces.


## References:

[1] S. Byun, D. Palagachev, Boundedness of the weak solutions to quasilinear elliptic equations with Morrey data, Indiana Univ, Math. J., 62, No. 5 (2013), 1565-1585.
[2] F. Leonetti, P.V. Petricca, Regularity for solutions to some nonlinear elliptic systems, Complex Variabl. Elliptic Equ., 56, No. 3 (2011), 1099-1113. [3] L. Softova, Lp-Integrability of the gradient of solutions to quasilinear systems with discontinuous coefficients, Differ. Integr. Equ., 26, No. 9-10 (2013), 1091-1104.

# The Navier-Stokes Equations - New Trends on Weak and Strong Solutions 

Werner Varnhorn<br>Institute of Mathematics, Kassel University, 34109 Kassel, Germany varnhorn@mathematik.uni-kassel.de


#### Abstract

The nonstationary nonlinear three-dimensional Navier-Stokes problem occupies a central position in the study of nonlinear partial differential equations, dynamical systems, scientific computation, and classical fluid dynamics. Because of the complexity and variety of fluid dynamical phenomena on the one hand, and the simplicity and exactitude of the equations' shape on the other hand, a strong depth and beauty is expected in the mathematical theory. It is a source of pleasure and fascination that many of the most important questions in the theory remain yet to be answered. So the famous American Clay Mathematics Institute created the Navier-Stokes Millennium Price Problem and offered one Million Dollar for its solution, stating: "Although the Navier-Stokes equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory, which will unlock the secrets hidden in the Navier-Stokes equations". The modern mathematical theory of the Navier-Stokes equations started with the pioneering work of Jean Leray [6] in 1933-34. Leray was the first to use methods of functional analysis for the treatment of partial differential equations. He developed the concept of weak solutions for the Navier-Stokes Cauchy problem and proved their existence globally in time long before Schwartz established the theory of distributions, and even before Sobolev systematically introduced the spaces bearing his name. Leray has laid the basis of the mathematical theory for the Navier-Stokes equations as we know it today, and he has introduced many tools and ideas still used constantly since then. The lecture introduces the Navier-Stokes equations from a historical and physical point of view, touches some fundamental mathematical problems of viscous incompressible fluid flow [1,4,8] and ends up with recent regularity results on weak [5] and strong [7] solutions in Sobolev and Besov spaces [2,3].


Keywords: Navier Stokes Equations, Weak Solutions, Strong Solutions, Serrin's Condition, Besov Spaces.

## References:

[1] N. Asanalieva, C. Heutling, W. Varnhorn, "Time delay and Lagrangian approximation for Navier-Stokes flow", Analysis 35 (2015), 213-229
[2] R. Farwig, H. Sohr, W. Varnhorn, "Besov space regularity conditions for weak solutions of the Navier-Stokes equations", J. Math. Fluid Mech.16(2014), 307-320

[3] R. Farwig, H. Sohr, W. Varnhorn, "Local strong solutions of the nonhomogeneous Navier-Stokes system with control of the interval of existence, Topol. Meth. Nonl. Anal. 46 (2015), 999-1012
[4] R. Farwig, C. Simader, H. Sohr, W. Varnhorn, "General properties of the Helmholtz decomposition in spaces of Lq-type. Contemp. Math., to appear
[5] E. Hopf, "Über die Anfangswertaufgabe für die hydrodynamischen Grundgleichungen", Math. Nachr. 4 (1951), 213-231.
[6] J. Leray, "Sur le mouvement d'un liquide visqueux emplissant l'espace", Acta Math. 63 (1934), 193-248.
[7] J. Serrin, "The initial value problem for the Navier-Stokes equations", Nonlinear Problems, R. E. Langer, ed., University of Wisconsin Press (1963), 69-98. [8] W. Varnhorn, "A Crank-Nicholson Method for Non-steady Stokes Flow", In: Proc. Topical Problems of Fluid Mechanics 2016, Inst. Thermomech. \& Acad. Sci. Czech Republic, Prague (2016), 237-240

# Neo-Equilibrium Point in the Network Game 

Ahmad Termimi Ab Ghani<br>School of Informatics and Applied Mathematics, Universiti Malaysia Terengganu, Malaysia<br>termimi@umt.edu.my


#### Abstract

Mavronicholas et al. [1] presented a network game as an undirected graph whose nodes are exposed to infection by attackers, and whose edges are protected by a defender. After that, MedYahya et al. [2] generalized the model so that they have many defenders instead of a single player. Then in [3], we introduced a new network game with the roles of players interchanged, and obtained a graph-theoretic characterization of pure Nash equilibria of our new model. Then in [4], we focus on mixed strategies and study the complexity of finding (mixed) Nash equilibria in the new game. In this paper, we provide a variation of Nash equilibrium with respect to graph properties.


Keywords: Pure Strategy, Mixed Strategy, Nash Equilibrium.

## References:

[1] Mavronicholas, M., Papadopoulou, V., Philippou, A., Spirakis, P.G.: A Network Game with Attacker and Protector Entities. Proc. of the 16th Annual International Symposium on Algorithms and Computation, 3827, 288-297, 2005.
[2] MedYahya Ould-MedSalem., Manoussakis, Y., Tanaka, K.: A Game on Graphs with Many non-Centralized Defenders. (EURO XXII) 22nd European Conference on Operations Research. July 8-11, Prague, 2007.
[3] Ahmad Termimi, A.G. and Tanaka, K.: Network Games with Many Attackers and Defenders. Proceedings of Research Institute for Mathematical Sciences (RIMS) K^oky^uroku Kyoto University. 1729, 146-151, 2011
[4] Ahmad Termimi, A.G., Kojiro Higuchi and Tanaka, K.: Values on generalized reachability games. Proc. of Research Institute for Mathematical Sciences (RIMS) K^oky^uroku Kyoto University. 1832, 143-153, 2013

# An Extension of Derivation on Prime Near-Rings 

Madugu Abdu and Moharram A. Khan<br>Department of Mathematics and Computer Science Umaru Musa Yaradua University, Katsina, P.M.B. 2218 Dutsin-ma Road, Katsina Sstate, Nigeria.

abdumadugu@yahoo.com, mkhan91@gmail.com


#### Abstract

In this talk, we discuss the study of derivation in near-rings that was initiated by Bell and Mason (1987) and found many results regarding the behavior of near-ring as a commutative ring. In this line of investigation several authors have studied commutativity of prime and semi prime-rings admitting suitably constrained derivations. From this observation, it is a natural to look for comparable results as near-ring. Our aim is to investigate identities and then extend some results on prime near-ring with derivations. Motivated by this, we establish the commutativity of prime near-ring $N$, if there exist positive integers $m, n$ such that $N$ admit a derivation $d$ satisfying the folloing identities:


(i) $d([x, y])+y^{m}[x, y] y^{n}=0 \forall x, y \in N$
(ii) $d([x, y])-y^{m}[x, y] y^{n}=0 \forall x, y \in N$
(iii) $d(x o y)+y^{m}(x o y) y^{n}=0 \forall x, y \in N$
(iv) $d(x o y)-y^{m}(x o y) y^{n}=0 \forall x, y \in N$.

Keywords: Commutativity, Derivation, Prime Near-Ring.

## References:

[1] K.I. Beidar, Y. Fong and X. K. Wang, Posner and Herstein's theorem for derivations of 3-prime near-rings, Comm. Algebra 24 (1996), 1581-1589.
[2] H. E.Bell, On derivations in near-ring II, Near-rings and Near-fields and KLoops, 426 (Kluwer Academic Publishers, Dordrecht), (1997), 191-197.
[3] H.E. Bell, and G.Mason, On derivations in near-rings, Near-rings and nearFields, Amsterdam, Math. Stud., 137 (1987), 31-35.
[4] M. A. Khan and M..S. Khan, On near-rings with derivations, Math.Pannonica, 17(2006), 131-138.
[5] X. K. Wang, X. K. Derivations in prime near-rings, Proc. Amer. Math. Soc., 121(1994), 361-366.


# Polynomial Inequalities on Unbounded Regions 

F. G. Abdullayev, G. A. Abdullayev<br>Mersin University<br>Mersin, Turkey<br>fabdul@mersin.edu.tr; gabdullayeva@yandex.com


#### Abstract

In this, we study the estimation of the modulus of algebraic polynomials on the boundary contour of unbounded regions with weight function, when the contour and the weight function have some singularities, wich respect to the their quasinorm in the weighted Lebesgue space and the exterior Riemann function. We obtain sharp estimations for modulus of polynomials on the every points of the unbounded region. Keywords: Algebraic Polynomials, Conformal Mapping, Quasi-Smooth Curve, Quasicircle.

\section*{References:} [1] Hille E., Szegö G., Tamarkin J.D., On some generalization of a theorem of A.Markoff , Duke Math., 3(1937), 729-739. [2] Stylianopoulos N., Strong asymptotics for Bergman polynomials over domains with corners and applications, Const. Approx., 38(2013), 59-100. [3] Abdullayev F.G., Özkartepe P., Uniform and pointwise polynomial inequalities in regions with cusps in the weighted Lebesgue space, Jaen Journal on Approximation, Vol.7, No:2, 2015, pp.231-261.


# Using Wavelet Denoising Methodology for Enhancing the Performance of Forecasting of ARIMA Methodology 

Qais Mustafa Abdulqader<br>Department of Hospital Management, Duhok Polytechnic University, Dohuk, Zakho, Iraq<br>qais.mustafa@dpu.edu.krd


#### Abstract

Many applications have been done in the field of using wavelet analysis for time series analysis such as the recent works of Mustafa and Alzubaydi [1] and also Oinam, Kumar and Patil [2]. In [3] Koo, Lee and Park suggested a method for short-term electric load forecasting that uses a wavelet transform and group method of data handling algorithm. In this paper, we aim to describe how wavelet denoising can be used in time series forecasting and improve the forecasting quality through applying both methodologies on electricity load data and comparing the efficiency of the main classical ARIMA methodology and wavelet denoising methodology.The comparison is depending on some statistical criteria.


Keywords: Wavelet Denoising, ARIMA Methodology, Time Series Forecasting, Electricity Load Data.

## References:

[1] Q. Mustafa and T. H. Alzubaydi, "Comparing the Box-Jenkins models before and after the wavelet filtering in terms of reducing the orders with application", Journal of Concrete and Applicable Mathematics. 11(2013), 190-198.
[2] S. Oinam, H. Kumar and S. B. Patil "Compression of time series signal using wavelet decomposition, wavelet packet and decimated discrete wavelet compression transforms techniques and their comparison", International Journal of Advanced Research in Computer and Communication Engineering, 2(2013), 1540-1544.
[3] B. G. Koo, H. S. Lee and J. Park "Short-term electric load forecasting based on wavelet transform and GMDH", Journal of Electrical Engineering and Technology, 10(2015), 832-837.

# The Use of Conformable Variational Iteration Method, Conformable Reduced Differential Transform Method and Conformable Homotopy Analaysis Method for Solving Different Types of Nonlinear Partial Differential Equations 

Omer Acan ${ }^{1,2^{*}}$ Omer Firat ${ }^{3}$ Yildiray Keskin ${ }^{1}$<br>${ }^{1}$ Department of Mathematics, Science Faculty, Selçuk University, Konya 42003, Turkey<br>${ }^{2}$ Department of Mathematics, Faculty of Arts and Science, Siirt University, Siirt, Turkey<br>${ }^{3}$ Department of Mathematics, Faculty of Arts and Science, Kilis 7 Aralik University, Kilis<br>*omeracan@yahoo.com,<br>ofirat@gmail.com, vildiraykeskin@yahoo.com


#### Abstract

In this presentation, we introduce conformable variational iteration method (C-VIM), conformable fractional reduced differential transform method (CFRDTM) and conformable homotopy analysis Method (q-HAM). These methods are a new version of well known variational iteration method (VIM), reduced differential transformation method (RDTM) and homotopy analaysis method (HAM) based on new defined conformable fractional derivative to solve linear and nonlinear fractional partial differential equations (PDEs). Firstly, we present some basic definitions, theorems and general algorithm for proposal methods to solve linear and nonlinear fractional PDEs. And then to better understand, the presented new methods are supported by some examples. Finally, the obtained results are illustrated by the aid of graphics and the tables. The applications show that these new techniques C-VIM, CFRDTM and q-HAM are extremely reliable and high accuracy and provide a significant improvement in solving linear and nonlinear fractional PDEs


Keywords: Fractional derivative, Conformable variational iteration method (CVIM), Conformable fractional reduced differential transform method (CFRDTM), Conformable homotopy analaysis method (q-HAM), Partial differential equations.

## References:

[1] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, J. Comput. Appl. Math. 264 (2014) 65-70.
[2] T. Abdeljawad, On conformable fractional calculus, J. Comput. Appl. Math. 279 (2015) 57-66.
[3] E. Ünal, A. Gödogan, Solution of Conformable Fractional Ordinary Differential Equations via Differential Transform Method, arXiv Prepr. 1602.05605 (2016) 1-14.
[4] O. Acan, O. Firat, Y. Keskin, G. Oturanc, Conformable Variational Iteration Method, New Trends Math. Sci. (2016) (In press).


Rough Semicontinuous Set Valued Maps<br>Fatma Gecit Akcay and Salih Aytar<br>Department of Mathematics, Süleyman Demirel University, Isparta, Turkey<br>salihaytar@sdu.edu.tr


#### Abstract

In this talk, we define the concepts of rough semi-continuity and rough continuity of set valued maps. Then we will investigate whether these definitions can be characterized in different ways or not. Finally, we will investigate whether rough continuity is preserved under certain operations or not.


Keywords: Rough Continuous; Set-Valued Map; Upper Semi-Continuous; Lower Semi-Continuous.

## References:

[1] Aubin, J. P. 1990. Set-Valued Analysis Chambridge, Mass: Birkhauser Boston Inc..
[2] Beer, G. 1993. Topologies on Closed and Closed Convex Sets. Boston: Kluwer Academic Publishers.
[3] Geletu. A. "Introduction to Topological Spaces and Set-Valued Maps". Ilmenau University of Technology Lecturer Notes.
[4] Kanibir, A., Reilly I. L. 2009. "Generalized contunuity for multifunctions", Acta. Math. Hungar, 122 (3), 283-292.
[5] Phu, H. X. 2002. "Rough continuity of linear operators", Numerical Functional Analysis and Optimization, 23, 139--146.
[6] Rockafellar, R.T., Wets R. J. B. 1997. Variational Analysis. Grundlehren der Mathematischen Wissenschaften Springer.

# On a New Subclass of Meromorphic Functions with Positive Coeffitients Defined by a Certain Integral Operator 

Arzu Akgul<br>Department of Mathematics, Kocaeli University,<br>Kocaeli, Turkey<br>akgulcagla@hotmail.com


#### Abstract

Many subclasses of meromorphic functions with positive coeffitient have been defined and studied in the past. The functions in this class are introduced by Junea and Reddy [5]. The aim of the present paper is to introduce a new subclass of meromorphic funtions with positive coeffitients by means of a certain integral operator introduced by Lashin [7] and a necessary and suffitient condition for a function $f$ to be in this class. We obtain distortion properties, radii of close-to-convexity, starlikeness, meromorphically convex linear combinations for the functions $f$ in this class.


Keywords: Meromorphic Function, Positive Coeffitients, Coeffitient Inequality, Integral Operators, Convex Linear Combination.

## References:

[1] E.Aqlan, J.M.Jhangiri and S.R.Kulkarni, "Class of K-uniformly convex and starlike functions", Tamkang J. Math. 35(2004), 1-7.
[2] W.G.Athsan and S.R.Kulkarni, , "Subclass of meromorphic functions with positive coeffitients defined by Ruscheweyh derivate I", J. Rajasthan Acad. Phy.Sci., 6(2) (2007), 129-140.
[3] S.K. Bajpai, "A note on a class of meromorphic univalent functions", Rev.Roumanie Math. Pure Appl., 22(1977), 295-297.
[4] R.M.Goel and N.S. Sohi, "On a class of meromorphic functions", Glasnik Mathematicci, 17 (1981).
[5] O.P.Junea and T.R.Reddy, "Meromorphic starlike univalent functions with positive coeffitients", Ann. Univ. Mariae Curie Sklodowska, Sect.A, 39 (1985), 65-76.
[6] I.B.Jung,Y.C.Kim, H.M. Srivastava, "The Hardy spaces of analytic functions associated with certain one parameter families of integral operators", J.Math.Anal.Appl. 176 (1), (1993), 138-147.
[7] A.Y.Lashin, "On certain subclasses of meromorphic functions associated with certain integral operators", Comput.Math.Appl.59(1), (2010), 524-531.

# Some Properties of Generalized Hausdorff Matrices 

F.Aydin Akgun and B.E.Rhoades<br>Department of Mathematical Engineering, Yildiz Technical University, Istanbul, Turkey Department of Mathematics, Indiana University, Bloomington, U.S.A. fakgun@yildiz.edu.tr, rhoades@indiana.edu


#### Abstract

The nonzero entries of an H-J matrix are defined by $$
h_{n k}(\mu ; \lambda)=\lambda_{k+1} \ldots \lambda_{n}\left[\mu_{k}, \ldots \mu_{n}\right]
$$


where [.] is the symmetric difference operator defined by $\left[\mu_{k}, \mu_{k+1}\right]=\left(\mu_{k}-\right.$ $\left.\mu_{k+1}\right) /\left(\lambda_{k+1}-\lambda_{k}\right)$ and, for $\mathrm{n}>1$,

$$
\left[\mu_{k}, \ldots, \mu_{n}\right]=\left\{\left[\mu_{k}, \ldots, \mu_{n-1}\right]-\left[\mu_{k+1}, \ldots, \mu_{n}\right]\right\} /\left(\lambda_{n}-\lambda_{k}\right)
$$

Hausdorff considered those methods for which $\lambda_{0}=0$, and Jakimovski[2] investigated such matrices for $\lambda_{0}>0$.
The other generalization we shall consider is the class of E-J matrices, which were defined independently by Jakimovski [2] and Endl [1]. The nonzero entries of an E-J matrix are

$$
h_{n k}^{(\alpha)}=\binom{n+\alpha}{n-k} \Delta^{n-k} \mu_{k}, 0 \leq k \leq n, n, \alpha \geq 0
$$

Thus, the H-J matrices reduce to the E-J matrices by setting $\lambda_{n}=\mathrm{n}+\alpha$, and the choice $\lambda_{n}=\mathrm{n}$ yields the ordinary Hausdorff matrices. The purpose of this paper is to show that, in spite of the richness of these integral domains of operators, there are no conservative H-J matrices with certain kinds of structure.
Keywords: H-J Matrices, E-J Matrices, Hausdorff Matrices.

## References:

[1] K. Endl, Abstracts of short communications and scientific program, Int. Congress of Math. 73(1960), p. 46.
[2] A. Jakimovski, The product of summability methods; new classes of transformations and their properties, I, II, Technical (Scientific) Note No. 2, Contract No. AF61(052)-187 (1959).

## Matrices Which Preserve Decreasing Sequences

F.Aydin Akgun, S. Mahmood and B.E.Rhoades<br>Department of Mathematical Engineering, Yildiz Technical University, Istanbul, Turkey Department of Mathematics, Indiana University, Bloomington, U.S.A.<br>fakgun@yildiz.edu.tr,mosamosa510@yahoo.com rhoades@indiana.edu

A sufficient condition is established in order for large classes of nonnegative triangular matrices to map positive monotone decreasing sequences into positive monotone decreasing sequences. A necessary condition is obtained, and examples are also given of some positive matrices which, because of their structure, map positive monotone decreasing sequences into positive monotone decreasing sequences.

Keywords: Monotone Decreasing Sequences, Factorable Matrices, Generalized Nörlund Matrices, Summability Matrices, Weighted Mean Matrices.

## References:

[1] G.Bennett, Lower bounds for matrices, II Canadian j. Math. 44(1992), 54-74.
[2] F. Hausdorff, Summationmethoden und Momentfolgen, II Math. Z. 9(1921), 280-299

# Some Mixed Means of Fourier Series in Lebesgue Spaces $\mathbf{L}_{\mathrm{p}, \boldsymbol{\omega}}\left(\mathbf{T}^{\mathrm{d}}\right)$ with Muckenhoupt Weights $\omega$ 

Ramazan Akgun<br>Department of Mathematics, Balikesir University, Balikesir, Turkey<br>rakgun@balikesir.edu.tr


#### Abstract

Mixed modulus of smoothness in Lebesgue spaces with Muckenhoupt weights are investigated. Using mixed modulus of smoothness we obtain Potapov type direct and inverse estimates of angular trigonometric approximation of functions in these spaces. Also we obtain equivalences between mixed modulus of smoothness and K-functional and realization functional. Fractional order modulus of smoothness is considered as well. Keywords: Direct/Inverse Theorems, Muckenhoupt Weights, Modulus of Smoothness, Fourier Series, Mixed Means.

\section*{References:} [1] R. Akgün, Mixed modulus of continuity in Lebesgue spaces with Muckenhoupt weights, In press, Turk J Math. [2] A. D. Nakhman and B. P. Osilenker, Estimates of weighted norms of some operators generated by multiple trigonometric Fourier series, Izvestiya Vysshikh Uchebnykh Zavedeni Matematika, 239 (1982), no. 4, 39-50 (Russian). [3] M. K. Potapov, Approximation by "angle" (in Russian). In: Proceedings of the Conference on the Constructive Theory of Functions and Approximation Theory, Budapest, 1969, Akademiai Kiado, 1972, pp. 371-399.


# On Iterative Processes and Spectral Problems of Generalized Difference Operator-Matrices 

Ali M. Akhmedov<br>akhmedovali@rambler.ru


#### Abstract

The main purpose of this work is to determine the fine spectrum of the operators $B(r, s, t)$ and $\Delta_{u v w}$ defined by a triple band matrices over some known sequences spaces. For it we have used new approach using some recurrence relation which arises in resolvents above operators. Analyzing the work [1] and [2] one can see that the receiving conditions for the spectra of the operators $B(r, s, t)$ and $\Delta_{u v w}$ not easy verifiable. In this work we have found new conditions for the spectrum which are more simple than conditions in [1] and [2].

\section*{References:} [1] Hasan Furkan, Huseyin Bilgic, Feyzi Basar, On the fine spectrum of the operators $B(r, s, t)$ over the sequence spaces $l_{p}$ and $b_{v p}(1\langle p\langle\infty)$, Computers and Mathematics with Applications, V.60, I.7, 2010, 2141-2151. [2] Adiyasuren Vandanjav, Batzorig Undrakh, On the fine spectrum of generazized third order difference operator $\Delta_{u v w}$ on the Banach space $c_{0}$, Gulf Journal of Mathematics, V.2, I.2, 2014, 94-106.


# On Compactness of Bigeneralized Topological Spaces 

Samer Al-Ghour<br>Department of Mathematicsand Statistics, Jordan University of Science and Technology, Irbid, Jordan<br>algore@just.edu.jo


#### Abstract

Via q-open sets and u-open sets; q-compactness, u-compactness, scompactness and q -compactness are introduced in bigeneralized topological spaces. Several relationships, examples and counterexamples regarding them are given. The study is focued on subspaces and continuity images of each of them. Keywords: Generalized Topology, Bigeneralized Topology, Compact, Continuous Functions.

\section*{References:} [1] J. C. Kelly, "Bitopological spaces", Proc. London Math. Soc. 13(1963), 71-89. [2] I. Dochviri and T. Noiri, "On some properties of stable bitopological spaces" Topology Proc. 45 (2015), 111--119. [3] S. Ózcag, "Bornologies and bitopological function spaces", Filomat 27 (2013), 1345-1349. [4] Á. Császár, "Generalized topology, generalized continuity", Acta Math. Hungar., 96(2002), 351--357. [5] C. Boonpok, "Weakly open functions on bigeneralized topological spaces" Int. J. Math. Anal. (Ruse) 4(2010), 891--897. [6] A. H. Zakari, "Almost homeomorphisms on bigeneralized topological spaces", Int. Math. Forum 8(2013), 1853--1861.


Hermitian Part, and Skew Hermitian Part of Normal Matrices

M.Al-Hawari<br>Irbid National University Irbid<br>analysis2003@yahoo.com


#### Abstract

In this work we find relations between a normal matrix, a Hermitian part , and skew Hermitian part of Normal Matrices. Also we present new results for this subject. Keywords: Normal Matrix, Hermitian Part, Skew Hermitian Part.

\section*{References:} [1] R.A. Horn. and C.R. Johnson. Matrix Analysis, Cambridge University Press, Cambridge, (2003) 2nd ed. [2] H. Schneider and G. Ph. Barker. Matrix and Linear Algebra. Dover Publication, (1989). [3] H. Schneider, Theorems on normal matrices, Quarterly J. Math. Oxford Ser. (1952). [4] J. H. Wilkinson. The Algebraic Eigenvalue Problem. Clarendon Press, Oxford, (1965). [5] N. A. Wiegmann, Normal products of matrices, Duke Math. J.(1948).


# On $(\mu, \lambda)-c$-Continuous Functions in Generalized Topological Spaces 

Ahmad Al-Omari<br>Department of Mathematics, Al al-Bayt University Jordan<br>omarimutah1@yahoo.com


#### Abstract

Let $(X, \mu)$ be a generalized topological space (GTS) on $X$ due to Császár. In this paper, we define and investigate the notions of $(\mu, \lambda)-\mathcal{C}-$ continuous. Let $\mu$ and $\lambda$ be generalized topologies on $X$ and $Y$, respectively. A function $f:(X, \mu) \rightarrow(Y, \lambda)$ is said to be $(\mu, \lambda)-C-$  $Y-U$ is $\lambda$-compact, then there exists $V \in \mu$ containing $X$ such that $f(V) \subseteq U$. The function $f$ is said to be $(\mu, \lambda)-C$-continuous on $X$ provided $f$ is $(\mu, \lambda)-{ }^{C}$-continuous at each point of $X$.The connections between these functions and other related functions are investigated. Keywords: Generalized Topology, $(\mu, \lambda)_{\text {-Continuity, Weak }}(\mu, \lambda)_{\text {_ }}$ Continuity, $\mu$-Compact

\section*{References:}


[1] A. Al-Omari and T. Noiri, A unified theory of contra- $(\mu, \lambda)$-continuous functions in generalized topological spaces,Acta Math. Hungar,135(1-2)2012, 31-41.
[2] A. Al-Omari and T. Noiri, A Unified Theory of Weakly Contra- $(\mu, \lambda)$ continuous Functions in Generalized Topological Spaces, Stud. Univ. Babe S Bolyai Math. 58 (2013), (1), 107--117
[3] Á, Császár, Generalized open sets, Acta Math. Hungar,75(1-2) 1997, $65-87$.
[4] Á, Császár, Generalized topology, generalized continuity, Acta Math. Hungar., 96 (4) (2002), 351-357.
[5] Á, Császár, Generalized open sets in generalized topologies, Acta Math. Hungar., 106 (2005), 53-66.
[6] Á, Császár, Modification of generalized topologies via hereditary classes, Acta Math. Hungar., 115(1-2) (2007), 29-36.
[7] Á, Császár, Remark on quasi-topologies, Acta Math. Hungar., 119 (1-2) (2008), 197-200.


# Fuzzy Hypergraphs: Medical Application 

Hanan A. Alolyian<br>King Saud University<br>Mathematics department<br>holayan@ksu.edu.sa


#### Abstract

We deal with fuzzy directed hypergraphs as a tool to model and solve some classes of problems arising in medical field. Important concepts are defined using a relational formalism and fuzzy theory. The representation of complex systems as Fuzzy hypergraphs is appropriate for the study of certain problems. Linear systems have proved their importance to solve many applied problems, combined them to hypergraphs. [1,2]. Fuzzy hypergraphs are generalization of fuzzy graphs, and quite often have proved to be a successful tool to represent and model concepts and structures in various areas of Computer Science and Discrete Mathematics [3,4]. Keywords: Fuzzy Theory, Relation Algebra, Hypergraphs Fuzzy Hypergraphs. References: [1] M. Akram and W. A. Dudek, Interval-valued fuzzy graphs, Computers and Mathematics with Applications, 61(2), 289-299, 2011. [2] M. Akram, W. A. Dudek, Intuitionistic fuzzy hypergraphs with applications, Information Sciences, 218, 182-193, 2013. [3] K. R. Bhutani and A. Battou, On M-strong fuzzy graphs, Information Sciences, 155(12), 103-109, 2003. [4] K. R. Bhutani and A. Rosenfeld, Strong arcs in fuzzy graphs, Information Sciences, 152, 319-322, 2003.


57-64, 2004.

# Numerical Solution of Fractional Second-Order System by Using Reproducing Kernel Algorithm 

Zuhier Altawallbeh ${ }^{1}$, Mohammed Al-Smadi ${ }^{2}$, Ali Ateiwi ${ }^{3}$,*<br>${ }^{1}$ Department of Mathematics, Tafila Technical University, Tafila 66110, P.O. Box 179, Tafila-Jordan<br>${ }^{2}$ Department of Applied Science, Ajloun College, Al-Balqa Applied University, Ajloun 26816, Jordan<br>${ }^{3}$ Department of Mathematics, Faculty of Science, Al-Hussein Bin Talal University, P.O.<br>Box 20, Ma'an-Jordan<br>zuhier1980@gmail.com


#### Abstract

This study proposes numeric-analytic technique, called fractional reproducing kernel method (FRKM), for providing approximate solution of system of differential equation of fractional-order with appropriate initial conditions in Caputo sense. The solution methodology is based on generating an orthogonal system from the obtained kernel function in Hilbert space. The analytical solution is formulated in form of a finite series whilst the n-term numerical solution is proved to converge uniformly to analytical solution. However, numerical examples are given to show the good performance of the FROM. The results indicate that the present algorithm is powerful tool for solving other fractional problems arising in physics, computer and engineering fields. For more details about reproducing kernel method and fractional calculus, we refer to [1-5]


Keywords: Reproducing Kernel Theory, Fractional Differential Equation, System of Initial Value Problem, Numerical Solution.

## References:

[1] R. Hilfer, Application of fractional calculus in physics, World scientific publishing Co., Singapore, 2000.
[2] X. J. Yang, Advanced Local Fractional Calculus and Its Applications, World Science Publisher, New York, 2012.
[3] I. Podlubny, Fractional Differential Equations, New York: Academic Press, 1999.
[4] O. Abu Arqub and M. Al-Smadi, Numerical algorithm for solving two-point, second-order periodic boundary value problems for mixed integro-differential equations, Applied Mathematical and Computation, 243 (2014), 911-922.
[5] O. Abu Arqub, M. Al-Smadi, S. Momani and T. Hayat, Numerical Solutions of Fuzzy Differential Equations using Reproducing Kernel Hilbert Space Method, Soft Computing, (2015), 1-20.

# Majorization by Starlike Functions 

Osman Altıntaș ${ }^{1}$, Öznur Özkan Kılıç ${ }^{2}$<br>1Department of Matematics Education, Baskent University, Baglica, TR-06810, Ankara, Turkey, oaltintas@baskent.edu.tr<br>2Department of Statistics and Computer Sciences, Baskent University, Baglica, TR 06810, Ankara, Turkey,<br>oznur@baskent.edu.tr


#### Abstract

The main object of this paper is to investigate some majorization problems involving the subclass $S(\alpha, A, B)$ of starlike functions in the open unit disk $U$. Relevant connections of the results presented here with those given by earlier workers on the subject are also indicated. Keywords: Analytic Function, Starlike Function, Convex Function, Subordination, Majorization, Quasi-Subordination.


## References:

[1] Altıntaş O., Owa S., Majorization and quasi-subordinations for certain analytic functions. Japan Acad. Ser. A Math. Sci. 68, 181-185.(1992)
[2] Altıntaş O., Özkan Ö, Srivastava, H. M., Majorization by starlike functions of complex order, Complex Variables Theory Appl. 46, 207-218.(2001)
[3] Altıntaş O., Srivastava, H. M., Some majorization problems associated with p-valently starlike and convex functions of complex order. East Asian Math. J. 17, No:2, 175-183. (2001)
[4] MacGregor T.H., Majorization by univalent functions. Duke Math. J. 34 , 95-102.(1967)
[5] Nehari Z., Conformal Mapping. McGraw-Hill Book Company, NewYork, Toronto and London, 1952.
[6] Owa S., Shen C.Y., Certain subclass of analytic functions. Math. Japan. 34 , 409-412.(1989)
[7] Srivastava, H. M., Altınta, s O., Serenbay S. K., Coefficient bounds for certain subclasses of starlike functions of complex order. Applied Math. Lett. 24(8), 1359-1363.(2011)

# A New Approximate Solution for Quadratic Riccati Differential Equation 

Nidal Ankira<br>Department of Mathematics, Faculity of Science and Information Technolog, Irbid National University, Irbid, Jordan<br>alanaghreh_nedal@yahoo.com


#### Abstract

An efficient algorithm based on the optimal homotopy asymptotic method (OHAM ) is introduced to obtain an approximate solutions of a class of nonlinear quadratic Ricati differential equations. Two examples are considered to demonstrate the efficiency and simplicity of the presented method. The obtained results show that only a few terms required to obtain an approximate solutions which is more accurate and effective than other solutions in litreture.




# An Algorithm for Solving Bi-Matrix Games 

Aicha Anzi, Ramzi Kasri, Hicham Lenouar and Radjef Mohammed Said Department of Operations Research,University of Bejaia, Bejaia, Algeria<br>anzi_aicha@yahoo.fr


#### Abstract

The search of Nash equilibrium is the most common problem in game theory. The equilibrium that consists in taking single decision (pure) may not exist, it will be then of interest to seek the mixed strategy equilibrium which constitutes a probability distribution over the entire set of strategies. For the latter, Nash [1] showed the existence in the case of normal form games. In this work, we aim to calculate this equilibrium for a bi-matrix game, that is to say a finite non zero-sum and normal form game. For this, we used a nonconvex optimization technique which is DC (Difference of Convex) optimization and DCA (DC Algorithm) [2]. First, we reformulated the bi-matrix game as a linear complementarity problem (LCP) [3]. Then, we transformed the LCP into an optimization problem for which we considered four formulations which lead to four DCA versions that we tested and compared on games whose equilibrium is known. We also conducted a comparison study with another algorithm for solving bi-matrix games.


Keywords: Bi-Matrix Games, LCP, DC Optimization, DCA.

## References:

[1] J. Nash, "Noncooperative games", Annals of Mathematics, 54(1951).
[2] H. A. Le Thi and T. Pham Dinh, "On solving Linear Complementarity Problems by DC programming and DCA", Comput. Optim. and App., 50(2011), 507-524.
[3] R. Savani, "Finding Nash equilibria of bimatrix games", PhD Thesis, London school of economics (2006).

# Boundary Stabilization of a Flexible Beam with a Tip Rigid Body Without Dissipativity 

Moulay Driss Aouragh<br>M2I Laboratory, MAMCS Group, FST Errachidia<br>P.O Box 509 Boutalamine, 52000 Errachidia<br>Morocco<br>d.aouragh@fste.umi.ac.ma


#### Abstract

A linear feedback control is designed regardless of dissipativity of the system of a exible structure modeled by an Euler-Bernoulli beam which is held by a rigid hub at one end and totally free at the other.To realize the uniform stabilization, the high derivative feedback control is usually required. However, on the other hand, the design of the high derivative feedback controllers in literature are mainly based on principle of passivity that makes the closed-loop system be dissipative so that the system is at least asymptotically stable by lyapunov function method. In applications, on the other hand, there are many ways of designing controllers that make system pratically uniformly stable but there is no dissipativity which usually brings the difficulty of theoretical proof for the uniform stability of the system. The approach used here is so called Riesz basis approach that is recently used to study the basis generation, exponential stability and distribution of eigenvalues of the Euler-Bernoulli beam equations.


Keywords: Boundary Stabilization, Exponential Energy Decay.

## References:

[1] T. Fukuda, Y. Kuribayashi, H. Hosogai and N. Yajima, Vibration mode estimation and control of exible solar battery arrays on solar cell outputs, Theoret. Appli. Mech. 33, 299-309, 1985.
[2] B. Z. Guo and R. Yu, The Riesz basis property of discrete operators and application to a Euler-Bernoulli beam equation with boundary linear feedback control, IMA J. of Math. Control and Information, 18, 241-251, 2001.

# Migrativity Property for Uninorms and Nullnorms 

Emel Asici<br>Department of Software Engineering, Faculty of Technology, Karadeniz Technical University, Trabzon, Turkey<br>emelkalin@hotmail.com


#### Abstract

Durante and Sarkoci[1] introduced the $\alpha$-migrativity of triangular norms. The migrativity property has been studied for $t$-norm in([3],[4],[5]), for tsubnorms in[6], for semicopulas, quasi-copulas and copulas in([2],[7]). In this paper the notions of $\alpha$-migrative uninorms over a fixed nullnorm and $\alpha$ migrative nullnorms over a fixed uninorm are introduced and studied. All solutions of the migrativity equation for all possible combinations of uninorms and nullnorms are analyzed and characterized. Finally, the migrativity of t-norms over uninorms and nullnorms are introduced and studied.


Keywords: Uninorm, Nullnorm, Migrativity.

## References:

[1] F.Durante and P.Sarkoci, "A note on the convex combinations of triangular norms", Fuzzy Sets Syst. 159(2008), 77-80.
[2] F.Durante and R.G.Ricci, "Supermigrative semi-copulas and triangular norms", Inf. Sci. 179(2009), 2689-2694.
[3] J.Fodor and I.J.Rudas, "An extension of the migrative property for triangular norms", Fuzzy Sets Syst. 168(2011), 70-80.
[4] J.Fodor and I.J.Rudas, "Migrative t-norms with respect to continuous ordinal sums", Inf. Sci. 181(2011), 4860-4866.
[5] Y.Ouyang, "Generalizing the migrativity of continuous t-norms", Fuzzy Sets Syst. 211(2013), 73-83.
[6] L.Wu, Y.Ouyang, "On the migrativity of triangular subnorms", Fuzzy Sets Syst. 226(2013), 89-98.
[7] R.Mesiar, H.Bustince and J.Fernandez, "On the $\alpha$-migrativity of semicopulas, quasi-copulas and copulas", Inf.Sci. 180(2010), 1967-1976.

Totally Free Square Complexes

Hasan Atik ${ }^{l}$<br>${ }^{1}$ Istanbul Medeniyet University,, Istanbul, Turkey<br>hasan.atik@medeniyet.edu.tr<br>Murat Atik ${ }^{2}$<br>${ }^{2}$ Karatekin University, Çankırı, Turkey<br>murat.atik@karatekin.edu.tr


#### Abstract

In this work, we show that category of totally free square complexes is cofibration category in the sense of Baues. We also explore homotopies for crossed squares and square complexes morphisms.


Keywords: Crossed Square, Cofibration Category.
References:
[1] Z. Arvasi and E. Ulualan, On algebraic models for homotopy 3-types, Journal of Homotopy and Related Structures Vol.1, No 1, pp.1-27,(2006).
[2] H.J. Baues, Combinatorial homotopy and 4-dimensional complexes, Walter de Gruyter, 15, 380 pages, (1991).
[3] R. Brown and P.J.Higgins, Tensor Products and Homotopies for !- groupoids and Crossed Complexes, J.P.A.A 47, 11-44 (1987)
[4] R. Brown and M. Golanski, A model structure for the homotopy theory of rossed complexes. Cah. Top. Geom. Di Cat, 11, 30 (1989) 61-82.
[5] D. Conduche, Modules croises generalises de longueur 2, Journal of Pure and Applied Algebra, 34, pp 155-178, (1984).
[6] G.J.Ellis,Crossed squares and combinatorial homotopy , Math.Z.,214,93-110,
[7] K.H.Kamps, Kan-Bedingungen und abstrakte Homotopie theorie, Math. Z., 124, pp 215-236, (1972). (1993).
[8] J.L. Loday, Spaces with finitely many non-trivial homotopy groups, J. Pure and Applied Algebra, 24, 179-202, (1982).
[9] Joao Faria Martin, Homotopies of 2-crossed complexes and the homotopy category of pointed 3-types, (2011), 46 pages.
[10] A. Mutlu, T. Porter,Freeness conditions for 2 -crossed modules and complexes.Theory and Applications of Categories, 4, No.8, pp.174-194, (1998).
[11] D.Quillen,Lecture Notes in Math.Homotopical Algebra, 11,185-206 (1967)
[12] Andrei Radulescu-Banu, Cofibrations in Homotopy Theory,43, (2009), 152 pages.
[13] D. Guin-Walery and J.L. Loday, Obsruction a l'excision en K-theories algebrique, In: Friedlander, E.M.,Stein, M.R.(eds.) Evanston conf. on algebraic K-Theory 1980. (Lect. Notes Math., vol.854, pp 179-216) Berlin Heidelberg New York: Springer (1981).
[14] J.H.C. Whitehead, Combinatorial homotopy II, Bull. Amer. Math. Soc., 55, pp 453-496, (1949).

# Trigonometric Approximation in Weighted Lorentz Spaces 

Ahmet Hamdi Avsar<br>Department of Mathematics, Necatibey Education Faculty, Balikesir University, Balikesir, Turkey ahmet.avsar@balikesir.edu.tr


#### Abstract

In this study, we obtain the degree of trigonometric approximation for a general class of lower triangular matrices with nonnegative entries and monotone rows in the weighted Lorentz sapaces with Muckenhoupt weights. We show that the degree of its approximation is $O\left(n^{-\alpha}\right)$. In [2], the similar results were obtained in Lebesgue spaces. The same degree of approximation were obtained by Chandra [1] using Nörlund mean transform of the Fourier series representation for $f$ in Lebesgue spaces. Keywords: Weighted Lorentz Space, Lower Triangular Matrice, Fourier Series, Muckenhoupt Weight.

\section*{References:} [1] Chandra, P., Trigometric approximation of function in Lp norm, J. Math. Anal. Appl., 275 (2002), 13-26. [2] Mittal, M.L., Rohades, B.E., Mishra, V.N., Singh, U.: Using infinite matrices to approximate functions of $\operatorname{Lip}(\alpha, p)$ class using trigonometric polynomials. J. Math. Anal. Appl. 326, 667-676 (2007).


# On Multiplicative Complex Integral 

Agamirza E. Bashirov and Sajedeh Norozpour Eastern Mediterranean University Gazimagusa, North Cyprus<br>agamirza.bashirov@emu.edu.tr<br>sajedeh.norozpour@cc.emu.edu.tr


#### Abstract

In 1972 Grossman and Katz [1] proposed alternative calculi to the calculus of Newton and Leibnitz. Among them multiplicative calculus is most popular. This pioneering work initiated numerous studies. What is the difference between Newtonian and multiplicative calculi? Both them describe the system of knowledges which is called calculus. Newtonian calculus describes it with reference to the linear function while multiplicative calculus with reference to the exponential function. Therefore, every theorem in one of them has an analog in the other one. Is it reasonable a consideration of a new calculus while Newtonian calculus is already well established? It is reasonable because different presentations of calculus provide different views. If proving some theorem is diffcult in one of them, it may be more visible and easy in another one. In such a way, in [2] it is proved the non-analyticity of some infinitely many times differentiable function in a rather compact form by means of multiplicative calculus while its prove by means of Newtonian calculus is rather complicated. Motivated from the exponential nature of complex numbers, in [3] complex differentiation was revised by means of multiplicative calculus. In the present presentation we are aiming to present the results related to multiplicative complex integration from [4]. Since the multivalued nature of complex logarithm, which has an underlying role in multiplicative calculus, complex multiplicative calculus is not one-to-one transformation of ordinary complex calculus. In particular, unlike ordinary complex integral, multiplicative complex integral has a multivalued nature as well. This makes its properties to be in the form of inclusion rather than in the form of equality. The Cauchy integral formula is not affected in the multiplicative case, that is, the multiplicative complex integral does not count residues, etc. All these demonstrate that we should expect nontrivial transformation of complex calculus to multiplicative case, that is not yet completely formalized. We are going to discuss these issues in our presentation. Keywords: Nwetonian Calculi, Multiplicative Calculi, Complex Multiplicative, Complex Integral.

\section*{References:} [1] Grossman, M., and Katz, R., Non-Newtonian Calculus, Lee Press, Pigeon Cove, MA, 1972. [2] Bashirov, A. E., Kurpinar, E., and • Ozyapici, A., Multiplicative calculus and its applications, Journal of Mathematical Analysis and Applications, 337(1), 3648, 2008. [3] Bashirov, A. E., Riza, M.,On complex multiplicative differentiation, TWMS Journal of Applied and Engineering Mathematics, 1(1), 75-85, 2011.




# The Spectrum and Trace of a Discontinuous Value Problem with Retarded Argument 

M. Bayramoglu and A. Bayramov<br>Institute of Mathematics and Mechanics of NAS of Azerbaijan<br>Azerbaijan State Pedagogical University, Baku<br>azadbay@gmail.com


#### Abstract

In this study a discontinuous boundary value problem with retarded argument and with transmission conditions at two points of discontinuity investigated.We obtained asymptotic formulas for eigenvalues and regularized trace of eigenvalues.


# Numerical Solution of Semilinear Elliptic Equation via Difference Schemes 

Elif Ozturk Betgmohammadı ${ }^{1}$, Esra Demirel ${ }^{2}$<br>${ }^{1}$ Department of Economics, Canakkale Onsekiz Mart University, Canakkale,Turkey eozturk@comu.edu.tr<br>${ }^{2}$ Department of Banking and Finance, Canakkale Onsekiz Mart University, Canakkale, Turkey<br>esrademirel@comu.edu.tr


#### Abstract

We consider $$
\left\{\begin{array}{l} -\frac{d^{2} u(t)}{d t^{2}}+A u(t)=f(t, u(t)), \quad 0<t<1 ; \\ u(0)==, \\ u(1)=\sum_{j=1}^{J} \alpha_{j} u\left(\lambda_{j}\right)+\psi, \quad 0<\lambda_{1}<\ldots<\lambda_{J}<1, \\ \sum_{j=1}^{J}\left|\alpha_{j}\right|<1 \end{array}\right.
$$

Bitsadze-Samarskii type nonlocal boundary value problem for semilinear elliptic equation. For approximate solution of this problem the first and second orders of accuracy difference schemes are presented. We have second order difference equations with respect to n with matrix coefficients. These difference equations are solved by modified Gauss elimination method and iteration method. Keywords: Semilinear Elliptic Equation, Difference Scheme, BitsadzeSamarskii Type Problem.

\section*{References:} [1] D. G. Gordeziani, "On solvability of one boundary problem for nonlinear elliptic equations", Bull. Georgian Acad. Sci., 68(1972), 289-292. [2] H. B. Keller, ".Approximation methods for nonlinear problems with application to two-point boundary value problems", Math. Comp., 29(1975), 464-474. [3] A. V. Bitsadze and A. A. Samarskii, "Some elementary generalizations of linear elliptic boundary value problems", Nauk SSSR, 185(1969), 739-740.


# The Mathematical Models for Pavement Design 

Mouloud Belachia, Nadjette Bouacha<br>Laboratory of Materials, Geotechnics, Habitat and Urban<br>University of 20 Aout 1955 - Skikda ALGERIA<br>belachia@yahoo.fr


#### Abstract

In general pavement design rules are based in part on the geotechnical knowledge at break, the characteristics of deformability of materials determined by laboratory tests and findings on the behavior of structures, other hand on Mathematical methods: rational approach lanes by a suitable mathematical model and finally the confrontation between technological knowledge and theoretical results. The examination of a set of analytical models that have succeeded in time, the simplest model like BOUSSINESQ the most complicated model like Burmister, led us to conclude that the main defect is to introduce many simplifying assumptions. It is therefore necessary to introduce the finite element method, because the development of the latter allows considering the use of more realistic models that have the opportunity to study edge loading problems and the ability to consider materials not of an elastic behavior.


Keywords: Mathematical Model, Pavement Design, Finite Element Method. References:
[1] P. Autret, A. Boissoudy, J. Baucheron Marchand and P. Alize iii practice. Proc. 5th Int. Conf. Structural Design of Asphalt Pavements, 1982.
[2] J. Boussinesq Potential application to the study of equilibrium and motion of elastic cops. Gauthier Villars, Paris, 1985.
[3] A. Messabhia. Using the definition of the technology and modeling structure works in a CAD system building. PhD Thesis, University of Savoie in Chambery, 1989.
[4] C. Peyronie, G. Caroff and C. Gilbert, Route - Design of pavements, edited by G. Jeuffroy and R. Sauterey, Presses de l'Ecole Nationale des Ponts et Chaussées, 1991.
[5] LCPC. Design and measurements of pavement structures. Technical Guide, LCPC-SETRA, 1994.
[6] S. Salasc, Contact phenomena in concrete pavements: theoretical modeling and experimental validation. Thesis: Ecole Centrale de Nantes, 1998.

# On Quadratic Integral Equations of Volterra Type in Fréchet Spaces 

Latifa Benhamouche<br>Department of Mathematics, University Saad Dahlab of Blida<br>Blida, Algeria<br>latifabenhamouche@gmail.com


#### Abstract

We investigate the existence of solutions of Quadratic integral equations of Volterra type by using Schauder-Tychonov fixed point theorem in the Fréchet Space of continuous functions on unbounded open set $\Omega$ of $R^{n}$. Many authors studied the solvability of several types of integral equations; most of their results were obtained in the Banach space of bounded and continuous functions on the nonnegative real half axis, by using fixed point theorems via measures of noncompactness (see for e.g [1-2] and references therein). The use of measures of noncompactness turns to be a good technique allowing not only to get the existence of solutions but also to obtain some characterizations of those solutions. However, some restrictive conditions are caused by applying such measures in such a space.Recently, in [3-4] authors defined the notion of sequence of measures of noncompactness and obtained some good existence results in the Fréchet Space of continuous functions on the real half axis. More recently authors in [5] developed some fixed point theorems in locally convex spaces and succeed to get existence result in the space $\mathrm{C}\left(R^{+}, R^{d}\right)$ without using the technique of measures of noncompactness. Our aim is to generalize their ideas to the case of unbounded open sets $\Omega$ of $R^{n}$ for a quit general quadratic Volterra type equation and to slightly relax the assumptions in [3].


Keywords: Quadratic Integral Equations, Schauder-Tychonov Fixed Point Theorem, Volterra Operator, Fréchet Spaces.

## References:

[1] M. A. Darwish, J. Banas, E. O. Alzahrani, The existence and attractivity of solutions of an Urysohn integral equation on an unbounded interval, Abstract and Applied Analysis, 2013, Article ID 147409, 9pages.
[2] J. Banas, D. O'Regan On existence and local attractivity of solutions of a quadratic Volterra integral equation of fractional order, J. Math. Anal. Appl. 2008, 573-582
[3] S. Dudek, L. Olszowy, Continuous dependence of the solutions of nonlinear integral quadratic volterra equation on the parameter, Journal of function spaces, 2015, Article ID 471235.
[4] L. Olszowy, Fixed point theorems in the Fréchet space $\mathrm{C}\left(R^{+}\right)$and functional integral equations on an unbounded interval, App. Math. comp. 218(2012) 9066-9074.
[5] F. Wang, H. Zhou,Fixed point theorems in locally convex spaces and a nonlinear integral equation of mixed type, Fixed Point Theory Appl. 2015, 2015:228

# On $A$-Statistical Convergence with Density of Moduli 

Stuti Borgohain and Ekrem Savas Department of Mathematics,Istanbul Ticaret University, Uskudar, Istanbul, Turkey<br>ekremsavas@yahoo.com


#### Abstract

Since the invention of statistical convergence by Fast [2] and Schoenberg [3] independently, many mathematicians have been working on several generalizations and applications of this notion. One can refer ([5] ,[4] , [1], [5]) to get more references of works on statistical convergence and its applications to different fields. We study the concept of density of moduli with respect to $A$-statistical convergence where $A$ is a non-negative regular matrix. Also we are trying to investigate some relation between the ordinary convergence and module statistical convergence on $A$-statistical convergence for evey unbounded modulus function. Morever we also study the newly introduced $f$-statistical on $A$ summability for a non-negative regular matrix $A$.


Keywords: Statistical Convergence, $f$ - Statistical Convergence, $A$-Summability. References:
[1] E. Kolk, "Matrix summability of statistically convergent sequences", Analysis, 13(1993), 77-83.
[2] H. Fast, "Sur la convergence statistique", Colloq. Math. 2(1951), 241-244.
[3] I. J. Schoenberg, "The integrability methods", Amer. Math. Monthly, 66(1959), 361-375.
[4] J. A. Fridy, "On statistical convergence", Analysis, 5(1985), 301-313.
[5] J. Connor and E. Savas, "Lacunary statistical and sliding window convergence for measurable functions", Acta Mmathematica Hungarica, 145(2)(2015), 416-432.
[6] T. Salat, "On statistically convergent sequences of real numbers", Math Slovaca, 30.2(1980), 139-150.

# The Boundary Integral Method for the Laplace Equation with Mixed and Oblique Conditions 

Hamza Bouguerne<br>Department of Mathematics, Applied Mathematics Laboratory, University Badji Mokhtar<br>Annaba, Algeria<br>m2ma.bouguerne@gmail.com


#### Abstract

The solvability of the mixed and oblique boundary value problem is estabilshed by the Boundary Integral Equation method. Based on the Green formula, we express the solution in terms of the boundary data. The key to the realization of this method is to translate the tangential derivative to the fundamental solution. A system of boundary intregral equation of second kind Fredholm type is obtained. By the Fredholm and Riez thoerem, the existence and the uniqueness of the solution is established.


Keywords: Boundary Integral Equation Method, Pseudo-Differential Operator, Tangential Derivative.

## References:

[1] M.Costabel, W.L.Wendland : Strong ellipticity of boundary integral operators J. Reine Angew. Math. 372, 39-63(1986).
[2] J.Giroire,J.C.Nedelec : Numerical solution of exterior Neumann problem using a double layer potentiel.Math.Comp.32(1968).
[3] J.JKohn, L.Nirenberg : On the algebra of pseudo di érential operators. Comm.Pure Appl.Math.18(1968).
[4] J.Treve : Introduction to pseudodi erential and fourier integral operators. Plenom press. New York. London (1964).
[5] C.A. Brebbia. J. C. F. Telles. L.C.Wrobel : Boundary Element Techniques. Springer. Verlag,Berlin(1984).
[6] D.Lesnic : The boundary Element method for solving the Laplace equation in two dimensions with oblique derivative boundary conditions, Communications in numerical methods in engineering, 23 : 1071-1080 (2007).

# Efficiency Conditions for Multiobjective Bilevel Optimization Problem Under Generalized Invexit 

Karima Bouibed, Hachem Slimani, Mohammed Said Radjef<br>LaMOS Research Unit, University of Bejaia, 06000 Bejaia, Algeria.<br>karima.bouibed@gmail.com


#### Abstract

In this paper, we consider an optimistic bilevel optimization problem (MBP) where the upper-level is a vector optimization problem and the lowerlevel is a scalar linear optimization problem. By using the Karush-KuhnTucker conditions associated to the lower-level problem (LLP)x, we reformulate the bilevel optimization problem into a nonlinear multiobjective single-level optimization problem with equality and inequality constraints (MP). Since the problem (LLP)x is linear, we have global (weakly or proprely) efficient solutions of (MP) correspond to global (weakly or proprely) efficient solutions of (MBP). We establish Fritz John type necessary efficiency conditions for (MBP) without using any constraint qualification. Furthermore, we obtain (Fritz John type) sufficient efficiency conditions for a feasible point of (MP) corresponds to a (weakly or proprely) efficient solution for the bilevel problem (MBP) under generalized invexity [2,3] and infineness [1].


Keywords: Multiobjective Bilevel Programming, KKT Conditions, Efficiency Conditions, Generalized Invexity.

## References:

[1] P.Sach, GM. Lee and D.S. Kim, Infine functions, nonsmooth alternative theorems and vector optimization problems. J Global Optim 27:51-81, 2003.
[2] H. Slimani and M.S. Radjef, Duality for nonlinear programming under generalized Kuhn-Tucher condition, Int. J. Optim. Theor. Methods Appl., 01:75-86, 2009.
[3] H. Slimani and M.S. Radjef, Fritz John type optimality and duality in nonlinear programming under weak pseudo-invexity, RAIRO Oper. Res., 49: 451-472, 2015.

# Convergence Result in the Approximation of Some Matrix Functions by Krylov Method (Application on an Inverse Problem) 

*A. Bouzitouna ${ }^{1}$, N. Boussetila ${ }^{2}$ and F. Rebbani ${ }^{3}$<br>${ }^{1,3}$ Preparatory School of Sciences and Engineering, B.P. 218 Annaba 23000, Algeria.<br>1, 2, 3 Applied Mathematics Laboratory, Badji Mokhtar University - Annaba. PB 12, 23000 Algeria.<br>${ }^{2}$ Department of Mathematics, FMISM, Guelma University, BP. 401, Guelma 24000, Algeria.<br>a.bouzitouna@epst-annaba.dz ${ }^{1}$, n.boussetila@ gmail.com ${ }^{2}$, f.rebbani@epst-annaba.dz ${ }^{3}$

Abstract: The inverse problems and PDE brought to matrix problems (matrix function) $u=f(A) u_{0}$, for the squared matrixes $A$ of big sizes; the numeric approximation of the vector $u$ received a considerable attention in the last years.
The objective of this work is to extend the results developed by the author [1], as part of the projection method of Krylov, in case $f(A) v$, where the function $f(Z)=e^{Z}$ and $A$ is a square symmetric matrix of large size. The results obtained in this investigation are based mainly on the Taylor's series and the estimated remaining integral. Our contribution will focus on the functions encountered in the quantification of certain academic inverse problems. Some results of convergence is given by applying this method on specific types of matrix functions $f(A)$.
In this work, we study the extent of the methods of Krylov subspace, a technique that has been recently proposed to approximate

$$
\begin{equation*}
u=f(A) v \tag{1.1}
\end{equation*}
$$

Note: The convergence criterion is exponentially depending on the size of the $K_{m}$ projection space and the norm of the matrix $A$; we can introduce the following inequality

$$
\begin{equation*}
\|f(A) v-\| v\left\|V_{m} f\left(H_{m}\right) e_{1}\right\| \leq 2\|v\| \frac{\lambda_{\max }^{m} e^{\lambda_{\max }}}{m!} \tag{1.2}
\end{equation*}
$$

Keywords: Krylov Subspace, Matrix Function, Inverse Problems, Iterative Method, Projection Method, Arnoldi, Lanczòs, Regularization.

## References :

[1] Y. Saad. "Analysis of some Krylov subspace approximations to the matrix exponential operator", SIAM. Journal on Numerical Analysis, 1992, Vol 29, p:209-228.

Stability Result of a Fractional Differential Equation

S. Bouzitouna ${ }^{I}$, R. Atmania ${ }^{2}$<br>${ }^{1}$ Preparatory School of Economics and Management Science, Annaba, Algeria.<br>1,2 Applied Mathematics Laboratory, Badji Mokhtar University Annaba, BP12, 23000 Annaba Algeria.<br>${ }^{1}$ bouzitouna.live@gmail.com , ${ }^{2}$ atmanira@yahoo.fr


#### Abstract

Fractional calculates mathematical analysis is the branch that studies the generalization of notions of derivations and integration are not necessarily entire orders (real or complex); fractional differential equation is considered an ingenious application of fractional calculus. In fact; fractional derivatives have a nonlocal character which made them a powerful tool for the description of the hereditary effects of different substancesand for modeling some dynamic processes. In this work, we are interested in generalized equation of Sturm-Liouville is a


 real second order linear differential equation of the forme:$$
\frac{d}{d x}\left[p(x) \frac{d y}{d x}\right]+q(x) y=-\chi w(x) y
$$

With the boundary conditions Connect the values $y(a), y^{\prime}(a), y(b)$ and $y^{\prime}(b)$.
Keywords: Fractional Calculus, Sturm-Liouville Equation, Caputo Derivative, Ulam Stability.

## References:

[1] A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, Theory and applications of fractional differential equations, in: North-Holland Mathematics Studies, vol. 204, Elsevier Science B.V., Amsterdam, 2006.
[2] J.Wang, L. Lv, Y. Zhou, Ulam stability and data dependence for fractional differential equations with caputo derivative,J. Appl. Math. Comput. (2011), No. 63, 1-10;
[3] R.W.Ibrahim, "Stability of fractional differential equation", I.J.O.M.S, (2013), 7-12.

# Analysis of Centrodes in Planar and Spherical Motion 

Serife Nur Bozdag, Ali Çallskan<br>Department of Mathematics, Ege University, Bornova, İzmir, Turkey<br>serife.nur.yalcin@ege.edu.tr, ali.caliskan@ege.edu.tr


#### Abstract

We can first trace kinematic geometry's influence on space kinematics in the works of Blashke and Müller [1]. Than lots of developments and novelty have been made in planar, spherical and spatial kinematics with studies of Hunt[2], Bottema and Roth[3]. In [4] Wang, Liu and Xiao introduced a new approach of a space curve adjoint to a ruled surface which is based on differential geometry. In this paper, we study and give some classifications of centrods of planar and spherical motion by an addjoint approach. We also present the properties of these centrodes which we believe that these properties can provide contributions to the kinematic differential geometry and theoretical mechanics.


Keywords: Kinematic geometry, rigid body motion, invariants, centrodes

## References:

[1] Blashke, W., Müller, H.R. : "Ebene Kinematik", München, Oldenbourg, 1956.
[2] K. H. Hunt, "Kinematic Geometry of Mechanism", Oxford University Press, Oxford, 1978.
[3] O. Bottema and B. Roth, "Theoretical Kinematics", North-Holland, New York, 1979.
[4] D. L., Wang, J., Liu and D.Z., Xiao, "Kinematic differential geometry of a rigid body in spatial motion-I. A new adjoint approach and instantaneous properties of a point trajectory in spatial kinematics", Mech. and Mach. Theory, 32-4, (1997), 419-432.

# Empirical Likelihood for Copula Moment Based Estimation Method 

Brahim Brahimi<br>Department of Mathematics, Laboratory of Applied Mathematics, Mohamed Khider<br>University, Biskra, Algeria<br>brah.brahim@gmail.com


#### Abstract

We applied the empirical likelihood method to the copula moment based estimation methods which originally proposed by [4], [5] and [6]. Several authors investigated the empirical likelihood see for instance [7], [1]. [2] and [3]. The advantage of this method is that the empirical likelihood has both effectiveness and flexibility of the likelihood method, and reliability of the nonparametric methods, it helps us to construct confidence intervals without estimating the asymptotic variance, so the complexity of the asymptotic variance for some estimator especially the CM based estimators and the construction of nonparametric confidence intervals via estimating the asymptotic variance is usually inaccurate.


Keywords: Archimedean Copulas; Asymptotic Distribution; Copula Models; Method of Moments; Semiparametric Models; Z-Estimator.

## References:

[1] Monti, A.C., 1997. Empirical likelihood confidence regions in time series models. Biometrika 80, 329-338.
[2] Newey, W.K. and Smith, R.J., 2004. Higher order properties of GMM and generalized empirical likelihood estimators. Econometrica 72, 219-255.
[3]Ogata, H., Taniguchi, M., 2010. An empirical likelihood approach for nonGaussian vector stationary processes and its application to minimum contrast estimation. Aust. N. Z. J. Stat. 52, 451-468.
[4] Owen, A.B., 1988. Empirical likelihood ratio confidence intervals for a single functional. Biometrika 75, 237-249.
[5] Owen, A.B., 1990. Empirical likelihood confidence regions. Ann. Statist. 18, 90-120.
[6] Owen, A.B., 2001. Empirical likelihood. Chapman \& Hall/CRC, Boca Raton, London, New York, Washington.
[7] Qin, J. and Lawless, J., 1994. Empirical likelihood and general estimating equations. Ann. Statist. 22, 300-325.

# Approximation of Entropic and Admissible Solution of the Hyperbolic Problems of Conservation Laws by the New Scheme Ziti's $\boldsymbol{\delta}$-Scheme in Several Dimensions. Application to Burgers Equation, Advection Convection Equation, Gaz Dynamics and Biological Problems 

Larbi Bsiss ${ }^{1}$ and Chérif Ziti ${ }^{2}$<br>Department of Mathematics, Moulay Ismail University, Meknès 50000, Marocco<br>1- lyrbi01@gmail.com<br>2-chziti@gmail.com


#### Abstract

Hyperbolic systems of conservations laws can usually be obtained by assuming that the phenomena under consideration evolves on the advection time scale and that other effects, like viscosity, dispersion, capillary, etc, can neglect. This leads to discontinuities, non-uniqueness and "unphysical" solutions. To keep the discontinuities but to avoid the other two possibilities solutions are considered in a weak sense together with some admissibility conditions. Following are the most common admissibility criteria for shocks waves in case of strictly hyperbolic systems:


1- Linearized stability analysis. (Lax conditions)
2- Existence of a stable viscous profile. (Liu's conditions)
3- Physical entropy derived from the second law of thermodynamics.
4- Requirement for hyperbolic equations to be a limit of the same equations perturbed by linear viscosity termes with the multiple of identity viscosity matrix.
5- Solution should be admissible for the equations derived as a weakly nonlinear asymptotic limit of the full physical system of equations.
For example, in case of polytropic gaz dynamics and other similar systems all of above criteria reject the expansion shocks waves.
As it is well known in numerical analysis that most of the numerical schemes have undesirable oscillations, especially near the domain's border, or near the physical phenomena (empty region, collapse, boundary layer, among others)(mathematically invisible) eg: the heat equation with a bad sign, Burgers equation(the solution loses its regularity).
In the case where the differential problem solution presents a singularity (shock, blow-up which cannot be numerically detected easily), the classical scheme cannot generally operate correctly and in the best case we are confronted with a very difficult algorithm, especially in several dimensions.
Generally, well using classical methods such as Finite differences, Finite elements, Particle methods and Spectral methods in hyperbolic problems usually give some oscillation or Gibs phenomena that are not detect and admissible shocks. Therefore we can't obtain any entropy solution. An analysis of such schemes was made in [3][4][5][6][7][8][13], but the extension to several dimensions ( $\mathrm{n}=2,3, \ldots$ ) is not encouraging [13]. As against the schemes of Godunov and Glimm give entropy solutions in one dimension but unfortunately they are based on solving the Riemann problem

involve a complication state, adding that the extension to several dimension does not give satisfactory results. The problem becomes more complicated if the problem is not strictly hyperbolic.
Our objective here is to construct a less complicated scheme compared to the classical methods by keeping their advantages and obtained the admissible solution in the most difficult situations without complications obtained from the selected meshing.
In this paper, we apply a new approximation method called ziti's $\delta$-scheme to strictly or not strictly hyperbolic problem in one or more dimensions which is able to resist to such oscillation near the singularity and enables us to detect a lot of physical phenomena. We test our method to some models and compare its results with the exact one and other classical methods eg: burgers equation [2] [9][10], gaz dynamic [1][2][9], advection convection problems, biology chemotactic problems [11][12].
We can conclude that our results are very striking. The ziti's $\delta$ - scheme that we obtained is faster, more efficient, rebut, easy to handle in several dimension and gives entropy solutions.
Keywords: Hyperbolic, Riemann Problem, Entropic Solution, Shock, Gibbs Phenomena, Gudunov, Glimm, Oscillation, Ziti's $\delta$ - Scheme.

## References:

[1] G.-S Jiang, D.Levy, G.-T Lin, S.Osher, and E.Tadmor, High Resolution nonoscillatory central Schemes with nonstaggerd drids for Hyperbolic Conservation Laws, 2147-2168. SIAM J. NUMER. ANAL. Vol. 35 No. 6, (1998)
[2] Ami Hartin, High Resolution Schemes for Hyperbolic Conservation Laws, 357-393. Journal of Computational Physics 49, (1983)
[3] A.Lerat and R.Peyret, Sur le choix de schémas aux différences du second ordre fournissant des profils de choc sans oscillation, 363-366. C.R.A.S., 277, (1973)
[4] A.Lerat and R.Peyret, Sur l'origine des oscillations apparaissant dans les profils de choc calculés par des méthodes aux différences, 759-762. C.R.A.S., 276, (1973)
[5] A.Lerat and R.Peyret, Noncentered schemes and shock propagation problems, 35-52. Computers and Fluids, 2, (1974)
[6] A.Lerat and R.Peyret, 4éme conference Internationale sur les ml'ethodes numériques en dynamique des fluides. Boulder (Colorado), juin (1974)
[7] A.Lerat and R.Peyret, Systèmes hyperboliques non linèaire. Rech. Aérosp. N.1974-2, Mars-Avril (1974)
[8] A.Lerat and R.Peyret, Proprités dispersives et dissipatives d'une classe de schemas aux differences pour les systèmes hyperboliques non linèaires. Rech. Aérosp. N.2:61-79, (1975)
[9] B.Tessieras, Résolution numérique, par des méthodes de poursuite de fronts, de systèmes hyperboliques de lois de conservation non lineaires, These de Docteur, U.Bordeaux I, (1983)
[10] D.Yoon and W.Hwang, Two-dmensional Riemann problem for Burgers' equation, 191-205. Bull.Korean Math.Soc.45, (2008)
[11] C.Ziti, Analyse et simulation numérique d'un système hyperbolique modélisant le comportement d'une population bactérienne, These de Docteur, U.Saint-Etienne (1987)
[13] C.Ziti, Analysis and numerical simulation for a non-strictly hyperbolic system arising in biology, 751-756, C.R. Acad. Sci. Paris, t.319, Série I, (1994)

# Certain Properties of a New Subclass of Close-to-Convex Functions 

Serap Bulut<br>Kocaeli University, Faculty of Aviation and Space Sciences, Kocaeli, Turkey<br>serap.bulut@kocaeli.edu.tr


#### Abstract

We introduce and investigate an interesting subclass $\mathcal{K}_{s}^{(k)}(\gamma, p)$ of analytic and $p$-valently close-to-convex functions in the open unit disk $\mathbb{U}$. For functions belonging to this class, we derive several properties as the inclusion relationships and distortion theorems. The various results presented here would generalize many known recent results.


Keywords: Analytic Functions, p-Valently Close-to-Convex Functions, Inclusion Relationships, Subordination Principle.

## References:

[1] M. K. Aouf, "On a class of p-valent starlike functions of order $\alpha$ ", Internat. J. Math. Math. Sci. 10 (4) (1987), 733--744.
[2] C.-Y. Gao and S.-Q. Zhou, "On a class of analytic functions related to the starlike functions", Kyungpook Math. J. 45 (2005), 123--130.
[3] T. Hayami, S. Owa and H.M. Srivastava, "Coefficient inequalities for certain classes of analytic and univalent functions", J. Ineq. Pure Appl. Math. 8 (4) (2007), Article 95, 1--10.
[4] J. Kowalczyk and E. Leś-Bomba, On a subclass of close-to-convex functions, Appl. Math. Lett. 23 (2010), 1147--1151.
[5] M.S. Liu, On a subclass of p-valent close-to-convex functions of order $\beta$ and type $\alpha$, J. Math. Study 30 (1997), 102--104.
[6] K.S. Padmanabhan, On a certain classes of starlike functions in the unit disc, J. Indian Math. Soc. 32 (1968), 89--103.
[7] B. Şeker, On certain new sublcass of close-to-convex functions, Appl. Math. Comput. 218 (2011), 1041--1045.

# Some Fixed Point Theorems Satisfying Generalized $\mathrm{B}_{\varphi}-$ Contraction on Metric Space and 2-Metric Space 

Abdurrahman Buyukkaya, Mahpeyker Ozturk<br>Department of Mathematics, Sakarya University, Sakarya, Turkey<br>abdurrahman.giresun@hotmail.com, mahpeykero@sakarya.edu.tr


#### Abstract

In this study, we defined the class of $\mathrm{B}_{\varphi}$ which includes nine-variable functions and strong comparison function and proved some fixed point theorems for the class of $B_{\varphi}$, which is a generalization of the class $A$ that introduced by Akram et al., on metric space and 2-metric space. The class of $B_{\varphi}$ is a analogue of the class of $B$, which is describe by Tran Van An et al.. Also our obtained results extend, generalize and improve the existing literatüre. Keywords: Fixed Point, Metric Space, A-Contraction, Generalized $\mathrm{B}_{\varphi^{-}}$ Contraction

\section*{References:} [1] Banach, S., Sur les operations dans les emsembles abstraits et leurs applications aux equations integrales, Fund. Math., 1, 133-181, 2012. [2] Akram, M., Zafar, AA., Siddiqui, AA., A general class of contractions: Acontractions, Novi Sad J. Math., 38(1), 25-33, 2002. [3] Gahler, S., 2-metrische raume und ihre topologiche struukture, Math.Nachr. 26, 953-956, 1963. [4] An, T.V., Dung, V.N., Hang, V.T., "General fixed point theorems on metric spaces and 2-metric spaces." Filomat, 28(10), 2037-2045, 2014.


# A Characterization of Curves According to Parallel Transport Frame in Euclidean n-Space IE $^{\mathrm{n}}$ 

Sezgin Buyukkutuk,, Ilim Kisi, Gunay Ozturk<br>Department of Mathematics, Kocaeli University, Kocaeli, Turkey<br>sezgin.buyukkutuk@kocaeli.edu.tr, ilim.ayvaz@kocaeli.edu.tr, ogunay@kocaeli.edu.tr


#### Abstract

In this study, we consider a regular curve in Euclidean n-space $\mathrm{IE}^{\mathrm{n}}$ whose position vector is written as a linear combination of its parallel transport frame vectors. We characterize constant ratio curves in terms of their curvature functions. Further, we obtain some results of $T$-constant type and $N$-constant type curves according to its Bishop curvatures in $\mathrm{IE}^{\mathrm{n}}$.


Keywords: Parallel Transport Frame, Position Vector, Constant Ratio Curves. References:
[1] L. R. Bishop, "There is more than one way to frame a curve", Amer. Math. Monthly, 82.3(1975), 246-251.
[2] B. Y. Chen, "Constant ratio hypersurfaces", Soochow J. Math, 28(2001), 353-362.
[3] B. Y. Chen, "Geometry of warped products as Riemannian submanifolds and related problems", Soochow Journal of Mathematics., 28.2(2002), 125-156.
[4] B. Y. Chen, "When does the position vector of a space curve always lies in its rectifying plane?", Amer. Math. Monthly, 110(2003), 147-152.
[5] B. Y. Chen, "More on convolution of Riemannian manifolds", Beitrage Algebra und Geom. 44(2003), 9-24.
[6] S. Cambie, W. Geomans and I.V.D Bussche, "Rectifying curves in the ndimensional Euclidean space", Turk J. Math, 40(2016), 210-223.
[7] S . Gürpınar, K. Arslan and G. Öztürk, "A Characterization of Constant-ratio Curves in Euclidean 3-space E ${ }^{3 \prime \prime}$, Acta Universitatis Apulensis, 44 (2015), 39-51. [8] F. Gokçelik, Z. Bozkurt, I. Gok, F.N. Ekmekci, Y. Yayl, "Parallel Transport Frame in 4-dimensional Euclidean Space $\mathrm{E}^{4,}$,Caspian J. of Math. Sci., 3.1(2014), 91-102.

# Initial Coefficients for a Subclass of Bi-Univalent Functions Defined by Salagean Differential Operator 

Murat Caglar<br>Department of Mathematics, Faculty of Science and Letters, Kafkas University, Kars, Turkey<br>mcaglar25@gmail.com


#### Abstract

In this paper, we investigate a new subclass $\Sigma^{n}(\tau, \gamma, \varphi)$ of analytic and bi-univalent functions in the open unit disk $U$ defined by Salagean differential operator. For functions belonging to this class, we obtain estimates on the first two Taylor-Maclaurin coefficient $\left|a_{2}\right|$ and $\left|a_{3}\right|$. Keywords: Analytic functions, Univalent functions, Bi-univalent functions, Subordination, Salagean differential operator.

\section*{References:} [1] M. Çağlar, H. Orhan and N. Yağmur, "Coefficient bounds for new subclasses of bi-univalent functions", Filomat 27:7 (2013), 1165-1171. [2] M. Lewin, "On a coefficient problem for bi-univalent functions", Proc. Amer. Math. Soc. 18 (1967), 63-68. [3] H. M. Srivastava, S. Bulut, M. Çağlar, N. Yağmur, "Coefficient estimates for a general subclass of analytic and bi-univalent functions", Filomat 27:5 (2013), 831-842. [4] E. Deniz, "Certain subclasses of bi-univalent functions satisfying subordinate conditions", J. Class. Anal. 2(1) (2013), 49-60. [5] H. M. Srivastava, D. Bansal, "Coefficient estimates for a subclass of analytic and bi-univalent functions", Journal of the Egyptian Mathematical Society 23(2) (2015) 242-246. [6] S. Bulut, "Coefficient estimates for new subclasses of analytic and biunivalent functions defined by Al-Oboudi differential operatör", Journal of Function Spaces and Applications, Volume 2013, Article ID 181932, 7 pages.


# Beyond the $\lambda$-Statistical Ward Continuity 

Huseyin Cakalli<br>Maltepe University, Maltepe, Istanbul, Turkey<br>huseyincakalli@maltepe.edu.tr


#### Abstract

A function is $\lambda$-statistically downward (resp.upward) continuous if it preserves $\lambda$-statistical downward (resp.upward) half quasi-Cauchy sequences, where a real sequence $(\alpha \mathrm{k})$ is called $\lambda$-statistically downward half quasi-Cauchy if $\lim _{n} \rightarrow \infty(1 / \mathrm{n})|\{\mathrm{k} \in \mathrm{In}: \alpha \mathrm{k}+1-\alpha \mathrm{k} \geq \varepsilon\}|=0$, and $\lambda$-statistically upward half quasiCauchy if $\lim _{\mathrm{n}} \mathrm{n} \rightarrow \infty(1 / \mathrm{n})|\{\mathrm{k} \in \mathrm{In}: \quad \alpha \mathrm{k}-\alpha \mathrm{k}+1 \geq \varepsilon\}|=0$ for every $\varepsilon>0$, where $(\lambda \mathrm{n})$ is a non-decreasing sequence of positive numbers tending to $\infty$ such that $\lambda n+1 \leq$ $\lambda n+1, \lambda 1=1, \mathrm{In}=[\mathrm{n}-\lambda \mathrm{n}+1, \mathrm{n}]$ for any positive integer n . It turns out that a function is uniformly continuous if it is $\lambda$-statistical downward continuous on a above bounded set.


Keywords: Statistical Convergence, $\lambda$-Statistical Convergence.

## References:

[1] H. Fast, "Sur la convergence statistique", Colloq. Math. 2 (1951), 241-244.
[2] I.J. Schoenberg, "The integrability methods", Amer. Math. Monthly, 66 (1959), 361-375.
[3] T. Salat, "On statistically convergent sequences of real numbers", Math Slovaca, 302 (1980), 139-150.
[4] J. A. Fridy, "On statistical convergence", Analysis, 5(1985), 301-313.
[5] A. Caserta and Lj.D.R. Kocinac, "On statistical exhaustiveness", Appl. Math. Lett., 2510 (2012), 1447-1451.
[6] H. Çakall,"Forward continuity", J.Comput.Anal. Appl.,13 2 (2011) 225-230. [7]H.Çakallı,"Statistical ward continuity",Appl.Math.Lett.,24,10(2011),17241728.
[8] H. Cakalli, "Statistical quasi-Cauchy sequences", Math. Comput. Modelling 54 (2011) 1620-1624.
[9] H. Cakalli, "A Variation on Statistical Ward Continuity", Bull. Malays. Math. Sci. Soc. DOI 10.1007/s40840-015-0195-0
[10] H.Cakalli, A. Sonmez, and C.G.Aras, " $\lambda$-statistical ward continuity", An. Stiint.Univ.Al.I.CuzaIasi.Mat.(N.S.).
[11] H. Cakalli, and E. Savas, "Statistical convergence of double sequences in topological groups", J. Comput. Anal. Appl. 122 (2010), 421-426.
[12] I. Canak and M. Dik, "New types of continuities", Abstr. Appl. Anal. 2010 (2010), Article ID, 258980, 6 pages.
[13] M. Mursaleen, " $\lambda$--statistical convergence",Math.Slovaca,50(2000),111-115. [14] R.F. Patterson, and E. Savas, Asymptotic equivalence of double sequences, Hacet. J. Math. Stat. 41 (2012), 487-497.


# On Abel Statistical Continuity 

Huseyin Cakalli*, E.İffet Taylan**<br>* Maltepe University, Maltepe, Istanbul, Turkey<br>huseyincakalli@maltepe.edu.tr<br>** Maltepe University, Edution Faculty, Maltepe, Istanbul, Turkey<br>iffettaylan@maltepe.edu.tr


#### Abstract

In this paper, we investigate a concept of Abel statistical continuity. A real valued function $f$ is Abel statistically continuous on a subset $E$ of $R$, the set of real numbers, if it preserves Abel statistical convergent sequences, i.e. ( $f\left(p_{k}\right)$ ) is Abel statistically convergent whenever $\left(p_{k}\right)$ is an Abel statistical convergent sequence of points in E. Some other types of continuities are also studied and interesting results are obtained.


Keywords: Statistical Convergence; Abel Series Method; Continuity.

## References:

[1] I. Canak and M. Dik, "New types of continuities", Abstr. Appl. Anal. 2010 (2010), Article ID, 258980, 6 pages.
[2] A. Caserta and Lj.D.R. Kocinac, On statistical exhaustiveness, Appl. Math. Lett., 2510 (2012), 1447-1451.
[3] A. Caserta and G. Di Maio and Lj.D.R. Kocinac, Statistical Convergence in Function Spaces, Abstr. Appl. Anal.. 2011 (2011), Article ID 420419, 11 pp
[4] H. Çakall, "Statistical ward continuity", Appl. Math. Lett., 24 10(2011), 1724-1728.
[5] H. Cakalli, "Statistical quasi-Cauchy sequences", Math. Comput. Modelling 54 (2011)
[6] H. Çakallı and M. Albayrak, New Type Continuities via Abel Convergence, Scientific World Journal, Volume 2014 (2014), Article ID 398379, 6 pages. [7] H. Cakalli, "A Variation on Statistical Ward Continuity", Bull. Malays. Math. Sci. Soc. DOI 10.1007/s40840-015-0195-0
[8] H. Çakallı and M.K. Khan, Summability in Topological Spaces, Appl. Math. Lett. 24, (2011), 348-352.
[9] H. Cakalli, and E. Savas, "Statistical convergence of double sequences in topological groups", J. Comput. Anal. Appl. 122 (2010), 421-426.
[10] J.A. Fridy, On statistical convergence, Analysis. 5, (1985), 301-313.
[11] M. Unver, Abel summability in topological spaces, Monatsh Math. DOI 10.1007/s00605-014-0717-0
[12] R.F. Patterson, and E. Savas, "Asymptotic equivalence of double sequences", Hacet. J. Math. Stat. 41 (2012), 487-497.


# A Variation on $\boldsymbol{N}_{\boldsymbol{\theta}}$ Ward Continuity 

Huseyin Cakalli*, Mikail Et**, Hacer Sengul***<br>*Maltepe University, Faculty of Arts and Sciences, Istanbul, Turkey hcakalli@gmail.com<br>**Department of Mathematics, Firat University, Elazığ, Turkey mikailet68@gmail.com<br>***Department of Mathematics, Siirt University, Siirt, Turkey hacer.sengul@hotmail.com


#### Abstract

The notion of $N_{\theta}$ convergence was introduced and studied by Freedman, Sember and Raphael[1]. The concept of ideal convergence (or Iconvergence) of real sequences was introduced by Nuray and Ruckle[2] and later on, lacunary $I$ - convergence of sequences was introduced and investigated by Tripathy, Hazarika and Choudhary[3]. Some further results connected with the notion of the $I$-convergence can be found Savaş and Das[4], Çakallı[5] and many others. In this paper, we shall introduce concept of strongly ideal lacunary quasi-Cauchyness of sequences of real numbers. Strongly ideal lacunary ward continuity is also investigated. Interesting results are obtained. Keywords: Summability, Continuity, $N_{\theta}$ Convergence.

\section*{References:} [1] A.R. Freedman, J.J. Sember and M. Raphael, "Some Cesaro-type summability spaces", Proc. London Math. Soc., 37(3), (1978), 508-520. [2] F. Nuray and W.H. Ruckle, "Generalized statistical convergence and convergence free spaces", J. Math. Anal. Appl., 245(2), (2000), 513-527. [3] B.C. Tripathy, B. Hazarika and B.Choudhary, "Lacunary I-convergent sequences", Kyungpook Math. J. 52(4), (2012), 473-482. [4] E. Savas and P. Das, "A generalized statistical convergence via ideals, Appl. Math. Lett., 24(6), (2011), 826-830. [5] H. Cakalli, "A variation on ward continuity", Filomat, 27(8), (2013), 15451549.


A Variation on Strongly Lacunary Ward Continuity

Huseyin Cakalli*, Huseyin Kaplan**<br>*Maltepe University, Maltepe, Istanbul, Turkey<br>huseyincakalli@maltepe.edu.tr<br>** Niğde University, Faculty of Science and Letters, Niğde, Turkey<br>hkaplan@nigde.edu.tr


#### Abstract

In this paper, the concept of a strongly lacunary $\delta$-quasi-Cauchy sequence is investigated. In this investigation, we proved interesting theorems related to strongly lacunary $\delta$-ward continuity, and some other kinds of continuities. A real valued function $f$ defined on a subset A of IR is called strongly lacunary delta ward continuous on A if it preserves strongly lacunary delta quasi-Cauchy sequences of points in A. It turns out that the set of strongly lacunary delta ward continuous functions is a closed subset of the set of continuous functions.


Keywords: Summability; Series and Sequences; Continuity and Related Questions.

## References:

[1] D. Burton, and J. Coleman, "Quasi-Cauchy Sequences", Amer. Math. Monthly, 117, 4, (2010), 328-333.
[2] A. Caserta and Lj.D.R. Kocinac, "On statistical exhaustiveness", Appl. Math. Lett., 2510 (2012), 1447-1451.
[3] H. Çakallı, "Forward continuity", J. Comput. Anal. Appl., 132 (2011) 225230.
[4] H. Çakallı, "Statistical ward continuity", Appl. Math. Lett., 24, 10, (2011), 1724-1728.
[5] H. Çakallı, "N-theta-Ward continuity", Abstr. Appl. Anal. 2012, Article ID 680456, (2012), 8 pp. doi:10.1155/2012/680456.
[6] H. Cakalli, and H. Kaplan, "A study on N-theta quasi-Cauchy sequences", Abstr. Appl. Anal. 2013 Article ID 836970, 4 pages, 2013. doi:10.1155/2013/836970
[7] H. Cakalli, and E. Savas, "Statistical convergence of double sequences in topological groups", J. Comput. Anal. Appl. 122 (2010), 421-426.
[8] I. Canak and M. Dik, "New types of continuities", Abstr. Appl. Anal. 2010 (2010), Article ID, 258980, 6 pages.
[9] A.R. Freedman, J.J. Sember, and M.Raphael, "Some Cesaro-type summability spaces", Proc.London Math. Soc., 337 (1978), 508-520.

# On the Smallest Bounding Disks of Graph-Directed Fractals 

Gokce Cakmak and Ali Deniz<br>Department of Mathematics, Anadolu University, Eskisehir, Turkey gokcecakmak@anadolu.edu.tr adeniz@anadolu.edu.tr


#### Abstract

The computation of a smallest disk that encloses the attractor is needed for more accurate computation of box-counting dimension and for approximation of the attractors. There are numerous algorithms to calculate smallest disk that encloses the IFS attractor including [2], [5], [6]. The most efficient one is introduced by Martyn and it is based on spanning points to which a brief definition will be given. In this work, the existence of the smallest disks containing the attractors of the graph-directed iterated function system (GIFS) is proven and the spanning point algorithm is generalized into GIFS.


Keywords: Fractals, Graph-Directed Iterated Function Systems, Smallest Bounding Disks.

## References:

[1] M. Barnsley, Fractals Everywhere, Boston: Academic Press, 1993.
[2] D. Canright, Estimating the spatial extent of attractors of iterated function systems, Computers and Graphics, 18(2) (1994), 231-238.
[3] S. Dubuc and R. Hamzaoui, On the diameter of the attractor of an IFS, C. R. Math. Rep. Acad. Sci. Canada, 1994.
[4] G. Edgar, Measure, Topology and Fractal Geometry, Springer, New York, 2008.
[5] T. Martyn, The smallest enclosing disc of an affine IFS fractal, Fractals, Vol. 17, No. 3 (2009) 269-281.
[6] J. Rice, Spatial Bounding of Self-Affine Iterated Function System Attractor Sets, Procedings of the Conference Graphics Interface, (1996), 107-115.

This work is supported by Anadolu University Research Fund under contract 1505F188.

# A Tauberian Theorem for (C,1,1) Summable Double Sequences of Fuzzy Numbers 

Ibrahim Canak, Umit Totur and Zerrin Onder Department of Mathematics, Ege University, Bornova, Izmir, Turkey

zerrin.onder11@gmail.com; ibrahim.canak@ege.edu.tr; utotur@adu.edu.tr


#### Abstract

In this study, we recall some notations, basic definitions and theorems for the fuzzy numbers. In the sequel, we determine necessary and sufficient Tauberian conditions under which convergence in Pringsheim's sense of a double sequence of fuzzy numbers follows from its (C,1,1) summability. Finally, we define the slow oscillation of a double sequence of fuzzy numbers in different senses and prove that the slow oscillation in some sense is a Tauberian condition for ( $\mathrm{C}, 1,1$ ) summability method. We also give a classical Tauberian theorem in Landau's type for ( $\mathrm{C}, 1,1$ ) summability method.


Keywords: Fuzzy numbers, Double sequences of fuzzy numbers, slow oscillation, summability ( $\mathrm{C}, 1,1$ ), Tauberian theorems, Tauberian conditions.

## References:

[1] F. Móricz, "Tauberian theorems for Cesàro summable double sequences", Studia Math., 110 (1994), 83-96.
[2] İ. Çanak, "Tauberian theorems for Cesàro summability of sequences of fuzzy number", J. Intell. Fuzzy Syst., 27.2(2014), 937-942.
[3] E. Savaş, "A note on double sequences of fuzzy numbers", Turkish J. Math., 20(1996), 175-178.
[4] Ö. Talo and F. Başar, "On the slowly decreasing sequences of fuzzy numbers", Abstr. Appl. Anal., Article ID 891986 (2013),1-7. doi:10.1155/2013/891986.
[5] Ö. Talo and C. Çakan, "On the Cesàro convergence of sequences of fuzzy numbers", Appl. Math. Lett., 25.4(2012), 676-681.

Chemical Applications of Graph Indices<br>Ismail Naci Cangul ${ }^{l}$, Aysun Yurttas ${ }^{l}$, Muge Togan ${ }^{1}$, Ahmet Sinan Cevik ${ }^{2}$<br>${ }^{l}$ Department of Mathematics, Uludag University, Gorukle, Bursa, Turkey<br>cangul@uludag.edu.tr, ayurttas@uludag.edu.tr, capkinm@uludag.edu.tr<br>${ }^{2}$ Department of Mathematics, Selcuk University, Konya, Turkey<br>ahmetsinancevik@gmail.com


#### Abstract

In chemistry, one often wants to represent chemical structures such as atoms or molecules in numerical form as completely as possible to study their structural properties. Molecular structures can be modelled by means of graphs, and graphs have several invariant numbers so that isomorphic graphs possess identical values. These values are called as topological indices or graph invariants.


In this talk, we will consider some extensively used topological indices and show how they have been applied to chemical problems.
Keywords: Graph Theory, Topological Index, Graph Theoretical Index, Molecular Graph, Chemometry.

## References:

[1] I. Gutman, K. C. Das, The First Zagreb Index 30 Years After, MATHCH Commun. Math. Comput. Chem. 50 (2004), 83-92.
[2] P. S. Ranjini, V. Lokesha, I. N. Cangul, On the Zagreb Indices of the Line Graphs of the Subdivision Graphs, Appl. Math. Comput., 218 (2011), 699-702.
[3] K. Ch. Das, A. Yurttas, M. Togan, I. N. Cangul, A. S. Cevik, The Multiplicative Zagreb Indices of Graph Operations, Journal of Inequalities and Applications, 90, doi:10.1186/1029-242X-2013-90, 2013, 1-14
[4] K. Ch. Das, I. N. Cangul, A. D. Maden, A. S. Cevik, On the Spectral Radius of Bipartite Graphs which are Nearly Complete, Journal of Inequalities and Applications, 121, doi:10.1186/1029-242X-2013-121, 2013
[5] M. Togan, A. Yurttas, I. N. Cangul, Some formulae and inequalities on several Zagreb indices of r-subdivision graphs, Enlightments of Pure and Applied Mathematics (EPAM), 1 (1), 2015, 29-45.
[6] V. Lokesha, S. B. Shetty, P. S. Ranjini, I. N. Cangul, Computing ABC, GA, Randic and Zagreb Indices, Enlightments of Pure and Applied Mathematics (EPAM), 1 (1), 2015, 17-28.

# Permuting $\boldsymbol{n}$-Derivations From Semilattice to Lattice 

Sahin Ceran and Utku Pehlivan<br>Department of Mathematics, Pamukkale University, Denizli, Turkey<br>sceran@pau.edu.tr, utkuphlvn@hotmail.com


#### Abstract

In this paper as a generalization of permuting tri-derivation on a lattice. We introduced the notion of permuting $n$-derivation from semilattices to lattice. We defined the isotone permuting $n$-derivation from semilattices to lattice and got some interesting results about isotoneness. We characterized the istributive and isotone lattices by permuting $n$-derivation from semilattices to lattice. Keywords: Semilattice, Lattice, Derivation, Permuting $n$-Derivation From Semilattices to Lattice.

\section*{References:} [1] A. Honda and M. Grabisch, "Entropy of capacities on lattices and set systems", Inform. Sci. 176 (2006), no. 23, 3472--3489. [2] R. Balbes and P. Dwinger, "Distributive Lattices", University of Missouri Press, Columbia, Mo., 1974. [3] C. Carpineto and G. Romano, "Information retrieval through hybrid navigation of lattice representations", International Journal of Human-Computers Studies 45 (1996), 553-578. [4] Y. Çeven and M. A. Öztürk, "On f-derivations of Lattices" Bull. Korean Math. Soc. 45 (2008), no. 4, 701--707.


# Existence of Solutions for Third Order Boundary Value Problems on an Infinite Interval 

Erbil Cetin<br>Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>erbil.cetin@ege.edu.tr


#### Abstract

In this work, we study existence of solutions for third order threepoint boundary value problems on an infinite interval. The existence results of a solution and three solutions are shown by using upper and lower solution method, the Schauder fixed point theorem and topological degree theory. Two examples are given to illustrate the main results.


Keywords: Third order boundary value problem, Green's function, upper and lower solution.

## References:

[1] C. Bai, C. Li, Unbounded upper and lower solution method for third-order boundary-value problems on the half-line, Electron. J. Differ. Equ., 119 (2009) 1-12.
[2] H. Lian, J. Zhao, Existence of unbounded solutions for third-order boundary value problem on infinite intervals, Discrete Dyn. Nat. Soc., Volume 2012, Article ID 357697, 14 pages.
[3] H.Shi, M. Pei, L. Wang, Solvability of a third- order three-point boundary value problem on half line, Bull. Malays. Math. Sci. Soc., 38 (2015) 909-926.
[4] B. Yan, D. O'Regan, R.P. Agarwal, Unbounded solutions for singular boundary value problems on the Semi-infinite interval: Upper and lower solutions multiplicity, J. Comput. Appl. Math., 197 (2006) 365-386.
[5] H. Lian, J. Zha, R.P. Agarwal, Upper and lower solutions method for nth order BVPs on an infinite interval, Boundary Value Problems 2014(2014), DOI 10.1186/1687-2770-2014-100.


# In DES Cryptologic System Using Graph Adjacency Matrice for Random Variables 

Ahmet Sinan Cevik, Mustafa Yonet<br>Department of Mathematics Selcuk University, Konya, Turkey<br>mustafayonet@hotmail.com


#### Abstract

In DES algorithm cryptologic system (cf. [1]) uses some random variables (cf. [2]). But this random variables are actually easy by comparing AES algorithm (see, for instance, [3]). In this case DES algorithm cannot be useable in coding theory. But if we change this random variables with "graph adjaceny matrices" (see [4] for the details), it will be revitalized the DES algorithm. By considering same priority of operations with removing S-Box and paritying bit in algorithm priority (cf. [5]), in this removable area, it will be used the symmetric property in graph adjacency matrices. Keywords: Des Algorithm, Graph Theory, Graph Adjaceny Matrices. References: [1] Cryptolgy Enterance Lesson Note , Akyıldız "DES algorithm" (2004) 11-19 [2] An Introduction to Mathematical Cryptography , HOffstein, Pipher \& Silverman "symmetric and asymmetric ciphers"(2000) 36-37 [3] Cryptolgy Enterance Lesson Note , Akyıldız "DES algorithm" (2004) 11-19 [4] A.Kaveh, Optimal Analysis os structures by concepts of symmetry and regularity, (2013) (17) [5] Cryptolgy Enterance Lesson Note , Akyıldız "DES algorithm" (2004) 11-19




# Inclusion Regions for Matrix Eigenvalues 

Mao-Ting Chien<br>Department of Mathematics, Soochow University<br>Taipei, Taiwan<br>mtchien@scu.edu.tw


#### Abstract

Let A be an n-square matrix, and R_i be the i-th deleted absolute row sum of A. The Gershgorin circle theorem [2] states that all the eigenvalues of A are contained in the Gershgorin region which is the union of n disks centered at the i-th diagonals with radius R_i. We choose special diagonal matrices X, apply $X^{\wedge}\{-1\} A X$ to the Gershgorin circle theorem, and obtain a new inclusion region of Gershgorin circles. An example is provided to illustrate that this new inclusion region is better than the regular Gershgorin region. Partial results of this talk are included in the paper [1]. Keywords: Matrix Eigenvalue, Gershgorin Circle, Inclusion Region.

\section*{References:} [1] M.-T. Chien, S.-T. Fang and Y.-X. Su, "A generalization of Gershgorin circles", to appear in Applied and Computational Mathematics. [2] R. A. Horn and C. R. Johnson, "Matrix Analysis", Cambridge Univ. Press, New York, 1985.


# Estimation Procedure for Multi-Parameter Archimedean Copulas Based on the Trimmed L-Moments Method 

A.Chine, F. Benatia<br>L.M.A Laboratory, Department of Mathematics, M. KHIDER University,Biskra, Algeria<br>fatahbenatia@hotmail.com


#### Abstract

A new semiparametric estimation method for multi-parameters Archimedian copulas based on the Trimmed L-moments theory [3,4] is proposed. By using [ $1,2,5,6$, , Consistency and asymptotic normality of the defined estimator are established. Extensive simulation study to compare estimators based on the L-moments, the maximum likelihood and the measures of concordance is carried out. We concluded that this method is quick and does not use the density function and therefore no boundry problems arise. Keywords:Trimmed L-moments; L-moments, Copulas, Dependence, Concordance measures; Semiparametric estimation. AMS 2010 Mathematics Subject Classification: 78M05, 62F12, 62H12, 62H20.

\section*{References:} [1] Bickel, P. J., Klaassen, C. A. J., Ritov, Y. and Wellner, J. A., 1993. Efficient and Adaptive Estimation for Semiparametric Models. Baltimore, MD: The Johns Hopkins University Press. [2] Deheuvels, P., 1979. La fonction de dépendance empirique et ses propriétés. Acad. Roy. Belg. Bull. Cl. Sci. 65, 274-292. [3] Diana, B. (2014) : Robust parameter estimation using L-moments, TLmoments and theorder statistics. American Journal of Applied Mathematics. [4] Elamir, E. A., Seheult, A. H. (2003) : Trimmed L-moments. Computational Statistics \& Data Analysis 43, 299.314 pages : 302-307,310. [5] Genest, C., 1987. Frank's family of bivariate distributions. Biometrika 74, 549-555. [6] Hosking, J.R.M., 1998. L-moments. in:Kotz, S., Read, C., Banks, D.L. (Eds.), Encyclopedia of Statistical Sciences, vol. 2. Wiley, NewYork, 357-362.


# Homology Groups of Certain Khalimsky Digital Spaces 

Ismet Cinar* and Ismet Karaca<br>Department of Mathematics, Ege University,<br>Bornova, Izmir, Turkey<br>ismet_cinar@hotmail.com ismet.karaca@ege.edu.tr


#### Abstract

Khalimsky's topology on the integers is a digital response of the Euclidean topology on the real line. They [2] investigated ordered topological spaces and generalized closed curves in the 1970s. Vergili and Karaca [6] have introduced the concept of singular homology groups of Khalimsky space. They have calculated the digital singular homology groups of certain digital spaces up to the dimension 2. They have also research the digital relative homology groups, Mayer-Vietoris theorem and Excision theorem on Khalimsky space. In this study, we will explain how to calculate digital singular homology groups of some Khalimsky digital space.


Keywords: Khalimsky Topology, Digital Topology, Singular Homology.

## References:

[1] E. H. Spainer, Algebraic Topology, Springer-Verlag, New York, (1966).
[2] E.Khalimsky, R. Kopperman, P. R. Meyer, Computer graphics and connected topologies on finite ordered sets, Topology and its Applications, 36(1)(1991), 117.
[3] L. Boxer, Digitally continuous functions, Pattern Recognition Letters, 15(1994), 833-839.
[4] T. Y. Kong and A. Rosenfeld, Digital topology - a brief introduction and bibliography, Topological algorithms for the digital image processing, Elsevier Science, Amsterdam, (1996).
[5] H. Arslan, I. Karaca, A. Oztel, Homology groups of n-dimensional digital images XXI. Turkish National Mathematics Symposium, (2008), B1-13.
[6] T. Vergili and I. Karaca, Some properties of homology groups of Khalimsky space, Mathematical Science Letter, 4(2015), 131-140.

# A Quintic B-Spline Based Differential Quadrature Method for Numerical Solution of Extended Fisher-Kolmogorov Equation 

Sumita Dahiya, R. C. Mittal<br>Indian Institute of Technology Roorkee, Roorkee, INDIA<br>sumita.iitr@gmail.com


#### Abstract

In this paper, the extended Fisher-Kolmogorov equation [1] is solved numerically by implementing a new differential quadrature technique that uses quintic B-spline as the basis functions for space integration. The derivatives are approximated using differential quadrature method [3]. The weighting coefficients are obtained by semi-explicit algorithm including an algebraic system with penta-diagonal coefficient matrix that is solved using the five-band Thomas algorithm. Stability analysis of method has also been done. The accuracy of the proposed scheme is demonstrated by applying on three test problems. Some theoretical properties of Fisher-Kolmogorov equation [2] like existence, uniqueness and regularity of have been discussed. The results are also shown graphically to demonstrate the accuracy and capabilities of this method and comparative study is done with results available in literature. The computed results are found to be in good agreement with the analytical solutions.


Keywords: Extended Fisher-Kolmogorov (EFK) Equation, Quintic B-Spline, Differential Quadrature Method, Stability

## References:

[1] Coullet P., Elphick C., and Repaux D., Nature of spatial chaos. Phys. Rev. Lett., 58 (1987) 431-434.
[2] Mittal R. C., Arora G., Quintic B-spline collocation method for numerical solution of the extended Fisher-Kolmogorov equation, Int. J. of Appl. Math and Mech. 6 (1) (2010) 74-85.
[3] Mittal R. C., Dahiya Sumita., Numerical Simulation on Hyperbolic Diffusion Equations using Modified Cubic B-spline Differential Quadrature Methods, Computers and Mathematics with Applications. 70 (5)2015 737-749.

# Certain Types of Open Covers and Selection Perinciples Using Ideals 

## Pratulananda Das

Department of Mathematics, Jadavpur university Kolkata-700032, West Bengal, India<br>pratulananda@yahoo.co.in


#### Abstract

In this note we make a new and very general approach to the study of open covers and selection principles by using the very general notion of ideals and investigate some of its consequences. Our results present a more general form of some statistical variants of open covers and related selection principles introduced by Di Maio and Kocinac.


# Imprimitive Action with Continued Fractions for the Suborbital Graphs 

Ali Hikmet Deger ${ }^{1}$, Ummugulsun Akbabal<br>${ }^{1}$ Department of Mathematics, Karadeniz Technical University, Ortahisar, Trabzon, Turkey ahikmetd@ktu.edu.tr<br>ummugulsun.akbaba@ktu.edu.tr


#### Abstract

The paths of minimal lengths on suborbital graphs for modular group $\Gamma$ is given in [1]. Modular group acts on the set of extended rational numbers $\widehat{\mathbb{Q}}$ transitively. By imprimitive action on the suborbital graphs, our major aim in the present study is to extend the minimal length problem to suborbital graphs fUr $\Gamma_{0}(N)$ and to examine continued fractions arising from this connection with convergence. Finally, we give some properties for the elements of Mobius group obtained by these relations.


Keywords: Impirimitive Action, Suborbital Graphs, Continued Fractions.

## References:

[1] A.H. Değer, M. Beşenk and B.Ö. Güler, "On suborbital graphs and related continued fractions", Applied Mathematics and Computation, 218(2011), 746750.
[2] G. A. Jones, D. Singerman and K. Wicks, "The modular group and generalized Farey Graphs", London Math. Soc. Lecture Notes, CUP, Cambridge, 160 (1991), 316-318.
[3] M. Akbaş, "On suborbital graphs for the modular group", Bull. London Math. Society, 33 (2001), 647-652.
[4] C.C. Sims, "Graphs and Finite Permutation Groups", Math. Z., 95(1967), 7686.
[5] R. Sarma, S. Kushwaha and R. Krishnan, "Continued fractions arising from F_1,2", Journal of Number Theory, 154 (2015), 179-200.

# Degree Sequences of Join and Corona Product of Graphs 

Sadık Delen, Ismail Naci Cangul<br>Department of Mathematics, Uludag University, Gorukle, Bursa, Turkey<br>matesadik@gmail.com, cangul@uludag.edu.tr


#### Abstract

There are several operations on graphs which gives new and larger graphs when applied to two given graphs. Two of them are join and Corona product. Degree sequence of a graph is a non-decreasing sequence consisting of degrees of the vertices of the graph. If there is a graph corresponding to a given degree sequence, then it is said that the degree sequence is realizable. This notion plays an important role in identifying the corresponding graph and classifying graphs according to some properties. In this talk, we determine the degree sequence of the join and Corona product of two given graphs in terms of the degree sequences of the two graphs. Also some properties are obtained. Keywords: Graph Theory, Degree Sequence, Graph Operation, Join, Corona Product.

\section*{References:} [1] Das, K. Ch., Xu, K., Cangul, I. N., Cevik, A. S., Graovac, A., On the Harary Index of Graph Operations, Journal of Inequalities and Applications, Recent Advances in General Inequalities, DOI: 10.1186/1029-242X-2013-339, 2013, 116. [2] Das, K. Ch., Yurttas, A., Togan, M., Cangul, I. N., Cevik, A. S., The Multiplicative Zagreb Indices of Graph Operations, Journal of Inequalities and Applications, 90, doi:10.1186/1029-242X-2013-90, 2013, 1-14 [3] Khalifeh, M. H., Azari, H. Y., Ashrafi, A. R.: The hyper-Wiener index of graph operations, Computers and Mathematices with Appl., 56 (2008), 14021407


# Bifurcation of Flow Patterns and Eddy Structure in an LShaped Cavity with Lids Moving in the Same Directions 

Ali Deliceoglu<br>Department of Mathematics, Erciyes University, Kayseri, Turkey<br>adelice@erciyes.edu.tr


#### Abstract

Flow development and eddy structure in an L-shaped cavity with lids moving in the same directions have been investigated using both tools from lowdimensional nonlinear dynamics and standard Galerkin finite element method. In particular, structural bifurcation near boundary singular points are used to make a local analysis of the velocity field based on a Taylor series expansion. The $\left(h_{1}, h_{2}\right)$ parameter space with a series of bifurcation curves are constructed to determine the sequence of flow structures by which eddies are generated in the L-shaped cavity.


Keywords: L-Shaped Cavity, Flow Structure, Bifurcations, FEM, Stabilization. Research supported by the TUBITAK under Grant No: 114F525.

## References:

[1] A. Deliceoğlu and S.H. Aydın, "Flow bifurcation and eddy genesis in an Lshaped cavity", Computers and Fluids, 73(2013), 24-46.
[2] A. Deliceoğlu and S.H. Aydın, "Topological flow structures in an L-shaped cavity with horizontal motion of the upper lid", J. of Computational and Applied Mathematics, 259(2014), 937-943.
[3] F. Gürcan, P.H. Gaskell, M.D. Savage, M.C.T., " Wilson, Eddy genesis and transformation of Stokes flow in adouble-lid driven cavity", Proc. Ins. of Mech Eng. Part-C J. Mec. Eng. Sci., 217(2003), 353-364.
[4] F. Gürcan, A. Deliceoğlu, P.G. Bakker, "Streamline topologies near nonsimple degenerate point close to a stationary wall using normal forms, J. Fluid Mech. 539(2005) 299-311.

# A Nonself-Adjoint Dirac Operators with a Spectral Parameter in the Boundary Condition and with Transmission Condition 

Isil Acik Demirci, Bilender P.Allahverdiev<br>Department of Mathematics, Faculty of Education, Mehmet Akif Ersoy University, Burdur, Turkey<br>isilacik@yahoo.com<br>Department of Mathematics, Suleyman Demirel University,<br>Isparta, Turkey<br>bilenderpasaoglu@sdu.edu.tr


#### Abstract

In this talk, we investigate nonself-adjoint Dirac operators boundary value transmission problems in Weyl's limit circle case in the Hilbert space. A self-adjoint dilation and a spectral model of these operators are constructed and the characteristic function is computed. Theorems on the completeness of the eigenvectors of the nonself-adjoint Dirac operators are proved. Keywords: Dissipative Singular Dirac System, Spectral Parameter in the Boundary Condition, Transmission Condition, Self-Adjoint Dilatation, Maximal Dissipative Operator, Functional Model, Characteristic Function, Completeness of the System of Eigenvectors and Associated Vectors.

\section*{References:} [1] B. P. Allahverdiev, "Spectral Analysis of Dissipative Dirac Operators with General Boundary Conditions", J. Math. Anal. Appl., 283.1(2003), 287-303. [2] B. P. Allahverdiev, "A Nonself-Adjoint 1D Singular Hamilton System with an Eigenparameter in the Boundary Condition". Potential Anal., 38.4(2013), 1031-1045. [3] A. Kablan, T. Özden, "A Dirac System with Transmission Condition and Eigenparameter in Boundary Condition", Hundawi Publlishing Corporation Abstract and Applied Analysis, 2013(2013), 6 pages. [4] B. S. Pavlov, "Spectral Analysis of A Dissipative Singular Schrödinger Operator in Terms of A Functional Model", Itogi Nauki Tekh. Ser. Sovrem. Probl. Math. Fundam.Napravleniya, 65(1996), 95-163, English Transl. Partial Differ. Equ., 8, Encycl. Math. Sci., 65(1991), 87-163.


# Close-to-Convexity of a Cross-Product of Bessel Functions 

Erhan Deniz<br>Department of Mathematics, KafkasUniversity, Kars, Turkey<br>edeniz36@gmail.com


#### Abstract

In this paper a necessary and sufficient condition is deduced for the close-to-convexityof a cross-product of Bessel and modified Bessel functions of the first kind and their derivatives by using a result of Shah and Trimble about transcendental entire functions with univalent derivatives and Mittag-Leffler expansions of this crossproduct, as well as a slightly modified version of a result of Lorch on the monotonicity of the zeros of the cross-product with respect to the order.


Keyword: Bessel Functions of the First Kind, Modified Bessel Functions of the First Kind, Close-to-Convex Functions, Starlike Functions, Zeros of CrossProduct of Bessel Functions, Mittag-Leffler Expansions.

## References:

[1] H. A. Al-Kharsanı, A. Baricz and T. K. Pogany, "Starlikeness of a crossproduct of Bessel functions" J. Math. Ineq., 10(3) (2016) 819-827.
[2] A. Baricz, M. Çağlar and E. Deniz, "Starlikeness of Bessel functions and their derivatives", Math. Ineq. Appl., 19(2) (2016), 439-449.
[3] A. Baricz, E. Deniz and N. Yağmur, "Close-to-convexity of normalized Dini functions", Math. Nachr., doi 10.1002/mana. 201500009 (in press).
[4] A. Baricz, S. Ponnusamy and S. Singh, "Cross-product of Bessel functions: monotonicity patterns and functional inequalities", Proc. Indian Acad. Sci., (Math. Sci.) (in press).
[5] A. Baricz and R. Szasz, "Close-to-convexity of some special functions and their derivatives", Bull. Malay. Sci. Soc., 39 (1) (2016), 427-437.
[6] A. Baricz and N. Yağmur, "Radii of starlikeness and convexity of a crossproduct of Bessel functions", Ramanujan J., (in press).
[7] L. Lorch, Monotonicity of the zeros of a cross-product of Bessel functions, Methods Appl. Anal., 1(1) (1994), 75-80.
[8] S. M. Shah and S. Y. Trimble, "Entire functions with univalent derivatives", J. Math. Anal. Appl., 33 (1971), 220-229.
[9] R. Szasz, "On starlikeness of Bessel functions of the first kind", In: Proceedings of the 8th Joint Conference on Mathematics and Computer Science, Komarno, Slovakia, 2010, 9 pp.

# Existence Criteria of Three Positive Solutions to Boundary Value Problems for p-Laplacian Dynamic Equations on Time Scales 

Abdulkadir Dogan ${ }^{1}$, Bunyamin Aydin ${ }^{2}$<br>${ }^{1}$ Department of Applied Mathematics, Faculty of ComputerSciences, Abdullah Gul University, Kayseri, Turkey<br>${ }^{2}$ Necmettin Erbakan Universitesi, Ahmet Kelesoglu Egitim Fakultesi, Konya, Turkey<br>abdulkadir.dogan@agu.edu.tr, baydin@konya.edu.tr.


#### Abstract

The study of dynamic equations on time scales goes back to its founder Stefan Hilger [1], and is new area of still fairly theoretical exploration in mathematics. In the recent years, boundary value problems for dynamic equations on time scales have received considerable attention. The usual notation and terminology for time scales as can be found in [2] will be used here. Agarwal and O'Regan [3] studied the existence of one or more solutions to nonlinear equations on time scales. They established by using either a nonlinear alternative of Leray-Schauder type or Krasnoselski's fixed point theorem in a cone. Anderson [4] established the existence of multiple positive solutions to the nonlinear second-order three-point boundary value problem on a time scale. He employed the Leggett-Williams fixed-point theorem in an appropriate cone to guarantee the existence of at least three positive solutions. He [5] considered the existence of at least double positive solutions of three-point boundary value problems for p -Laplacian dynamic equations on a time scale by applying a new double fixed-point theorem. In this paper, we consider existence criteria of three positive solutions of three-point boundary value problems for p-Laplacian dynamic equations on time scales. To show our main results, we apply the well known Leggett-Williams fixed-point theorem.


Keywords: Time Scales, Boundary Value Problem, p-Laplacian, Positive Solutions, Fixed Point Theorem

## References:

[1] S. Hilger, "Analysis on measure chains a unifed approach to continuous and discrete calculus", Results Math. 18(1990), 18-56.
[2] M. Bohner and A. Peterson, "Dynamic equations on time scales: An Introduction with Applications", Birkhauser, Boston, Cambridge, MA 2001.
[3] R.P. Agarwal and D. O'Regan, " Nonlinear boundary value problems on time scales", Nonlinear Anal., 44 (2001), 527-535.
[4] D. R. Anderson, "Solutions to second-order three-point problems on time scales", J. Difference. Equ. Appl., 8 (2004), 673-688.
[5] Z. He, "Double positive solutions of three-point boundary value problems for pLaplacian dynamic equations on time scales", J. Comput. Appl. Math., 182 (2005), 304-315.

# A Study on Some New Results Arising from (p,q)-Calculus 

Ugur Duran ${ }^{1, *}$, Mehmet Acikgoz ${ }^{2}$ and Serkan Araci ${ }^{3}$<br>${ }^{1}$ Department of Mathematics, Faculty of Arts and Science, University of Gaziantep, TR-27310 Gaziantep, Turkey duran.ugur@yahoo.com<br>*Corresponding Author<br>${ }^{2}$ Department of Mathematics, Faculty of Arts and Science, University of Gaziantep, TR-27310 Gaziantep, Turkey acikgoz@gantep.edu.tr<br>${ }^{3}$ Department of Economics, Faculty of Economics, Administrative and Social Science, Hasan Kalyoncu University, TR-27410 Gaziantep, Turkey mtsrkn@hotmail.com


#### Abstract

In this paper, we give some new investigations and results associated with post quantum calculus, denoted $(p, q)$-calculus. We develop the theory of the $(p, q)$-analogue of the chain rule for $(p, q)$-derivative. Also, we introduce a new $(p, q)$-analogue of the exponential function and investigate some its properties, and using this function, we derive the addition property for $(p, q)$-exponential functions. We also derive many useful results involving ( $p, q$ )-binomial coefficients and $(p, q)$-antiderivative. Finally, we consider $(p, q)$-analogue of some elementary functions including trigonometric functions and hyperbolic functions and research their properties and relations between them. This may be a good consideration in developing the $(p, q)$-calculus in combinatorics, number theory and other fields of mathematics.


Keywords: $\quad q$-Calculus; $\quad(p, q)$-Calculus; $\quad(p, q)$-Exponential Functions; Trigonometric Functions; Hyperbolic Functions.

## References:

[1] M. Arik, E. Demircan, T. Turgut, L. Ekinci, M. Mungan, Fibonacci oscillators, Z. Phys. C: Particles and Fields, 55 (1991), 89-95.
[2] R. Chakrabarti and R. Jagannathan, A (p,q)-oscillator realization of two-parameter quantum algebras, J. Phys. A: Math. Gen., 24 (1991), L711.
[3] U. Duran, M. Acikgoz, S. Araci, H. M. Srivastava, A certain (p,q)-derivative operator and associated divided differences, accepted for publication in J. Comput. Theor. Nanosci. (2016).
[4] V. Kac and P. Cheung, Quantum calculus, New York: Springer, (2002).
[5] T. Kim, q-extension of the Euler formula and trigonometric functions, Russ. J. Math. Phys., vol. 14, no. 3, pp. 275-278, 2007.
[6] P. N. Sadjang, On the fundamental theorem of (p,q)-calculus and some (p,q)-Taylor formulas, arXiv:1309.3934 [math.QA] (2013).
[7] H. M. Srivastava and J. Choi, Zeta and q-Zeta functions and associated series and integrals, Elsevier Science Publishers, Amsterdam, London and New York, 2012.

# Upper and Lower Solutions for Fourth Order Boundary Value Problems 

Serife Müge Ege<br>Department of Mathematics, Ege University,<br>Bornova, Izmir, Turkey<br>serife.muge.ege@ege.edu.tr


#### Abstract

In this work, existence of unbounded solutions for fourth order three point boundary value problem on a half line is handled. For this purpose Schauder's fixed point theorem and upper-lower solution method are applied. Sufficient conditions are given for existence of at least one solution and at least three solutions. An illustrative example is given to show the importance of results.


Keywords: Three Point Boundary Value Problem, Upper and Lower Solutions, Schauder's Fixed Point Theorem.

## References:

[1] E. Çetin and R. P. Agarwal, "Existence of solutions for fourth order threepoint boundary value problems on a half line", Elect. J. Qualitative Theory of Diff. Equ., 62 (2015), 1-23.
[2] R. P. Agarwal and E. Çetin, "Unbounded solutions of third order Three-point boundary value problems on a half line", Adv. Nonlinear Anal., 43 (2015), 1-15. [3] B. Yan, D. O’Regan, R.P. Agarwal, "Unbounded solutions for singular boundary value problems on the semi-infinite interval: Upper and lower solutions multiplicity", J. Comput. Appl. Math., 197 (2006), 365-386.
[4] A. Cabada, "The method of lower and upper solutions for second, third, fourth and higher order boundary value problems", J. Math. Anal. Appl., 185 (1994), 302-320.
[5] Y. Zhao, H. Chen, C. Xu, "Existence of multiple solutions for three-point boundary-value problems on infinite intervals in Banach spaces", Electron. J. Differ. Equ., 44 (2012), 1-11.

# Invariant Subspace Analysis of the Fractional Modifed Kuramoto-Sivashinsky Equation 

E. H. El Kinanil, 2<br>${ }^{1}$ A.A Group, Mathematical Department Moulay Ismïl University, Faculty of Sciences and Technics Errachidia, BP 509, Morocco.<br>${ }^{2}$ Université Moulay Ismail Ecole Nationale Supérieure des Arts et Métiers (ENSAM), Marjane 2, B.P. 15290, Meknès, Maroc.


#### Abstract

In this work, the invariant subspace method [1,2] is applied to the time fractional modified Kuramoto-Sivashinsky partial differential equation [3]. The obtained reduced system of nonlinear ordinary fractional equations is solved by the Laplace transform method and with using of some useful properties of Mittag-Leffler function[4]. Then, some exact solutions of the time fractional nonlinear studied equation are found.


Keywords: Invariance Subspace Method, Caputo Fractional Derivative, Fractional Modified Kuramoto-Sivashinsky Equation, Mittag-Lefller Function.

## References:

[1] R. Gazizov and A. Kasatkin, "Construction of exact solutions for fractional order differential equations by the invariant subspace method", Computers and Mathematics with Applications, vol. 66, no. 5, 2013, 576-584.
[2] V. Galaktionov and S. Svirshchevskii, "Exact solutions and invariant subspaces of nonlinear partial differential equations in mechanics and physics". Chapman and Hall/CRC applied mathematics and nonlinear science series, 2007. [3] T. Hocherman and P. Rosenau," On KS-type equations describing the evolution and rupture of a liquid interface", Phys. D, 67, 113-125, 1993.
[4] J. C. Prajapati,"Certain properties of Mittag-Leffler function with argument $\mathrm{x} \alpha, \alpha>0$ ", Italian journal of pure and applied mathematics 30,411-416, 2013.

# On the Study of the Holditch Theorem for the Non-Linear Three Points in $C_{p}$ 

Tulay Erisir and Mehmet Ali Gungor<br>Department of Mathematics, Sakarya University, Sakarya, Turkey<br>tsoyfidan@sakarya.edu.tr, agungor@sakarya.edu.tr


#### Abstract

The generalized complex number system and generalized complex plane were studied by Yaglom[1,2] and Harkin[3]. The Steiner area formula and the Holditch theorem giving the relationship between the areas formed by linear points in the generalized complex plane $C_{p}$ were given by Erisir[4]. In this paper, we study using non-linear points a new generalization of the Holditch theorem given by [4] in the generalized complex plane $C_{p}$. While we obtain the Holditch theorem, we use the the length of the enveloping curves of lines formed by points.


Keywords: Generalized Complex Number, Generalized Complex Plane, Holditch Theorem

## References:

[1] I. M. Yaglom, "A Simple non-Euclidean Geometry and its Physical Basis", Springer-Verlag, New-York, (1979).
[2] I. M. Yaglom, "Complex Numbers in Geometry", Academic Press, New York, (1968).
[3] A. A. Harkin and J. B. Harkin, "Geometry of Generalized Complex Numbers", Math. Mag., 77.2(2004), 118-129.
[4] T. Erisir, M. A. Güngör and M. Tosun, "A New Generalization of the Steine Formula and the Holditch Theorem", Adv. Appl. Clifford Algebr., DOI 10.1007/s00006-015-0559-4, (2015).


# The Stability Analysis of a Delay Differential Equation with State Dependent Delay 

Sertac Erman, Ali Demir<br>Department of Mathematics, Kocaeli University, Umuttepe, 41380 Izmit - Kocaeli, Turkey 106133002@kocaeli.edu.tr, ademir@kocaeli.edu.tr


#### Abstract

In this paper, we investigate the stability of a differential equation with state-dependent delay under some conditions on delay term. New neccesary and sufficient criterions are elaborated for the asymptotic stability of the differential equations with state dependent delay. Moreover, the asymptotic stability of it is illustrated for a special delay function


Keywords: Asymptotic Stability, State Depended Delay, Delay Differential Equation.

## References:

[1] R. D. Driver, "Existence theory for a delay-differential system", Contrib Differential Equations , 1(1963),317--336
[2] R. D. Driver, "A two-body problem of classical electrodynamics: the onedimensional case", Ann Physics, 21(1963), 122--142
[3] R. D. Driver, M. J. Norris, "Note on uniqueness for a one-dimensional twobody problem of classical electrodynamics", Ann Physics, 42(1967), 347--351..
[4] E. Winston, "Uniqueness of solutions of state dependent delay differential equations", J Math Anal Appl, 47(1974), 620--625
[5] K. L. Cooke, Asymptotic theory for the delay-differential equation u_(t)=-$\mathrm{au}(\mathrm{t}-\mathrm{r}(\mathrm{u}(\mathrm{t}))$ ), J Math Anal Appl, 19(1967), 160--173.

# Generation of the Trigonometric Cubic B-Spline Collocation Solutions for Generalized Burgers-Fisher Equation 

Ozlem Ersoy, Idris Dag<br>Department of Mathematics-Computer, Eskişehir Osmangazi University, Eskişehir, Turkey<br>ozersoy@ogu.edu.tr


#### Abstract

In this study Generalized Burgers-Fisher equation which includes several known evolution equations as particular cases has been solved numerically using the collocation finite element method, based on Crank Nicolson for the time integration and trigonometric cubic B-spline functions for the space integration. The accuracy of the method is measured by calculating the discerete error norms between the analytical and numerical solutions. Keywords: Finite Element Method, Collocation Method, Trigonometric Cubic B-Spline, Generalized Burgers-Fisher Equation.

\section*{References:} [1] T. Zhao, C. li, Z. Zhang, Y. Wuai "Chebyshev-Legendre pseudospectral method for the generalized Burgers-Fisher equation", Applied Mathematical Modelling 36(2012), 1046-1056. [2] R. Mohammadi, "Spline solution of the generalized Burgers'-Fisher equation", Applicable Analysis, 91.12(2012), 2189-2215. [3] Z. Rong-Pei and Z. Li-Wei, "Direct discontinuous Galerkin method for the generalized Burges-Fisher equation", Chin. Phys. B, 21.9(2012), 1-4. [4] R. Zhang, X. Yu, G. Zhao, "The local discontinuous Galerkin method for Burger's- Huxley and Burger's-Fisher equations", Appl. Math. Comput, 218.17(2012), 8773-8778. [5] I. Dag and O. Ersoy, "Numerical solution of generalized Burgers-Fisher equation by exponential cubic B-spline collocation method", AIP Conference Proceedings, 1648.370008 (2015).


# An Exponential Cubic B-Spline Finite Element Method for Solving the Nonlinear Coupled Burgers' Equation 

Ozlem Ersoy, Idris Dag<br>Department of Mathematics-Computer, Eskişehir Osmangazi University, Eskisehir, Turkey<br>ozersoy@ogu.edu.tr


#### Abstract

The exponential cubic B-spline functions together with Crank Nicolson are used to solve numerically the nonlinear coupled Burgers' equation using collocation method. This method has been tested by three different problems. The proposed scheme is compared with some existing methods. We have noticed that proposed scheme produced a highly accurate results. Keywords: Finite Element Method, Collocation Method, Exponential Cubic BSpline, Coupled Burgers' Equation.


## References:

[1] B. J., McCartin, "Theory of exponential splines", J. Approx. Theory, 66.1(1991), 1-23.
[2] R. Mohammadi, "Exponential B-spline solution of Convection-Diffusion Equations", Applied Mathematics, 4(2013), 933-944.
[3] J. Liu and G. Hou, "Numerical solutions of the space- and time-fractional coupled Burgers equations by generalized differential transform method", Applied Mathematics and Computation, 217(2011), 7001-7008.
[4] M. Kumar and S. Pandit, "A composite numerical scheme for the numerical simulation of coupled Burgers' equation", Computer Physics Communications 185(2014), 809-817.
[5] R. C. Mittal and G. Arora, "Numerical solution of the coupled viscous Burgers' equation", Commun Nonlinear Sci Numer Simulat, 16(2011), 13041313.

# On Weakly Sections in Sequence Spaces 

Merve Temizer Ersoy, Hasan Furkan, Bilal Altay * Department of Mathematics, Kahramanmaras Sutcu Imam University, Kahramanmaras, Turkey mervetemizer@hotmail.com, hasanfurkan@hotmail.com<br>*Department of Primary Education, Inonu University, Malatya, Turkey<br>bilal.altay@inonu.edu.tr


#### Abstract

The n-th section of a sequence $x=\left(x_{k}\right)$ is the sequence $\sum_{k=1}^{n} x_{k} e^{k}$, where $e^{k}$ is the sequence with 1 in the k-th position and 0 elsewhere. The sectional convergence (AK), boundedness (AB), weak convergence (SAK), functional sectional convergence (FAK), unconditionally bounded (UAB), unconditionally section convergence (UAK) and other sectional property were examined [1-6]. In this paper we deal with the weakly absolutely convergence, weakly bounded and weakly bounded variation sections of a sequence in sequence spaces. Then we give some relation between classical sectional property and weakly sections in sequence spaces.


Keywords: Topological Sequence Space, Weakly Absolutely Convergent Series, Weakly Bounded Variation Series, FK Spaces, BK Spaces.

## References:

[1] K. Zeller, " Allgemeine Eigenschaften von Limitierungsverfahren", Math. Z. 53 (1951), 463-487.
[2] J. Sember, M. Raphael, "The unrestricted section properties of sequences", Can. J. Math. Vol. 26(2) (1979), 331-336.
[3] Sargent, W.L.C, "On sectionally bounded BK-spaces", Math. Z. 83 (1964), 57-66.
[4] D. J. H. Garling, " On topological sequence spaces", Proc. Camb. Phil. Soc., 63 (1967), 997-1019.
[5] J. Sember, " On unconditional section boundedness in sequence spaces", Rocky Mountain J. Math. 7 (4) (1977), 699-706.
[6] K-G., Grosse-Erdmann, " On, -Invariant Sequence Spaces", J. Math. Anal. Appl.,( 262), (2001), 112-132.

# Dispersive Properties of Conservative Schemes for Three Coupled Nonlinear Schrödinger Equation 

Sevim Ertug and Ayhan Aydin<br>Department of Mathematics, Attlm University, Incek, Ankara, Turkey<br>ayhan.aydin@atilim.edu.tr


#### Abstract

Three coupled nonlinear Schrödinger (3-CNLS) equation is a system of partial differential equation (PDE) with second order dispersion and cubic nonlinearity. It has some conserved quantity such as the mass (or charge) and energy. We have proposed two numerical methods that preserve the discrete version of mass and the energy of the equation. The recent growth in the field of numerical solution of PDE has led to the development of numerical methods that preserve qualitative structure of the equation. However, litte attention has been given to the dispersive properties of the PDE and its numerical solution. Numerical errors in the dispersion relation and the group velocities can lead to the propagation of the numerical wave with different velocity and can lead to spurious solutions. Therefore, it is important to preserve the sign of the group velocity in order to understand the behavior of the numerical solution. In this study, we examine the dispersive properties of the mass and energy conserving numerical methods for the 3-CNLS equation.


Keywords: Three Coupled Nonlinear Schrödinger Equation, Geometric Integration, Dispersion.

## References:

[1] C .M. Schober, T. H. Wlodarczyk, "Dispersive properties of multisymplectic integrators", J. Computational Phys., 227(2008), 5090-5104.
[2] L. N. Trefhethen, "Group velocities in finite difference schemes", SIAM Rev., 24(1982), 113-136.

# A Model of Nickel-Iron Alloy Electrodeposition on Rotating Disk Electrode: the Global Existence in the Quadratic Case 

Nadia Idrissi Fatmi<br>Laboratory LIPOSI, ENSA KHOURIBGA<br>Hassan I University<br>Settat, Morocco<br>nadidrissi200133@gmail.com


#### Abstract

To better understand the nickel-iron electrodeposition process, we are interested in the one-dimensional model. This model addresses dissociation, diffusion, electomigration, convection and deposition of multiple ion species. We study the global existence of solutions that are here different ion concentrations in the mixture as well as the electric potential. The classic techniques, based on the $\mathrm{C}^{\alpha}$ estimations, to prove the existence and the positivity of solutions fall in defect and news techniques must be developed. We present them here and we obtain global existence and positivity of classical solution for our model in the quadratic case, without any restriction of growth on the non linear terms.


Keywords: Poisson--Nernst--Planck Equations, Electrochemical Systems, Butler--Volmer Reaction Kinetics, Classical Solution.

## References:

[1] N. Alaa, N. Idrissi Fatmi, J.R. Roche, A. Tounsi, Mathematical Analysis for a model of Nickel-Iron alloy Electrodeposition on rotating disk electrode: parabolic case, International Journal of Mathematics and Statistics, Vol 2 (2008), 30-48.
[2] H. Amann, M. Renardy, Reaction-Diffusion Problems in Electrolysis, Birkhauser Verlag, Basel 1994.
[3] Krause, L. Arulnayagam, and M. Pritzker, J. Electrochem. Soc. 144, 960, 1997.
[4] M. Matlosz, J. Electrochem. Soc. 140, 2272 ,1993.
[5] R : M. Ramasubramanian, S. N. Popova, B. N. Popova, R. E. White, and K.M. Yin, J. Electrochem. Soc. 143, 2164, 1996.
[6] Schultz, and M. Pritzker, J. Electrochem. Soc. 145, 2033 ,1998.

# Mathematical Analysis for a Class of Quasilinear Elliptic Equations with Nonlinearity in the Gradient and Lí-Data 

Nadia Idrissi Fatmi<br>Laboratory LIPOSI, ENSA KHOURIBGA<br>Hassan I University<br>Settat, Morocco<br>nadidrissi200133@gmail.com


#### Abstract

In this paper we show the existence of weak solutions for some quasilinear elliptic equations with Dirichlet boundary conditions. The nonlinearity we consider here has critical growth with respect to the gradient and the data is in $\mathrm{L}^{1}$. Keywords: Quasilinear Elliptic Equations, Weak Solution, Dirichlet Boundary Condition, Critical Growth with Respect to the Gradient.

\section*{References:} [1] N. Alaa, N. Idrissi Fatmi and M. Pierre. Quasilinear elliptic degenerate equations with nonlinearity in the gradient and $L^{1}$ data, International Journal of Mathematics and Statistics, Volume 7, Number W10, pp. 62-69, (2010). [2] H. Amann, M.G. Grandall, On some existence theorems for semi linear equation equations. Indiana Univ. Math.J, 27(1978), 779,790. [3] P. Baras, M. Pierre, Problèmes Paraboliques Semi-Linéaires Avec Données Mesures, Applicable Analysis, 18 (1984), pp. 11-49. [4] L. Boccardo, T. Gallouet, Nonlinear Elliptic and Parabolic Equations Involving Measure Data, Journal of Functional Analysis, 87 (1996), pp. 149-768. [5] L. Boccardo, F. Murat, J. P. Puel, Existence results for some quasilinear parabolic equations. Nonlinear Analysis Theory Method and Applications 13 (1989), 373-392. [6] P.L. Lions, Résolution de problèmes elliptiques quasilinéaires. Arch. Rational Mech. Anl. 74 (1980), 335-353.


Homoclinical Structure of Hybrid Systems with Impacts

Mehmet Onur Fen ${ }^{1}$, Fatma Tokmak Fen ${ }^{2}$<br>${ }^{1}$ Department of Mathematics,Middle East Technical University, 06800 Ankara, Turkey<br>monur.fen@gmail.com<br>${ }^{2}$ Department of Mathematics, Gazi University, Teknikokullar, 06500 Ankara, Turkey fatmatokmak@gazi.edu.tr


#### Abstract

In this study, we investigate the homoclinical structure of impulsive systems which are influenced by a discrete map with homoclinic and heteroclinic orbits. An example supporting the theoretical results is presented.


Keywords: Hybrid Systems, Homoclinic Motion, Heteroclinic Motion.

## References:

[1] M. Akhmet, "Principles of Discontinuous Dynamical Systems", Springer, New York, 2010.
[2] M.U. Akhmet, "Homoclinical structure of the chaotic attractor", Commun. Nonlinear Sci. Numer. Simulat. 15(2010), 819-822.
[3] M.U. Akhmet, "Hyperbolic sets of impact systems", Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal. 15 (Suppl. S1) (2008), 1-2, in: Proceedings of the 5th International Conference on Impulsive and Hybrid Dynamical Systems and Applications, Beijing, Watan Press, 2008.
[4] V. Avrutin, B. Schenke, L. Gardini, "Calculation of homoclinic and heteroclinic orbits in 1D maps", Commun. Nonlinear Sci. Numer. Simulat. 22(2015), 1201-1214.
[5] F. Battelli, M. Feckan, "Chaos in singular impulsive O.D.E.", Nonlinear Analysis, Theory, Methods \& Applications 28 (1997), 655-671.
[6] J. Hale, H. Kocak, "Dynamics and Bifurcations", Springer-Verlag, New York, 1991.

# Multiple and Nodal Solutions for Nonlinear Problems 

Michail E. Filippakis<br>Department of Digital Systemss, University of Piraeus, Piraeus, Greece mfilip@unipi.gr


#### Abstract

We consider a nonlinear elliptic Neumann problem driven by a nonhomogeneous differential operator, which is strictly monotone and incorporates as special cases the p-Laplacian, the (p,q)-differential operator and the generalized p-mean curvature differential operator. Using variational methods coupled with suitable truncation and comparison techniques and Morse theory (critical groups), we prove the existence of at least three nontrivial smooth solutions, one positive, the second negative and the third nodal.


Keywords: Nonhomogeneous Differential Operator, Nonlinear Regularity, Nonlinear Strong Maximum Principle, Extremal Constant Sign Solutions, Nodal Solution, Local Minimizer.

## References:

[1] S.Aizicovici, N.S. Papageorgiou, V.Staicu, Degree theory for operators of monotone type and nonlinear elliptic equations with inequality constraints, Memoirs of AMS 196, No. 915 (2008).
[2] S.Aizicovici, N.S. Papageorgiou, V.Staicu, Existence of multiple solutions with precise sign information for superlinear Neumann problems, Annali Mat.Pura Appl., 188 (2009), 679-719.
[3] S.Aizicovici, N.S. Papageorgiou, V.Staicu, The spectrum and an index formula for the Neumann $\$ \mathrm{p} \$$-Laplacian and multiple solutions problems with crossing nonlinearities, Discrete Cont. Dynam. Systems, 25 (2009), 431-456.
[4] H.Attouch, G.Buttazzo, G.Michaille, Variational Analysis in Sobolev and BV Spaces, MPS-SIAM Series in Optimization, Philadelphia, (2006).
[5] K.C.Chang, Infinite Dimensional Morse Theory and Multiple Solution Problems, Birkhauser, Boston, (1993).
[6] E.DiBenedetto, $\mathrm{C}^{\wedge}\{1+\mathrm{a}\}$ local regularity of weak solutions of degenerate elliptic equations, Nonlinear Analysis, 7 (1993), 827-850.
[7] J.I.Diaz-J.E.Saa, Existence et unicite de solutions positives pour certaines equations elliptiques quasilineaires, NCRAS,Paris,t. 305 (1987), 521-524.
[8] N.Dunford, J.Schwartz, Linear Operators I, Wiley-Interscience, New York, (1958).
[9] L.Gasinski-N.S.Papageorgiou, Nonsmooth Critical Point Theory and Nonlinear Boundary Value Rroblems, Chapman and Hall/CRC Press, Boca Raton, (2005).
[10] L.Gasinski-N.S.Papageorgiou, Nonlinear Analysis, Chapman and Hall /CRC Press, Boca Raton, (2006).

# Estimation of the Scatter Matrix of an Elliptically Symmetric Distributions． Orthogonally Invariant Estimators． 

Dominique Fourdrinier，Fatiha Mezoued and Martin T．Wells<br>Université de Rouen，France，Ecole Nationale de Statistique et d＇Economie Appliquée， Alger，Algeria<br>Cornell University，USA<br>dominique．Fourdrinier＠univ－rouen．fr，famezoued＠yahoo．fr，mtw 1 ＠cornell．edu


#### Abstract

Let $(\mathrm{X}, \mathrm{U})=\left(\mathrm{X}, \mathrm{U}_{1}, \ldots, \mathrm{U}_{\mathrm{n}}\right)$ be $n+1$ random vectors having an elliptically contoured distribution国国國 joint density of the form $$
|\Sigma|^{-(n+!) / 2} f\left((x-\theta \quad)^{t} \Sigma^{-1}(x-\theta)+\sum_{i=1}^{n} u_{i}^{t} \Sigma^{-1} \mathbf{u}_{\mathrm{i}}\right)
$$


Where $X$ and $U_{i}$＇s are $p \times 1$ vectors and $S=U U^{\mathrm{t}}$ and both $\theta$ and $\Sigma^{-1}$ are unknown．We provide orthogonally invariant estimators of the precision matrix $\sum$ ．According to the fact that the matrix $S$ is invertible or not，natural estimators are of the form
$a S^{-1}$ or $a S^{+}$where $a$ is a positive constant and where $S^{-1}$ and $S^{+}$are respectively the inverse and the Moore－Penrose inverse of $S$ ．We propose improved estimators under the loss $\operatorname{tr}\left(\left(\hat{\Sigma}^{-1}-\Sigma^{-1}\right)^{2}\right)$ ．So here，we generalize the works in［1］，［2］，［3］，［4］which dealt with the Gaussian case．
Keywords：Elliptically Symmetric Distributions；Precision Matrix；Berger Class．

## References：

［1］D．K．Dey，＂Improved estimation of multinormal precision matrix＂，Statistics and probability letters，6：125－128， 1987.
［2］T．Kubokawa，M．S．Srivastava，＂Estimation of the precision matrix of a singular wishart distribution and its application in high－dimentional data＂， Journal of multivariate Analysis，99：1906－1928， 2008.
［3］H．Tsukuma，Y．Konno，＂On improved estimation of normal precision and discriminant coefficients＂，Journal ofmultivariate Analysis，97（1）：1477－ 1500，2006．
［4］H．Tsukuma，＂Estimating the inverse matrix of scale parameters in an elliptically contoured distribution＂，J．Japan．Statist．Soc，35（1）：21－39，2005．

# New Summability Method and Its Applications 

Mubariz T. Garayev**, Mehmet Gurdal*, Ulas Yamanci*<br>*Department of Mathematics, Suleyman Demirel University, Isparta, Turkey<br>**Department of Mathematics, College of Science, King Saud University, Riyadh, Saudi Arabia mgarayev@ksu.edu.sa, gurdalmehmet@sdu.edu.tr ulasyamanci@sdu.edu.tr


#### Abstract

By using Berezin symbol technique, we prove some results for Borel summability. Also, we give a Tauberian type theorem for Borel summability. Keywords: Borel Summabiliy, Berezin Symbol, Diagonal Operator, Fock Space. References: [1] M.T. Karaev, "(e)-convergence and related problem", C. R. Math. Acad. Sci. Paris, 348(2010), 1059-1062. [2] M.T. Karaev, "Tauberian-type theorem for (e)-convergent sequences", C. R. Math. Acad. Sci. Paris, 351 (2013), 177-179. [3] B. Sawyer and B. Watson, "Borel's Methods of Summability: Theory and Applications", Oxford University Press Inc., New York, 1994. [4] K. Stroethoff, "The Berezin transform and operators on spaces of analytic functions", Banach Center Publ., 38(1997), 361-380.


# Some New Ideal Convergent Double Sequence Spaces and Weighted Lacunary I-Statistical Convergence for Double Sequences 

Ergin Genc ${ }^{1}$ and Sukran Konca ${ }^{2}$<br>Department of Mathematics, Bitlis Eren University, 13000, Bitlis, Turkey<br>ergin_genc44@hotmail.com; skonca@beu.edu.tr


#### Abstract

In this work, we aim to introduce some new double ideal convergent sequence spaces and investigate some of their topological paraperties. Keywords: I-Statistical Convergence, Ideal Convergence, Weighted Mean, Double Sequence, Sequence Space.


## References:

[1] H. Fast, "Sur la convergence statistique", Colloq. Math. 2(1951), 241-244.
[2] I. J. Schoenberg, "The integrability methods", Amer. Math. Monthly, 66(1959), 361-375.
[3] J. A. Fridy, "On statistical convergence", Analysis, 5(1985), 301-313.
[4] P. Kostyrko , M. Macaj , T. Salat, "I - convergence and extremmal I-limit points", Mathematica Slovaca 55 (2005) 443-464
[5] P. Kostyrko, M. Macaj, T. Salat, M. Sleziak , "I - convergence and extremmal I-limit points", Mathematica Slovaca 55 (2005) 443-464
[6] S. Konca, "Some new ideal convergent sequence spaces and weighted lacunary I-statistical convergence", 2016, Submitted.
[7] S. Konca, M. Basarir, "Riesz lacunary almost convergent double sequence spaces defined by Orlicz functions", Facta Universitatis Ser.Math. Inform. 31 (1), 2016.
[8] S. Konca, "Weighted lacunary statistical convergence of double sequences in locally solid Riesz spaces", Filomat, 2016, preprint.

# Linear Combinations of L-Functions Satisfying the Same Riemann-Type of Functional Equation 

Dorin Ghisa<br>Department of Mathematics, Glendon College, York University<br>Toronto, Canada<br>dghisa@yorku.cam


#### Abstract

Linear combinations of L-functions satisfying Riemann-type of functional equations do not preserve in general this property. One important example in which the property is preserved is that of the Davenport and Heilbronn function, which represents a very particular linear combination of two very particular L-functions satisfying different functional equations. Its importance resides in the fact that off critical line non trivial zeros have been found for this function. In this paper are showing that infinitely many similar functions can be obtained and the question of multiplicity of the zeros of those functions is answered. On the other hand, if some linearly independent Lfunctions satisfy the same Riemann-type of functional equation, then any linear combination of them will satisfy the respective equation. The following question is answered. Suppose that all those functions satisfy RH. Is it possible to find a linear combination of them exhibiting off critical line non trivial zeros?


# Common Fixed Points of $(\alpha, \beta)$-Implicit Graph Contraction via Cyclic Admissible Pair in Modular Spaces 

Ekber Girgin, Mahpeyker Ozturk<br>Department of Mathematics, Sakarya University, Sakarya, Turkey girginekber@hotmail.com, mahpeykero@sakarya.edu.tr.


#### Abstract

Motivated by Aydi et al. (RACSAM, 109:367-384, 2015), we introduce the notion of $(\alpha, \beta)$-implicit graph contraction in the setting of modular spaces. Further, the obtained results encompass various generalizations of implicit contractions. Keywords: Implicit Contraction, Cyclic Admissible Pair, Common Fixed Point. References: [1] V. Popa, "Fixed point theorems for implicit contractive mappings", Stud. Cercet. Ştiint., Ser. Mat .Univ. Bacau, 7:129-133, 1997. [2] H. Aydi, M. Jellali, E. Karapınar, Common fixed points for generalized $\alpha$ implicit contractions in partial metric spaces: Consequences and application, RACSAM, 109:367-384, 2015. [3] M. Öztürk, M. Abbas , E.Girgin, " Fixed points of mappings satisfying contractive condition of integral type in modular spaces endowed with a graph", Fixed Point Theory and Applications, 2014, 2014:220, doi:10.1186/1687-1812-2014-220. [4] M. Öztürk, M. Abbas, E.Girgin, "Common fixed point results of a pair of generalized ( $\psi, \varphi$ )-contraction mappings in modular spaces", 2016, 2016:19, doi: 10.1186/s13663-016-0503-x.


# Improvement of a Secure Authentication Scheme for Session Initiation Protocol Based on ECC 

Debasis Giril ${ }^{1}$, Tanmoy Maitra ${ }^{2}$ and P. D. Srivastava ${ }^{3}$<br>${ }^{l}$ Department of Computer Science and Engineering<br>Haldia Institute of Technology, Haldia-721657, India<br>${ }^{2}$ Department of Computer Science and Engineering Jadavpur University, Kolkata, West Bengal-700032, India<br>${ }^{3}$ Department of Mathematics<br>Indian Institute of Technology Kharagpur<br>Kharagpur-721302, India<br>debasis_giri@hotmail.com, tanmoy.maitra@live.com, pds@maths.iitkgp.ernet.in


#### Abstract

In 2015, Chaudhry et al. proposed asmart card based authentication scheme to remove the security loopholes of Tu et al.'s scheme. However, this paper shows that Chaudhry et al.'s scheme has inefficient login and password change phases, and does not preserve user's anonymity. Therefore, this paper proposes an improved scheme to eliminate the aforementioned drawbacks. After performing analysis, this paper states that the proposed scheme is efficient and has batter tradeoff among several measurement costs along with security.


Keywords: Smart Card, Password, Attack, Authentication, Elliptic Curve, Session Initiation Protocol.

## References:

[1] M.G. Martine, E. Henrikson, and D. Mills,"Private header (P-header) extensions to the session initiation protocol (SIP) for the 3rd-generation partnership project (3GPP)", IETF RFC3455, 2003.
[2]T. Maitra, and D. Giri,"An efficient biometric and password-based remote user authentication using smart card for telecare medical information systems in multi-server environment", Journal of Medical Systems,38.12(2014),doi:10.1007/s10916-014-0142-x.
[3] R. Amin, T. Maitra, and S.P. Rana,"An improvement of wang.et. al.'s remote user authentication scheme against smart card security breach", International Journal of Computer Applications, 75.13(2013), 37-42.
[4] D. Giri, T. Maitra, R. Amin, and P.D. Srivastava,"An efficient and robust rsabased remote user authentication for telecare medical information systems", Journal of Medical Systems, 39.1(2014), doi:10.1007/s10916-014-0145-7.
[5] R. Amin, T. Maitra and D.Giri, "An Improved Efficient Remote User Authentication Scheme in Multi-server Environment using Smart Card", International Journal of Computer Applications, 69.22(2013), 1-6.
[6] N.Koblitz, "Elliptic curve cryptosystems", Mathematics of Computation, 48.177(1987), 203-209.


## Lower Envelopes in Vector Spaces

Nihat Gokhan Gogus<br>Sabancı University, Istanbul, Turkey nggogus@sabanciuniv.edu


#### Abstract

The lower envelopes of certain functions appear quite naturally in functional analysis, extremal and dual extremal problems, optimization, in the theory of uniform algebras and in potential theory. This notion was used in [1] in a very special form in the context of pluripotential theory. We investigate the continuity properties of lower envelopes in the abstract setting of infinite dimensional spaces. One can start with any set in a topological space $A$ and assign to each point $a$ in this set a fiber $J_{a}$, that is, a class of elements from the dual space $X^{*}$ of some vector space $X$. Then one can construct a new function on $A$ via lower envelopes by taking the infimum over all numbers of the form Re $\left(x^{*}, x\right)$, where $x$ in $X$ is fixed, and $x^{*}$ changes over the fiber $J_{a}$ for any $a$ in $A$. Let $\left(x^{*}, x\right)_{a}$ be the optimal value. For example, one can think of $x$ as a function which we minimize subject to some set of constraints $J_{a}$, where $a$ runs in some sample space $A$. We completely characterize conditions which guarantee global or local continuity in $A$ of these optimal values $\left(x^{*}, x\right)_{a}$.


Keywords: Extremal and Dual Extremal Problems, Lower Envelopes.

## References:

[1] N. G. Göğüş, Continuity of plurisubharmonic envelopes, Ann. Polon. Math. 86 (2005), no. 3, 197-217.

# Matrix Operators on the Series Space $\left|\bar{N}_{p}^{\boldsymbol{\theta}}\right|(\mu)$ and Applications 

Fadime Gökce, Mehmet Ali Sarıgol<br>Department of Mathematics, Pamukkale University, Denizli, Turkey<br>fgokce@pau.edu.tr


#### Abstract

In the study, we introduce the notion of generalized absolute summability method which includes almost well known summabilities. By showing that series space $\left|\overline{\mathrm{N}}_{\mathrm{p}}^{\theta}\right|(\mu)$ is a FK-space with AK-property with respect to its natural paranorm. We characterize some matrix operators on the space.


Keywords: Absolute Weighted Summability, Matrix Transformations, Sequence Space, Bounded Operators.

## References:

[1] Boos, J. and Cass, P., "Classical and modern methods in summability", Oxford University Press, New York, (2000)..
[2] Grosse-Erdmann, K.G., "Matrix transformations between the sequence spaces of Maddox", J. Math. Anal. Appl. 180 (1993) 223--238..
[3] Maddox, I.J.," Elements of functinal analysis", Cambridge University Press, London, New York, (1970)
[4] Maddox, I.J., "Some properties of paranormed sequence spaces", J. London Math. Soc. 1 (1969), 316-322
[5] Maddox, I.J., "Paranormed sequence spaces generated by infinite matrices", Proc. Cambridge Philos. Soc. 64 (1968), 335-340.
[6] Malkowsky, E. and Rakocevic, V., "On matrix domains of triangles", Appl. Math. Comp. 189 (2) (2007), 1146-1163.
[7]Orhan, C. and Sarıgöl, M.A., "On absolute weighted mean summability", Rocky Moun. J. Math. 23 (3), (1993), 1091-1097.
[8]Sarıgöl, M.A., "Spaces of series summable by absolute Cesàro and matrix operators", Comm. Math. Appl. (to appear).
[9] Sarıgöl, M.A., "An inequality for matrix operators and its applications", Journal of Classical Analysis, 2 (2013), 145-150.
[10] Sarıgöl, M.A., "Matrix transformations on fields of absolute weighted mean summability", Studia Sci. Math. Hungar., 48 (3), (2011), 331-341.
[11] Sarıgöl, M.A., "On some absolute summability methods", Bull. Calcutta Math. Soc., 83 (1991), 421-426.

# Numerical Investigation on the Effect of the Rotation Intensity of a Tornadic Wind 

Xixiong Guo, Jun Cao<br>Department of Mechanical and Industrial Engineering, Ryerson University, Toronto, Ontario, Canada<br>jcao@ryerson.ca


#### Abstract

A tornadic wind is essentially considered as an airflow that simultaneously translates and rotates. Numerical simulations of this kind of hybrid flow remain inadequate due to many numerical difficulties, one of the major challenges consisting in the establishment of a set of boundary conditions that are, for the tornado-obstacle interaction scenario, both rational in physics and simple in numerical implementation. Inspired by the success of immersedboundary (IB) lattice Boltzmann method (LBM) for simulations of fluidstructure interaction problems, this study proposes a new outlet of the IB-LBM framework for investigation of tornadic wind effects, featuring a reformed interpretation of the Rankine-Combined Vortex Model (RCVM) that considerably facilitates the boundary condition setup. Moreover, the main purpose of this study is to examine the tornadic wind loadings in relation to the rotation intensity of a tornado, and presents a practical Newton's bi-section-like method for determining the critical rotation intensity beyond which the aerodynamic coefficients no longer increase when Reynolds number rises. This critical rotation intensity serves to characterize tornadic winds, such that the tornado with a rotation density below its critical value can be considered as mainly dominated by the translation part and, otherwise, the dominance no longer belongs only to the translational component of the tornado. Since it has been rather conventional that, when studying tornadoes, Reynolds number is determined using only the translation velocity as characteristic velocity, the present tornado study intends to suggest, through a number of numerical test series, that more attention be paid to the insufficiently explored rotational component, which physically tends to play a more dominant role when an intensive rotation is present in a tornado scenario.


Keywords: Tornadic Wind; Rankine-Combined Vortex Model; Immersed Boundary Lattice Boltzmann Method (IB-LBM); Rotation Intensity; Wind Loads; Aerodynamic Coefficients.

## References:

[1] Selvam R.P., Millett P.C.\} Computer modeling of tornado forces on buildings. Wind and Structures. 2003. 6(3),209-220

# Compact Finite Difference Solutions of Soybean Hydration Model as Stefan Problem 

Seda Gulen ${ }^{1,2}$,Turgut Ozis ${ }^{l}$<br>${ }^{l}$ Department of Mathematics, EgeUniversity, Bornova,İmir, Turkey<br>${ }^{2}$ Department of Mathematics,Namık Kemal University, Merkez,Tekirdağ,Turkey<br>${ }^{1}$ sedagulenn@gmail.com


#### Abstract

Many physical problems arising in engineering and science include volume variation or movement of system boundaries. The mathematical description of these problems can be expressed by moving boundary problems or Stefan problems. Soybean hydration model is considered as a Stefan problem with a one moving boundary or two moving boundaries. In this work, we present numerical solutions of soybean hydration model has one moving boundary which expresses the movement of radius and variable diffusivity coefficient. The solutions are obtained by using boundary immobilization method combined with compact finite difference schemes. Numerical results are compared Nicolin, et al. [1] and obtained good agreement with minimal computational effort.


Keywords: Finite Differences, Compact Schemes, Stefan Problem, Hydration Model

## References:

[1] D.J.Nicolin, G.E.C. da Silva, R.M.M.Jorge and L.M.M.Jorge, "Numerical Solution of a Nonlinear Diffusion Model for Soybean Hydration with Moving Boundary", Int. J. Food Eng., 11(2015), 587-595.
[2] S.K. Lele, Compact Finite Difference Schemes with Spektral-like Resolution, J. Comput. Physics., 103(1992), 16-42.
[3] J. Crank, Free and Moving Boundary Problems, Clerandon Press, 1984.


# Enneper-Type Surfaces in Three Dimensional Minkowski Space 

Erhan Guler, Semra Saracoglu Celik, Omer Kisi<br>Department of Mathematics, Bartın University, Bartın, Turkey<br>ergler@gmail.com, ssaracoglu@bartin.edu.tr,okisi@bartin.edu.tr


#### Abstract

In this work, we study on an Enneper-type minimal surfaces using Weierstrass representation in the three dimensional Minkowski space. We compute implicit equations, degree and class of the surfaces. Keywords: Enneper Surface, Minimal Surface, Weierstrass Representation, Degree, Class.


# A Banach-Stone Type Theorem for Isometries on $\boldsymbol{L}^{p}$ Spaces 

Banu Gunturk<br>Faculty of Engineering, Baskent University, Ankara, Turkey<br>bgunturk@baskent.edu.tr<br>Bahaettin Cengiz<br>Faculty of Engineering, Baskent University, Ankara, Turkey<br>bcengiz@baskent.edu.tr


#### Abstract

In this article we prove that for any positive measure $\mu$ and a real number $p, 1 \leq p<\infty, p \neq 2$, there is a one-to-one correspondence between the set of all surjective linear isometries on the Banach space $L^{p}(\mu)$ and the set of all onto homeomorphisms on the Stonean space of the measure algebra the given measure space determines. To be more specific, we extend the well-known Banach-Stone theorem for spaces of scalar continuous functions to $L^{p}$ spaces. Keywords: Isometries of $\boldsymbol{L}^{p}$ Spaces, Banach-Stone Theorem, Complete Boolean Algebra, Topological Homeomorphism.




# On Some Codes Over R2 

Murat Guzeltepe, Alev Altinel<br>Department of Mathematics, Sakarya University, Esentepe, Sakarya, Turkey<br>mguzeltepe@sakarya.edu.tr, alevaltinell@gmail.com


#### Abstract

Linear codes over rings have played an increasingly prominent role in the coding theory literature due to the fact that they are easier to construct, encode and decode since the publication of [1] classifying codes over $\square_{4}$. A method was given in [1] that it leads to a new direction in coding theory. Since then, many types of finite rings have been studied in coding theory by using the same method, i.e., by introducing a Gray map to obtain a connection between $\square_{4}$-codes and their binary images. Some of these publications are about $\square{2^{k}}^{-}$ linear codes [2], Type II codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}$ [3], etc. Also, cyclic codes over rings studied by many authors ([4], [5], [6]). All of the rings have a common case, which is all finite chain and also principal ideal ring. Studying over a finite chain give you a chance to define generator and check matrices for the codes in standard forms as in [7]. On the other hand, the ring $\mathrm{F}_{2}+u \mathrm{~F}_{2}+\nu \mathrm{F}_{2}+u \nu \mathrm{~F}_{2}$ have been studied in [8] and [9] aparts from the others since it is neither a finite chain nor principal. Let $R_{2}$ denotes the ring $\mathrm{F}_{2}+u \mathrm{~F}_{2}+v \mathrm{~F}_{2}+w \mathrm{~F}_{2}+u v \mathrm{~F}_{2}+u w \mathrm{~F}_{2}+v w \mathrm{~F}_{2}+u v w \mathrm{~F}_{2}$ throughout this paper. The ring $R_{2}$ has two important properties, which the ring is neither finite chain nor principal ideal ring. In this paper, linear and cyclic codes are studied over the ring $R_{2}$. Initially, the structure of the ring $R_{2}$ is examined and linear codes are defined over this ring. The ideal decomposition of linear codes over $R_{2}$ are obtained. Moreover, Lee weight and a Gray map are defined for these codes. The $R_{2}$-linearity of binary codes over the ring $R_{2}$ and the codes corresponding to pre-images of Reed Muller codes are given. Also, the dual codes and the complete weight enumerators are defined for $R_{2}$-linear codes. Furthermore, cyclic codes over $R_{2}$ are characterized when $n$ is odd.


## References:

[1] Hammons A. R., Kumar V., Calderbank A. R., Sloane N. J. A., Solé P., "The $\square_{4}$-linearity of Kerdock, Preparata, Goethals and related codes", IEEE Trans. Inform. Theory, 40, 301-319 (1994).
[2] Carlet C., " $\square_{2^{k}}$-linear codes", IEEE Trans. Inform. Theory, 44, 1543-1547 (1998).

[3] Dougherty S. T., Gaborit P., Harada M., Solé P., "Type II codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}$ ", IEEE Trans. Inform. Theory, 45, 32-45 (1999).
[4] Abualrub T., Siap I., "Cyclic codes over the rings $\square_{2}+u \square_{2}$ and $\square_{2}+u \square_{2}+u^{2} \square_{2}$ ", Des. Codes Crypt., 42, 273-287 (2007).
[5] Blackford T., "Cyclic codes over $\square_{4}$ of only even length", In: Proceddings of the international workshop on coding and cryptography, 83-92, WCC 2001, Paris, France (2001).
[6] Bonnecaze A., Udaya P., "Cyclic codes and self dual codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}$ ", IEEE Trans. Inform. Theory, 45, 1250-1255 (1999).
[7] Huffman W. C., "Decompositions and extremal Type II codes over $\square_{4}$ ", IEEE Trans. Inform. Theory, 44, 800-809 (1998).
[8] Yildiz B., Karadeniz S., "Linear codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}+v \mathrm{~F}_{2}+u v \mathrm{~F}_{2}$ ", Des. Codes Crypt., 54, 61-81 (2010). (DOI: 10.1007/s10623-010-9309-8)
[9] Yildiz B., Karadeniz S., "Cyclic codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}+v \mathrm{~F}_{2}+u v \mathrm{~F}_{2}$ ", Des. Codes Crypt., 58, 221-234 (2011). (DOI: 10.1007/s10623-010-9399-3)

Closure Algebras of Metric Spaces

Ahmet Hamal<br>Department of Mathematics, Ege University,<br>Bornova, lzmir, Turkey<br>ahmet.hamal@ege.edu.tr


#### Abstract

We will be mainly concerned with the Theorem of McKinsey and Tarski in [3] which says: The closure algebra of a separable metric space with no isolated points is dissectable. We explain that the proof given in [3] for this theorem has a gap. Then, we give the proofs of a more general theorem in [5] and a special case of the theorem in [4] by use of a simple and precise language. Keywords: Separable Metric Space, Dissectable Closure Algebra, Isolated Point.

\section*{References:} [1] R. Engelking, General Topology, Heldermann, Berlin, 1989. [2] A. Hamal, On the logic of closure algebra, Bull. Sect. Logic Univ. Lodz 40 (2011), no. 3-4, 147-163. [3] J. C. C. McKinsey and A. Tarski, The algebra of topology, Annals of Mathematics, 45 (1944), pp.141-191. [4] A. Tarski, Der Aussagenkalkül und die Topologie, Fund. Math., 31 (1938), pp.103-134. [5] H. Rasiowa and R. Sikorski, The Mathematics of Metamathematics, Panstwowe Wydawnictwo Naukowe, (1963).




# Positive Solutions for Semipositone Fractional Boundary Value Problems 

Nuket Aykut Hamal, Fulya Yoruk Deren, Tugba Senlik Cerdik<br>Department of Mathematics, Ege University, Izmir, Turkey<br>nuket.aykut@ege.edu.tr, fulya.yoruk@ege.edu.tr, tubasenlik@gmail.com


#### Abstract

This paper deals with the existence of positive solutions for boundary value problems of semipositone fractional differential equations. The arguments are based upon a fixed point theorem in cones.


Keywords: Fractional differential equations, positive solutions, semipositone.

## References:

[1] D. Foukrach, T. Moussaoui, S. Ntouyas, "Existence of positive solutions for semipositone fractional boundary value problems'", Journal of Fractional Calculus and Applications, Vol. 5(1) Jan. 2014, pp. 85-96.
[2] R. Ma, "Positive Solutions for Semipositone m-point Boundary-value Problems", Acta Mathematica Sinica, 2004, Vol.20, No.2, pp. 273-282.
[3] C. Zhai, C. Yang, "Multiple Positive Solutions for Semipositone m-point Boundary-value Problems", Acta Mathematicae Applicatae Sinica, Vol.27, No. 3 (2011) 419-426.
[4] X. Li, S. Liu, W. Jiang, "Positive solutions for boundary value problem of nonlinear fractional functional differential equations", Applied Mathematics and Computation, 217 (2011) 9278-9285.


# On $\boldsymbol{k}$-Quasi Class $\boldsymbol{Q}^{*}$ Operators 

Valdete Rexhëbeqaj Hamiti, Shqipe Lohaj and Qefsere Gjonbalaj<br>Faculty of Electrical and Computer Engineering, University of Prishtina "Hasan Prishtina", Prishtinë, 10000, Kosova<br>valdete_r@hotmail.com, shqipe.lohaj@uni-pr.edu, qefsere.gjonbalaj@uni-pr.edu


#### Abstract

Let $T$ be a bounded linear operator on a complex Hilbert space $H$. In this paper we introduce a new class of operators: $k$-quasi class $Q^{*}$ operators. An operator $T$ is said to be $k$-quasi class $Q^{*}$ if it satisfies $\left\|T^{*} T^{k} x\right\|^{2} \leq$ $\frac{1}{2}\left(\left\|T^{k+2} x\right\|^{2}+\left\|T^{k} x\right\|^{2}\right)$, for all $x \in H$, where $k$ is a natural number. We prove the basic properties of this class of operators. We proved that $T$ is operator of $k$-quasi class $Q^{*}$ if and only if $T^{* k}\left(T^{* 2} T^{2}-2 T T^{*}+I\right) T^{k} \geq 0$. We give the relation between this class of operator and the class of $k$-quasi $-*-$ paranormal operator, the relation between this class and the class $Q^{*}$ operator. Morever, we proved that, for every operator $T$ if $\left\|T^{*}\right\| \leq \frac{1}{\sqrt{2}}$, then $T$ is operator of $k$-quasi class $Q^{*}$.


Keywords: $k$-Quasi Class $Q^{*}, k-$ Quasi $-*-$ Paranormal, Class $Q^{*}$. References:
[1] S. C. Arora and J. K. Thukral, "On a class of operators",Glasnik Math. 21(41) (1986), 381-386.
[2] D. Senthilkumar, P. MaheswariNaik and D. Kiruthika,"Quasi class $Q^{*}$ composition operators", International Journal of Math. Sci. and Engg. Appls. (IJMSEA), ISSN 09739424, Vol. 5, No IV, July, 2011, pp. 1-9.
[3] B. P. Duggal, C. S. Kubrusly, and N. Levan, "Contractions of class $Q$ and invariant subspaces", Bull. Korean Math. Soc. 42(2005), No. 1, pp. 169-177.
[4] T. Furuta, "On The Class of Paranormal Operators", Proc. Jap. Acad. 43(1967), 594598.
[5] J. K. Han, H. Y. Lee, and W. Y. Lee, "Invertible completions of $2 \times 2$ upper triangular operator matrices", Proceedings of the American Mathematical Society, vol. 1282000, 119-123.
[6] Ilmi Hoxha and Naim L. Braha, "A note on $k$-quasi - *-paranormal operators", Journal of Inequalities and Applications 2013, 2013:350.
[7] Salah Mecheri, "Bishops property $\beta$ and Riesz idempotent for $k$-quasi -* -paranormaloperators", Banach J. Math. Anal., 6(2012), No. 1, 147-154.
[8] Salah Mecheri, "On quasi $-*$-paranormal operators", Banach J. Math. Anal., 3(2012), No. 1, 86-91.
[9] S. M. Patel, "Contributions to the study of spectraloid operators", Ph. D. Thesis, Delhi University 1974.
[10] D. Senthilkumar and T. Prasad, "Composition Operators of Class $Q^{* *}$, Int. Journal of Math. Analysis, Vol. 42010, no. 21, 1035-1040.
[11] T. Veluchamy and A. Devika, "Some properties of quasi $-*-$ paranormal operators", Journal of Modern Mathematics and Statistics, 1 (1-4), 35-38, 2007.

# General Robin Boundary Value Problems for Elliptic Operational Differential Equations with Variable Operators 

Rabah Haoua and Ahmed Medeghri<br>Laboratoire de Mathématiques Pures et Appliquées, Université de Mostaganem 2700, Algérie<br>Haoua.rabah27@gmail.com ahmed_medeghri@yahoo.com


#### Abstract

In this paper we give some new results on abstract second order differential equation of elliptic type with variable operator coefficients and general Robin boundary conditions, in the framework of Hölder spaces. Here, we do not assume the differentiability of the operators resolvent. However, we suppose that the family of variable operators verifies the Labbas-Terreni assumption inspired by the sum theory and similar to the Acquistapace-Terreni one. We use Dunford calculus, interpolation spaces and semigroup theory in order to obtain existence, uniqueness and maximal regularity results for the solution of the problem.


Keywords: Abstract Differential Equation of Elliptic Type, Robin Boundary Conditions, Analytic Semigroup, Maximal Regularity, Dunford Operational Calculus.

## References:

[1] A. V. Balakrishnan.: Fractional Powers of Closed Operators and the Semigroups Generated by them. Pacif. J. Math. 10 (1960), 419-437.
[2] M. Cheggag, A. Favini, R. Labbas, S. Maingot and A. Medeghri.: Abstract differential equations of elliptic type with general Robin boundary conditions in Hölder spaces, Applicable Analysis, Vol, 91, No. 8, August 2012, 1453-1475. [3] R. Labbas.: Problèmes aux Limites pour une Equation Différentielle Abstraite de Type Elliptique, Thèse d'état, Université de Nice, 1987.

# Solutions of Fuzzy Fractional Heat-Like and Wave-Like Equations by Variational Iteration Method 

Atimad Harir, Said Melliani, L. Saadia Chadl<br>Department of Mathematics, Faculty of Sciences and Technics University,<br>Beni Mellal, Morocco<br>atimad.harir@gmail.com


#### Abstract

This paper applies the sufficient condition for the Buckley-Feuring solution to exist by the variation iteration method are used for find the exact fuzzy solution of the fuzzy fractional heat-like and wave-like equations with variable coefficients and fuzzy parameters. Some examples are given to show the reliability and the efficiency of the sufficient condition.


Keywords: Fuzzy Fractional Heat-Like and Wave-Like Equations, Variational Iteration Method, Fuzzy Number.

## References:

[1] S. Arshad, V. Lupulescu, On the fractional differential equations with uncertainty, Nonlinear Analysis 74 (2011) 3685--3693
[2] J. J. Buckley, Y. Qu, \{solving fuzzy equations: a new solution concept, \} Fuzzy Sets and Systems \{50\} (1992) 1--14.
[3] J. J. Buckley, T. Feuring, \{Fuzzy differential equations,\} Fuzzy Sets and Systems \{ 110\} (2000) 43--54.
[4] J. J. Buckley, T. Feuring, \{Introduction to fuzzy partial differential equations, \} Fuzzy Sets and Systems \{105\} (1999) 241--248.
[5] L.S.Chadli, A.Harir, S.Melliani, \{Solutions of fuzzy wave-like equations by variational iteration method,\} in Ann. Fuzzy Math. Inform., Vol. 8, No 4, (October 2014), 527-547.
[6] L.S.Chadli, A.Harir, S.Melliani, \{Exact fuzzy solution of the fuzzy heat-like equations, \} in Ann. Fuzzy Math. Inform., Vol. 10, No 1, (July 2015), 29-44.
[7] G. Wu, E.w.M.Lee,\{Fractional variational iteration method and its application, \} Physics Letters A \{374\} (2010) 2506--2509

# Absolute Cesàro Series Spaces and Matrix Operators 

G. Canan Hazar, M. Ali Sarıg̈l<br>Department of Mathematics, Pamukkale University, Denizli, Turkey<br>gchazar@pau.edu.tr,msarigol@pau.edu.tr.


#### Abstract

In this paper we derive a series space $\left|C_{\lambda, \mu}\right|_{k}$ using the well known absolute Cesàro summability $|C, \lambda, \mu|_{k}$ of Das [2], compute its $\beta$-dual, give some algebraic and topological properties, and characterize matrix operator defined on that space. So we generalize some results of Bosanquet [1], Flett [3], Mehdi [6], Orhan and Sarıgöl [10] and Sarıgöl [7,9]. Keywords: Sequence Spaces, Absolute Cesàro Summability, Matrix Transformations, Dual Spaces, BK Spaces.


## References:

[1] Bosanquet, L.S., Note on convergence and summability factors I, J. London Math. Soc. 20 (1945), 39-48.
[2] Das, G., A Tauberian theorem for absolute summability, Proc. Cambridge Philos. 67 (1970), 321-326.
[3] Flett, T.M., On an extension of absolute summability and some theorems of Littlewood and Paley, Proc. London Math. Soc. 7 (1957), 113-141.
[4] Kogbetliantz, E., Sur lesseries absolument sommables par la methods des moyannes arithmetiques, Bull. des Sci. Math. 49 (1925), 234-256.
[5] Maddox, I.J., Elements of functional analysis, Cambridge University Press, London, New York, (1970).
[6] Mehdi, M.R., Summability factors for generalized absolute summability I, Proc. London Math. Soc.(3), 10 (1960), 180-199.
[7] Sarıgöl, M.A., Absolute Cesàro summability spaces and matrix operators on them, Comm. Math. Appl. (in press).
[8] Sarıgöl, M.A., Extension of Mazhar's theorem on summability factors, Kuwait Journal of Sciences, 42(2), (2015), 28-35.
[9] Sarıgöl, M.A., Matrix operators on A_\{k\}, Math. Comp. Model. 55 (2012), 17631769.
[10] Orhan, C. and Sarıgöl, M.A., On absolute weighted mean summability, Rocky Moun. J. Math. 23 (3), (1993), 1091-1097.
[11] Sarıgöl, M.A., On two absolute Riesz summability factors of infinite series, Proc. Amer. Math. Soc., 118, (1993), 485-488.
[12] Peyerimhoff, A., Summierbarkeitafaktoren für absolut Cesàro-summiarbare Reiben, Math. Z., 59 (1954),417-424.
[13] Stieglitz, M. and Tietz, H., Matrixtransformationen von Folgenraumen Eine Ergebnisüberischt, Math Z., 154 (1977), 1-16.
[14] Wilansky, A., Summability Through Functional Analysis, North-Holland Mathematical Studies, vol. 85, Elsevier Science Publisher, 1984.

# Some New Numerical Approximations for Time Fractional Schrödinger Equations 

Betul Hicdurmaz<br>Department of Mathematics, Istanbul Medeniyet University, Uskudar, Istanbul, Turkey<br>betul.hicdurmaz@medeniyet.edu.tr


#### Abstract

In the present study, a Time Fractional Schrödinger Differential Equation (TFSDE) is considered. Here, it is important to note that increasing number of computational studies on TFSDE is not a surprise due to its physical applications in quantum mechanics. However, exact solutions of TFSDEs can be given only in terms of special functions and special functions are not easy to analyze or compute. So, numerical approximations for TFSDEs are studied more frequently in recent years ([1]-[3]). In the present study some stable numerical approximations are established for a TFSDE using finite difference method. Keywords: Finite Difference Scheme, TFSDE, Stability. References: [1] R. Garrappa, I. Moret, M. Popolizio, "Solving the time-fractional Schrödinger equation by Krylov projection methods", J. Comput. Phys., 293 (2015), 115-134. [2] N. J. Ford, M. M. Rodrigues, N. Vieira, "A numerical method for the fractionalpe equation of spatial dimension two", Fract. Calc. Appl. Anal., 16 (2013), 2, 454-468. [3] A. Ashyralyev, B. Hicdurmaz, "On the numerical solution of fractional Schrödinger differential equations with the Dirichlet condition", Int. J. Comput. Math., 89 (2012), 1927-1936.




# New Numerical Approximations for the Nonlinear Population Model 

Betul Hicdurmaz<br>Department of Mathematics, Istanbul Medeniyet University, Uskudar, Istanbul, Turkey<br>betul.hicdurmaz@medeniyet.edu.tr<br>Emine Can<br>Department of Physical Engineering, Istanbul Medeniyet University, Uskudar, Istanbul, Turkey emine.can@medeniyet.edu.tr


#### Abstract

Different population models are considered with different numerical approaches in the literature. The present study aims to present a new and efficient numerical method for the nonlinear population model which is based on the polynomial approximations via a pseudospectral approach. Keywords: Population Dynamics, Polynomial Approximation, Pseudospectral Method.


## References:

[1] M. Dehghan, M. Shahini, "Rational pseudospectral approximation to the solution of a nonlinear integro-differential equation arising in modeling of the population growth", Appl. Math. Model., 39 (2015), 5521-5530.
[2] K. G. Tebeest, "Numerical and Analytical solutions of Volterra's population model", SIAM Rev. 39 (1997), 3, 484-493.
[3] F. M. Scudo, "Vito Volterra and theoretical ecology", Theor. Popul. Biol. 2 (1971), 1, 1-23.


# Conbinatoric Bijection 

Fadila Hocini, Laboratory of Applied Mathematics, Department of Mathematics A/Mira University, Bejaia, Algeria fadilahocinie@hotmail.fr


#### Abstract

Our work focus in enumerative combinatoric to determinate card of set of sequence given by $(\mathrm{n}+1)$ terms $\left(\mathrm{x}_{\mathrm{i}}\right)_{\mathrm{i} \geq 0}$ such as $\mathrm{x}_{0}=\mathrm{x}_{\mathrm{n}}=0$, and $\mathrm{d}_{\mathrm{i}}=\mathrm{x}_{\mathrm{i}}-\mathrm{x}_{\mathrm{i}-1}$ for $i \in\{-1,0,1\}$, we construct one bijection between two particulars sets. Set of forets (graph) and set of paths (graph).


Keywords: Combinatoric; Foret Graph; Path Graph.

# To Compute Topological Complexity Numbers Using Steenrod Squares 

Melih Is* and Ismet Karaca<br>Department of Mathematics, Ege University, Bornova, Izmir, Turkey melihis2010@gmail.com ismet.karaca@ege.edu.tr


#### Abstract

Topic of this poster is about topological robotics [3] that closely related with algebraic topology [7]. We first introduce the concept of 'topological complexity' number [2], and then give some examples about topological complexity of certain topological structures. One can recall that topological complexity is a numerical homotopy invariant and similar in spirit to the Lusternik-Schnirelmann category ' $\operatorname{cat}(\mathrm{X})$ ' [1]. We also state some examples about computing topological complexity with using $\operatorname{cat}(\mathrm{X})$. After we associate topological complexity number to cohomology operations [4, 5] in this poster, we apply to computing topological complexity some properties of steenrod squares on topological robotics.


Keywords: Topological Complexity, Steenrod Operations, Motion Planning Algorithm, Configuration Spaces, Lusternik-Schnirelmann Theory.

## References:

[1] Cornea, O., Lupton, G., Oprea, J., and Tanre, D. (2003) ' Lusternik Schnirelmann category, Vol. 103 of Math. Surveys Monogr., Providence, RI, Amer. Math. Soc.
[2] M.Farber, Topological Complexity of Motion Planning. Discrete and Computational Geometry 29 (2003), 211-221.
[3] M. Farber, Invitation to Topological Robotics, Zurich lectures in Advanced Mathematics, E. M. S. (2008).
[4] M. Farber and M. Grant, Robot motion planning, weights of cohomology classes, and cohomology operations, Proc. Amer. Math. Soc. 136 (2008), 33393349.
[5] M.Grant, Topological complexity, fibrations and symmetry, Topology Appl. 159 (2012), 88-97.
[6] Robert E. Mosher and Martin C. Tangora, Cohomology operations and applications in homotopy theory, Harper \& Row, Publishers, New York-London, 1968.
[7] J. R. Munkres, "Elements of Algebraic Topology", Persues Books Publishing, (1984).


# On Darboux Vector in Lorentzian 5-Space 

Esen Iyigun<br>Department of Mathematics, Uludag University, Görükle, Bursa, Turkey esen@uludag.edu.tr


#### Abstract

In this work, we introduce Darboux vector in Lorentzian 5-space. We give some characterizations of the vector in the space. Also, we consider some special cases in the space.


Keywords: Non-Null Curve, Darboux Vector, Lorentzian 5-Space.

## References:

[1] B. O'neill, "Semi-Riemannian Geometry with Applications to Relativity", Academic Pres. New York, 1983.
[2] E. İyigün and K. Arslan, "On harmonic curvatures of curves in Lorentzian nspace", Commun. Fac. Sci.Univ. Ank. Series A1. 54.1(2005), 29-34.
[3] G. Öztürk, K. Arslan and H. H. Hacısalihoğlu, "A characterization of ccrcurves in $\mathrm{R}^{\mathrm{m} "}$, Proceedings of the Estonian Academy of Sciences, 57.4 (2008), 217-224.
[4] R. Uribe-Vargas, "On Singularities, 'Perestroikas' and diferential geometry of space curves", L'Enseigement Mathematique, 50(2004), 69-101.
[5] M. Turgut, , J.L. Lopez-Bonilla and S. Yılmaz, "On Frenet-Serret invariants of non-null curves in Lorentzian Space L ${ }^{5}$ ", World Academy of Science, Engineering and Technology, 55(2009), 638-640.
[6] H. Kocayiğit, G. Öztürk, B. K. Bayram, B. Bulca and K. Arslan, "Characterization of curves in $\mathrm{E}^{2 \mathrm{n}+1}$ with 1-type Darboux vector", Mathematica Moravica, 17.2(2013), 29-37.

# Weak Module ( $\sigma, \tau$ )-Amenability of Triangular Banach Algebras of Order Three 

Hulya Inceboz, Berna Arslan<br>Department of Mathematics, Adnan Menderes University, Aydin, Turkey<br>hinceboz@adu.edu.tr, byorganci@adu.edu.tr


#### Abstract

The notion of module amenability for a class of Banach algebras, which could be considered as a generalization of Johnson's amenability, was introduced by M. Amini in [1]. The weak module amenability of triangular Banach algebras of the form $$
{ }_{2}=\left\{\left[\begin{array}{ll} a & m \\ 0 & b \end{array}\right]: a \in A, b \in B, m \in M,\right.
$$ where A and B are Banach algebras and M is a Banach $\mathrm{A}, \mathrm{B}$-module, is studied by A. Pourabbas and E. Nasrabadi in [2], and they showed that the weak module amenability of $T_{2}$ is equivalent with the weak module amenability of the corner algebras A and B .

In this work, we investigate the structure of the first module $(\sigma, \tau)$ cohomology group of the triangular Banach algebra T of order three with coefficients in its dual space $\mathrm{T}^{*}$. Also we give some theorems related with the module $(\sigma, \tau)$-amenability and weak module $(\sigma, \tau)$-amenability of the triangular Banach algebras of order three, and get some results for semigroup algebras.


Keywords: Triangular Banach Algebras of Order Three, Weak Module ( $\sigma, \tau$ )Amenability, Inverse Semigroups.

## References:

[1] M. Amini, "Module amenability for semigroup algebras", Semigroup Forum, 69(2004), 243-254.
[2] A. Pourabbas and E. Nasrabadi, "Weak module amenability of triangular Banach algebras", Math. Slovaca, 61.6(2011), 949-958.

# Theoretical and Numerical Methods for Two-Phase Flow Modeling 

Hyeonseong Jin<br>Department of Mathematics, Jeju National University, Jeju, South Korea<br>hjin@jejunu.ac.kr


#### Abstract

We are concerned on governing equations and numerical methods describing turbuent fluid mixing behavior effectively. We propose a multiphase closure model for turbulent mixing of compressible fluids. The model closures are validated in a numerical study. Also we study numerical methods of macro and micro phenomena for turbulent fluid mixing driven by acceleration forces. We propose methods for verification and validation of simulations for chaotic, multiscale flows.


Keywords: Multiphase Flow, Turbulence, Numerical Simulations, Averaged Equations

## References:

[1] W. Bo, H. Jin, D. Kim, X, Liu, H. Lee, N. Pestieau, Y. Yu, J. Glimm and J. Grove. Comparison and validation of multi-phase closure models. Comput. Math Appl. 56: 1291-1302, 2008.
[2] H. Jin and J. Glimm. Weakly compressible two-pressure two-phase flow. Acta Mah. Sci. 29B(6):1497-1540, 2009.
[3] H. Jin, J. Glimm and D.H. Sharp. Compressible two-pressure two-phase flow models. Phys. Lett. A. 353: 469-474, 2006.
[4] H. Jin, J. Glimm and D.H. Sharp. Entropy of averaging for compressible twopressure two-phase flow models. Phys. Lett. A. 360: 114-121, 2006.

# What is a Multiset? 

Helmut Jürgensen<br>Department of Computer Science and Mathematics, The University of Western Ontario, London, Canada, hij@csd.uwo.ca


#### Abstract

A set is a collection of distinct elements - what is a multiset? It is a set in which an element can occur several times, a set in which multiple copies of an element can occur. But that is not a set! The notion of a multiset is useful in many practical situations, but contravenes established mathematical formalisms. In its simplest version, a multiset is specified by a set of pairs $\left\langle x, m_{x}\right\rangle$ where $x$ belongs to some basic underlying set $X$ and $m_{x}$ is the number of occurrences of $x$ in the multiset. This definition does not distinguish between different occurrences of the element $x$ in the multiset (for instance, there can be 3 students called Peter in a class; they are individuals, but indistinguishable when the class is considered as a multiset). The multiplicity is usually assumed to be a non-negative integer. Multisets are often also called bags in the literature. Multisets in the usual sense are similar to fuzzy sets and finite probability spaces. Multiplicities are really cardinal numbers -- not just non-negative integers. Operations on multisets need to take this property into account. For finite multiplicities this is not a problem, except when one tries to define the complement of a multiset. For infinite multiplicities, however, even the operations of intersection and union pose a difficulty. To cope with infinite multiplicities and with levels of distinguishability, we propose a substantially different definition of multisets, using families, that is, sets and mappings, as the basic concept. This framework works even when the multiplicity of an element is an arbitrarily large cardinal number. A special case of it reduces to the usual concept, when only finite multiplicities are considered. The need for a non-standard theory of multisets -- including the case of infinite multiplicities -- arose in our recent research on biologically based computations where several levels of differentiation between individuals in a multiset were required. We base our definition of multisets on the notion of universe (e.g. in the sense of Grothendieck) and the resulting notions of set, mapping, and family. In this general setting "multiplicity" is a derived notion. We outline options for the definition of operations on multisets and relations between multisets with appropriate examples. We formulate an intuitively motivated set of axioms concerning multisets and show that our concepts satisfy the axioms. The common concept of multisets forms a simple special case.




Keywords: Multisets, Bags, Set theory.

## References:

[1] C. Alsina, M.J. Frank, B. Schweizer: Problems on associative functions. Aequationes Math. 66 (2003), 128--140.
[2] I. Angelelli: Leibniz's misunderstanding of Nizolius' notion of 'multitudo'. Notre Dame J. Formal Logic 6 (1965), 319--322.
[3] K. Beyer et al.: Distinct-value synopses for multiset operations. Comm. ACM 52 (2009), 87--95.
[4] W.D. Blizard: Multiset theory. Notre Dame J. Formal Logic 03 (1989), 36-66. Negative membership. Notre Dame J. Formal Logic 31 (1990), 346--368. The development of multiset theory. Modern Logic 1 (1991), 319-352, 2 (1991), 219, 7 (1997), 434. Dedekind multisets and function shells. Theoret. Comput. Sci. 110 (1993), 79--98.
[5] A. Grothendieck, J.L. Verdier: Prefaisceaux. Lecture Notes in Mathematics 269, 1--217. Springer-Verlag, Berlin, 1972.
[6] J. Hickman: A note on the concept of multiset. Bull. Austral. Math. Soc. 22 (1980), 211--217.
[7] J. Lake: Sets, multisets and functions. J. London Math. Soc. 12 (1976), 323-326.
[8] G.P. Monro: The concept of multiset. Zeitschr. f. math. Logik und Grundlagen d. Math. 33 (2) (1987), 171--178.
[9] V.N. Red'ko et al.: The current state of multiset theory from an essential viewpoint. Kibernetika (2015) (1), 171--178, in Russian. English translation: Cybernetics 15 (1) (2015), 150--156.
[10] D. Singh et al.: An overview of the applications of multisets. Novi Sad J. Math. 37 (2007), 73--92.

# On the Exact Distribution of the Product of Two Independent Hypoexponential Random Variables 

Therrar Kadri<br>Department of Mathematics, Lebanese International University, Khyara, Lebanon<br>therrar@hotmail.com<br>Khaled Smaili<br>Department of Appled Mathematics, Lebanese University, Zahle, Lebanon<br>ksmeily@hotmail.com<br>Seifedine Kadry<br>School of Engineering, American University of the Middle East,<br>Eguaila, Kuwait<br>skadry@gmail.com<br>Ali El-Joubbeh<br>Department of Pure Mathematics, Lebanese University,<br>Zahle, Lebanon<br>ali_joubbeh@ul.edu.lb


#### Abstract

The product distributions are of interest in many areas of the sciences. The study of the product of same family and finding their exact density expression was examined by many authors. Some solved the problem and others gave approximations. In this paper, we consider the product of two independent Hypoexponential distributions which has many applications in stochastic PERT Network and many others. We find the exact expressions for the probability density function, the cumulative distribution function, moment generating function, the reliability function and hazard function, which was proved to be a linear combination of the K distribution. Finally, we will apply our results application in stochastic PERT Network.


Keywords: Product Distribution, Hypoexponential Distribution; Erlang Distribution; K Distribution; Probability Density Function; Cumulative Distribution Function; Reliability Function; Hazard Function.

## References:

[1] Abramowitz, Milton; Stegun, Irene A., eds. (1965), "Chapter 13", Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, New York: Dover, p. 504, ISBN 978-0486612720, MR 0167642
[2] Ali M. M, Pal M, and Woo J. On the Ratio of Inverted Gamma Variates. Austrian Journal of Statistic.

[3] Bhargava, R. P., Khatri, C. G. (1981). The distribution of product of independent beta random variables with application to multivariate analysis. Annals of the Institute of Statistical Mathematics 33 287-296.
[4] Bu-Salih, M. S. (1983). Distributions of the product and the quotient of power- function random variables. Arab Journal of Mathematics 4 77-90. 2007;36(2):153-159.
[5] Coelho C.A, Mexia J.T. On the Distribution of the Product and Ratio of Independent Generalized Gamma-Ratio. Sankhy ${ }^{-}$a: The Indian Journal of Statistics. 2007;69(2):221-255.
[6] Grimmett, G. R.; Stirzaker, D.R. (1982). Probability and Random Processes. New York: Wiley. ISBN 978-0471731351. Retrieved 24 September 2012.
[7] Harter, H. L. (1951). On the distribution of Walds classi.cation statistic. Annals of Mathematical Statistics 22 58-67.
[8] Jakeman, E. and Pusey, P. N. (1978) "Signi.cance of K-Distributions in Scattering Experiments", Physical Review Letters, 40, 546.550 doi:10.1103/PhysRevLett.40.546.
[9] Jakeman, E. and Tough, R. J. A. (1987) "Generalized K distribution: a statistical model for weak scattering," J. Opt. Soc. Am., 4, (9), pp. 1764-1772.
[10] Kadri, T., Smaili, K. (2015). Convolutions of Hyper-Erlang and of Erlang distributions. International Journal of Pure and Applied Mathematics, 98(1), 8198.
[11] Kadri T., Smaili, K. and Kadry S. (2014). Markov modeling for reliability analysis using Hypoexponential distribution. In Kadry, S., \& El Hami, A. Numerical methods for reliability and safety assessment. New York, USA: Springer.doi: 10.1007/978-3-319-07167-1_23
[12] Malik, H. J., Trudel, R. (1986). Probability density function of the product and quotient of two correlated exponential random variables. Canadian Mathematical Bulletin 29 413-418.
[13] Nadarajah S. The linear combination, product and ratio of Laplace random variables. Statistics. A Journal of Theoretical and Applied Statistics. 2007;41(6):535-545.
[14] Pham-Gia T. Turkkan N. Operations on the Generalized F-Variables, and Applications. A Journal of Theoretical and Applied Statistics. 2002;36(3):195209. doi:10.1080/02331880212855.
[15] Podolski, H. (1972). The distribution of a product of $n$ independent random variables with generalized Gamma distribution. Demonstratio Mathematica 4 119-123.
[16] Rathie, P. N., Rohrer, H. G. (1987). The exact distribution of products of independent random variables. Metron 45 235-245.
[17] Redding, N. J. (1999) Estimating the Parameters of the K Distribution in the Intensity Domain [1]. Report DSTO-TR-0839, DSTO Electronics and Surveillance Laboratory, South Australia.

# Finite Dimensional Chebyshev Subspaces of Classical Banach Spaces 

Aref K. Kamal<br>Department of Mathematics, Sultan Qaboos University,<br>Al Khoud 123, Muscat, Oman<br>akamal@squ.edu.om


#### Abstract

The set A of the normed linear space $X$ is said to be proximinal in $X$ if for each $x \in X$ there is $y_{0} \in A$ such that the distance $d(x, A)=\inf \{\|x-y\| ; y \in A\}=$ $\left\|x-y_{0}\right\|$. In this case $y_{0}$ is called a best approximation for x from A . The set A is called a Chebyshev subset of $X$ if for each $x \in X$, the best approximation for $x$ from A is unique. Finite dimensional Chebyshev subspaces were the center of attention of mathematician for some time. In this talk the speaker investigate the existence of finite dimensional Chebyshev subspaces in some classical Banach spaces like $\ell_{1}, \ell_{\infty}, L_{1}[0,1]$, and $L_{\infty}[0,1]$. Some of the results like the case of $L_{1}[0,1]$, is known but the speaker produce an easy proof for it. Keywords : Chebyshev Subspaces, $\ell_{1}, \ell_{\infty}, \mathrm{L}_{1}[0,1]$, and $\mathrm{L}_{\infty}[0,1]$, Banach Lattice, Hobby Rice Theorem.

\section*{References:} [1] N.I. Ahiezer, M.G Krein, " Some Questions in the Theory of Moments", Transl. Math. Monographs, Vol. 2, Amer. Math. Soc, Providence, R.I (1962) [2] R. R. Phelips, Chebyshev subspaces of finite dimension in L1, Proc Amer. Math. Soc, 17 (1966), 646-652 [3] I. Schoenberg And C. Yang, On the unicity of solutions of problems of best approximation, Ann. Mat. Pura. Appl. 54 (1961), 1-12. [4] I. Singer, "Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces" Springer-Verlag, Berlin, 1970.


# On the Moments of Semi-Markovian Inventory Model When the Demand Distribution Belongs to the General Class of Regularly Varying Distributions with Infinite Variance 

A.B.Kamislik ${ }^{\text {l, }}$, T. Kesemen ${ }^{2}$, E.Senol ${ }^{3}$<br>${ }^{1}$ Recep Tayyip Erdogan University, Department of Mathematics, 53020, Rize Turkey. matheasli@gmail.com<br>${ }^{2,3}$ Karadeniz Technical University, Department of Mathematics, 61080 Trabzon, Turkey, tkesemen@gmail.com,esenol@hotmail.com


#### Abstract

It is well known that regular variation is one of the important phenomenon encountered in many different areas of applied probability theory. Certainly one of these application areas is investigation of inventory models. There are plenty of studies which provide empirical examples for existence of regularly varying demands. The main purpose of the current study is to investigate the asymptotic behavior of the ergodic moments of the process $\mathrm{X}(\mathrm{t})$ which represents a semi-Markovian inventory model when the demands have any arbitrary distribution function from the regularly varying subclass of heavy tailed distributions with tail parameter $1<\alpha<2$. Previous studies in the literature the considered process $\mathrm{X}(\mathrm{t})$ has been investigated under the assumptions heavy tailed Pareto distributed demand and uniform distributed interference of chance i.e. when the random variables $\left\{\eta_{n}\right\}_{n=1,2,3, \ldots}$ which represent the amount of demands have Pareto distribution with $\bar{F}(x)=P\left\{\eta_{1} \leq x\right\}=\left(\frac{b}{x}\right)^{\alpha}, x \geq b, b>0$, $1<\alpha<2$ [6]. The most important difference of this study from the other studies in the literature is, we investigate the current problem with the class of regularly varying distributions rather than a single distribution like Pareto. In order to obtain renewal function generated by the regularly varying random variables, we used a special asymptotic expansion [5]. We show that considered process is ergodic under some weak conditions. Finally we obtained a general formula which provides two term asymptotic expansion for the $n^{\text {th }}$ order moments ( $\mathrm{n}=1,2,3, \ldots$ ) of the ergodic distribution which covers all regularly varying subclass with infinite variance.


Keywords: Semi-Markovian Inventory Model, Regular Variation, Ergodic Moments, Asymptotic Expansion

## References:

[1] Aliyev, R.T. (2016). On a stochastic process with a heavy tailed distributed component describing inventory model of type ( $\mathrm{s}, \mathrm{S}$ ). Communications in Statistics-Theory and Methods, DOI: 10.1080/03610926.2014.1002932.

[2] Bingham, N.H., Goldie, C.M., Teugels, J.L. (1987). Regular Variation. Cambridge University, Press, Cambridge.
[3] Feller, W. (1971). Introduction to Probability Theory and Its Applications II. John Wiley, New York.
[4] Geluk, J.L. (1997). A Renewal Theorem in the finite-mean case. Proceedings of the American Mathematical Society 125(11), 3407-3413.
[5] Kamışlık, A.B., Kesemen, T., Khaniyev, T., Küçük, Z., Moments of an inventory model of type ( $\mathrm{s}, \mathrm{S}$ ) with heavy tailed and infinitely varying demands. Informs Applied Probability Society Conference 2015. Abstract Book p:70.
[6] Khaniyev, T., Kokangul, A., Aliyev, R. (2013). An asymptotic approach for a semi-Markovian inventory model of type ( $\mathrm{s}, \mathrm{S}$ ). Applied Stochastic Models in Business and Industry. 29:5, 439-453.

# Inverse Problem of Elliptic Equation with Nonlocal Boundary Conditions 

Fatma Kanca<br>Department of Management Information Systems, Kadir Has University, 34083, Istanbul, Turkey<br>fatma.kanca@khas.edu.tr


#### Abstract

In this paper, the inverse problem of finding a time dependent coefficient in a second order elliptic equation is investigated. The existence and uniqueness of the classical solution of the problem under consideration are established. Numerical tests using the finite-difference scheme combined with an iteration method is presented and the sensitivity of this scheme with respect to noisy overdetermination data is illustrated.


Keywords: Elliptic Equation, Inverse Problem, Nonlocal Boundary Condition, Integral Overdetermination Condition.

## References:

[1] Y. T. Mehraliyev, F. Kanca, "An Inverse Boundary Value Problem for a Second Order Elliptic Equation in a Rectangle", Mathematical Modelling and Analysis 19 (2014) 241-256.
[2] M. Ismailov, F. Kanca, "An inverse coefficient problem for a parabolic equation in the case of nonlocal boundary and overdetermination conditions", Mathematical Methods in the Applied Science, 34 (2011), 692-702.
[3] V. V. Solov'ev, "Source and coefficient inverse problems for an elliptic equation in a rectangle", Computational Mathematics and Mathematical Physics 47 (8) (2007) 1310-1322.
[4] N. I. Ionkin, "Solution of a boundary-value problem in heat conduction with a nonclassical boundary condition", Differential Equations, 13 (1977), 204-
[5] D. N. Hao, T. N. T. Quyen, "Convergence rates for total variation regularization of coefficient identification problems in elliptic equations I", Inverse Problems 27 (2011).

# Groebner Shirshov Basis of Aut (Fn) for the Word Problem 

Esma Kangal*, Nurten Urlu, Ahmet Sinan Cevik<br>Department of Mathematics, Selçuk University, Selçuklu, Konya, Turkey<br>esmakangal@selcuk.edu.tr, n.urlu91@gmail.com.tr, sinan.cevik@selcuk.edu.tr


#### Abstract

In combinatorial group theory, Max Dehn introduced the decision problems such as the word problem, the conjugacy problem and the isomorphism problem in 1911. Especially, there are many studies on the word problem [1], [2], [3] in the literature. Novikov and Boone show that there exists a finitely presented group whose word problem is recursively unsolvable. Therefore to show a finitely presented group has solvable word problem is important. In here, we study on the word problem of the automorphism group $\operatorname{Aut}(\mathrm{Fn})$ of a free group with rank n. In particular, we use the method of Groebner Shirshov basis by using the presentation $\operatorname{Aut}(\mathrm{Fn})$ given by [4] to solve this problem.


Keywords: Word Problem, Automorphism Group of a Free Group, Rewriting System.

## References:

[1] E. G. Karpuz, A. S. Çevik, "The Word and Generalized Word Problem for Semigroups under Wreath Products", Bulletin Mathematique de la Societe des Sciences Mathematiques de Roumanie, Tome 52(100), No. 2, (2009), 151-160.
[2] E.G. Karpuz., F. Ateş, A.S. Çevik,., I.N. Cangül, A.D. Güngör (Maden), "The Next Step of the Word Problem over Monoids", Applied Mathematics and Computation, 218(3) (2011), 794-798.
[3] F. Ateş, E. G. Karpuz, C. Kocapınar, A. S. Çevik "Gröbner-Shirshov Bases of Some Monoids", Discrete Mathematics, 311(12) (2011) 1064-1071.
[4] H. Armstrong, B. Forrest., K. Vogtmann., "A Presentation for Aut(Fn)", Journal of Group Theory, 11 (2008), 267-276.


# The Existence of Positive Solutions for Fractional-Order Boundary Value Problems on Finite Interval 

Ilkay Yaslan Karaca and Serenay Abali<br>Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>ilkay.karaca@ege.edu.tr, serenayabal@gmail.com


#### Abstract

In this study, five functionals fixed point theorem and GuoLakshmikantham fixed point theorem are used to research the existence of positive solutions for fractional-order nonlinear boundary value problems on finite interval. As applications, the examples are given to illustrate the main results. Keywords: Fractional, Positive Solution, Finite Interval, Five Functionals Fixed Point Theorem, Guo - Lakshmikantham Fixed Point Theorem.

\section*{References:} [1] Agarwal, Ravi P., O'Regan Donal, Meehan, M., 2004, Fixed Point Theory and Applications, Cambridge University Press. [2] Avery, R.I., 1998, A Generalization of the Leggett-Williams Fixed Point Theorem, Math. Sci. Res. Hot-Line, 2:9-14pp. [3] Kilbas, A.A., Srivastava, H.M., Trujillo J.J., 1999, Theory and Applications of Fractional Differential Equations, North-Holland Mathematics Studies 204, 69-79p. [4] Liu, X., Jia M., Ge W., 2013, Multiple Solutions of a p-Laplacian Model Involving a Fractional Derivative, Advances in Difference Equations. [5] Guo D., Lakshmikantham V., Ames W. F., Nonlinear Problems in Abstract Cones, Nonlinear Problems in Abstract Cones, Academic Press, New York, 1988.


# Positive Solutions of Impulsive Time-Scale Boundary Value Problems with p-Laplacian on the Half-Line 

Ilkay Yaslan Karaca and Aycan Sinanoglu<br>Department of Mathematics, Ege University, 35100 Bornova, Izmir, Turkey<br>ilkay.karaca@ege.edu.tr aycansinanoglu@gmail.com


#### Abstract

Boundary value problem on infinite intervals appear often in applied mathematics and physics. There are many papers concerning the existence of solutions on the half-line for the boundary value problem, see [1-8]. Due to the fact that an infinite interval is noncompact, the discussion about boundary value problem on the half-line is more complicated, in particular, for p- Laplacian impulsive boundary value problem on infinite intervals, few works were done, see [5]. Especially, the corresponding theory for m-point impulsive boundary value problem on infinite interval on time scale is not investigated till now. In this study, four functionals fixed point theorem is used to investigate the existence of positive solutions for second-order time-scale boundary value problem of impulsive dynamic equations on the half-line.


Keywords: Four Functionals Fixed Point Theorems, Impulsive Dynamic Equation, Positive Solutions, Boundary Value Problems, Time Scale.

## References:

[1] G. Chai, Existence of positive solutions of boundary value problem for second-order functional differential equations on infinite intervals, Fixed Point Theory, 13 (2012), 423437.
[2] X. Chen, X. Zhang, Existence of positive solutions for nonlinear systems of secondorder differential equations with inregral boundary conditions on an infinite interval in Banach Space, Electron. J. Differential Equations, 2011 (2011), no. 15419 pp.
[3] X. Chen, X. Zhang, Existence of positive solutions for singular impulsive differential equations with integral boundary conditions on an infinite interval in Banach Space, Electron. J. Qual.Differ. Equ., 2011 (2011), no. 2818 pp.
[4] Y. Guo, C. Yu, J. Wang, Existence of three positive solutions for m-point boundary value problem in infinite intervals , Nonlinear Anal., 71 (2009), 717-722.
[5] Z. Hao, L. Ma, Existence of positive solutions for multi-point boundary value problem on infinite intervals in Banach Space, Abstr. Appl. Anal., 2012 (2012), Art. ID 107276, 18pp.
[6] I. Y. Karaca, F. Tokmak, Existence of three positive solutions for m-point time scale boundary value problem on infinite intervals, Dynam. Systems Appl., 20 (2011), 355-367. [7] X. Zhao, W. Ge, Multiple positive solutions for time scale boundary value problem on infinite interval, Acta Appl. Math., 106 (2009), 265-273.
[8] X. Zhao, W. Ge, Unbounded positive solutions for m-point time-scale boundary value problem on infinite intervals, J. Appl. Math. Comput., 33 (2010), 103-123.

# The Existence of Positive Solutions for Fractional-Order Nonlinear Boundary Value Problems on Infinite Interval 

Ilkay Yaslan Karaca and Dondu Oz<br>Department of Mathematics, Ege University, 35100 Bornova, Izmir, Turkey<br>\section*{ilkay.karaca@ege.edu.tr, donduayhan@gmail.com}


#### Abstract

In this study, six functionals fixed point theorem and Leray-Schauder nonlinear alternative theorem are used to investigate the existence of positive solutions for fractional-order nonlinear boundary value problems on infinite interval. As applications, the examples are given to illustrate the main results. Keywords: Fractional, Infinite Interval, Six Functionals Fixed Point Theorem, Leray-Schauder Nonlinear Alternative Theorem.

\section*{References:} [1] Agarwal,Ravi P., O'Regan Donal, Meehan, M., 2004, Fixed Point Theory and Applications, Cambridge University Press. [2] Avery,R., Henderson,J. and O'Regan, D., 2008, Six functionals fixed point theorem, Communications in Applied Mathematics, 12(1):69-81p. [3] Dalir, M. and Bashaur, M., 2010, Applications of fractional calculus, Mathematical Sciences, Vol. 4, 1021-1032. [4] Kilbas, A.A., Srivastava, H.M., Trujillo J.J., 1999, Theory and Applications of Fractional Differential Equations, North-Holland Mathematics Studies 204, 69-79p. [5] Liang,S., Zhang,J., 2011, Existence of Three Positive Solutions of m-point Boundary Value Problems for Some Nonlinear Fractional Differential Equations on an Infinite Interval, Computers and Mathematics with Applications, 33433354p. [6] Podlubny, I., 1999, Fractional Differential Equations, Academic Press.




# Some Properties of Digital Persistent Homology Groups 

Ismet Karaca and Ozgur Ege<br>Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>Department of Mathematics, Celal Bayar University, Muradiye, Manisa, Turkey<br>ismet.karaca@ege.edu.tr,ozgur.ege@cbu.edu.tr


#### Abstract

Digital topology is a growing area with significant properties in mathematics. In this theory, digital homology groups were defined in [1] and extended in [2]. Then it was given a different approach and some properties for digital homology groups [3,4]. Digital persistent homology is a way to describe some properties of a digital image $(X, \kappa)$ with filtration and can be viewed as an extension of digital homology groups. In this talk, we introduce some properties of the digital persistent homology groups and calculate digital persistent homology groups of some digital images.


Keywords: Digital Image, Persistent Homology, Image Processing.

## References:

[1] H. Arslan, I. Karaca and A. Oztel, "Homology groups of n-dimensional digital images", XXI. Turkish National Mathematics Symposium, B(2008), 1-13.
[2] L. Boxer, I. Karaca and A. Oztel, "Topological invariants in digital images", Journal of Mathematical Sciences: Advances and Applications, 11(2)(2011), 109-140.
[3] I. Karaca and O. Ege, "Cubical homology groups of digital images", International Journal of Information and Computer Science, 1(7)(2012), 178187.
[4] O. Ege and I. Karaca, "Fundamental properties of simplicial homology groups for digital images", American Journal of Computer Technology and Application, 1(2)(2013), 25-42.
[5] E. Spanier, "Algebraic Topology", McGraw-Hill, New York (1966), 108110.


# Calibration Problem with Unknown Operator 

Belaide Karima<br>Laboratory of Applied Mathematics,<br>Department of Mathematics<br>A/Mira University, Bejaia, Algeria<br>k_tim2002@yahoo.fr


#### Abstract

In this paper, we consider a linear equation $A x=u$, where $A$ is a unknown compact operator in Hilbert space $H_{-}\{1\}$. To solve this problem arising from many experimental fields of science, we propose an iterative method with Gaussian errors which converges almost completely. Keywords: Inverse Problem; Linear Operator; Tikhonov Regularization; Iterative Method.

\section*{References:} [1] A. G. Ramm, Dynamical systems method for ill-posed problem equations with monotone operators, Communication in Nonlinear Science and Numerical Simulation, 10, (2005) 35--940. [2] A. G. Ramm, Inverse Problems, Mathematical and Analytical Techniques with Applications to Engineering, Springer, 2005. [3] A. N. Tikhonov, A. Leonov, A. Yagola. Nonlinear Ill-Posed Problems, vols. 1,2, Chapman and Hall, London, 1998.


# Euler-Lagrange Dynamical Equations on 3-Dimensional Normal Almost Contact Geometry 

Zeki Kasap<br>Faculty of Education, Department of Elementary Education, Pamukkale University, Denizli/ Turkey<br>zekikasap@hotmail.com


#### Abstract

We consider Euler-Lagrange equations on almost contact manifolds. On the other hand, one way of solving problems in classical and analytical mechanics is through use of the Euler-Lagrange equations. The purpose of the present paper is to solve the problems of classical mechanics with 3-dimensional real number space on an almost contact manifold by using Euler-Lagrange equations.


Keywords: Paracontact Manifold, Mechanical System, Dynamic Equation, Lagrangian Formalism.

## References:

[1] M.M. Tripathi, E. Kilic, S.Y. Perktas and S. Keles, Indefinite almost paracontact metric manifolds, International Journal of Mathematics and Mathematical Sciences, 2010, 1-19.
[2] S.Kr. Srivastava, D. Narain and K. Srivastava, Properties of $\varepsilon$ - $\square \square$ paracontact manifold, VSRD-TNTJ, Vol. 2 (11), 2011, 559-569.
[3] M. Girtu, An almost 2-paracontact structure on the cotangent bundle of a Cartan space, Hacettepe Journal of Mathematics and Statistics, Volume 33, 2004, 15-22.
[4] M. Ahmad and J-B. Jun, Submanifolds of an almost r-paracontact Riemannian manifold endowed with a semi-symmetric non-metric connection, Journal of The Chungcheong Mathematical Society, Volume 22, No: 4, 2009, 653-664,
[5] I. Kupeli Erken, Some classes of 3-dimensional normal almost paracontact metric manifolds, Honam Mathematical J., 37, No: 4, 2015, 457-468.
[6] A. Bucki, Product submanifold almost r-paracontact Riemannian manifold of p-Sasakian type, Soochov Journal of Mathematics, Vol.24, No.4, 1998, 255-259.
[7] Z. Kasap and M. Tekkoyun, Mechanical systems on almost para/pseudo-Kähler-Weyl manifolds, IJGMMP, Vol. 10, No.5, 2013, 1-8.

# Weyl-Euler-Lagrange Movement Equations on Almost Paracontact Metric Manifold 

Zeki Kasap<br>Faculty of Education, Department of Elementary Education, Pamukkale University, Denizli/ Turkey<br>zekikasap@hotmail.com


#### Abstract

A preferred method to solve the problems of classical mechanics is using the Lagrangian mecanics. Classical field theory uses a simple solution method of Euler-Lagrangian dynamic equation. Weyl presented a global transformation on metrics. In the present paper, we introduce Weyl-EulerLagrange equations on almost paracontact metric manifolds. Also, at the end of the study differential equations the obtained had been solved by Maple computation program.


Keywords: Paracontact Manifold, Mechanical System, Dynamic Equation, Lagrangian Formalism.

## References:

[1] M. Manev and M. Staikova, On almost paracontact Riemannian manifolds of type (n,n), J. Geom., 72, (2001), 108-114.
[2] M.M. Tripathi, E. Kilic, S.Y. Perktas and S. Keles, Indenite almost paracontact metric manifolds,

International Journal of Mathematics and Mathematical Sciences, (2010), 119.
[3] B. Pandey, On indenite almost paracontact metric manifold, International Mathematical Forum, Vol. 6, No. 22, (2011), 1071-1078.
[4] J. Weyczko, Para-CR structures on almost paracontact metric manifolds, arXiv:1202.6383v2, (2012), 1-18.
[5] J. Weyczko, Slant curves in 3-dimensional normal almost paracontact metric manifolds, http://arxiv.org/abs/1212.5839v1, 2012.
[6] Y. Gunduzalp and B. Sahin, Paracontact semi-Riemannian submersions, Turkish Journal of Mathematics, 37, (2013), 114-128.
[7] Z. Kasap, M. Tekkoyun, Mechanical systems on almost para/pseudo-KählerWeyl manifolds, IJGMMP, Vol.10, No.5, (2013), 1-8.

# Generalized Berinde-Type Contractions in Partially Ordered $\boldsymbol{G}_{\boldsymbol{p}}$-Metric Spaces 

Meltem Kaya and Hasan Furkan<br>Department of Mathematics, Kahramanmaras Sutcu Imam University, Kahramanmaras, Turkey<br>meltemkaya55@hotmail.com, hasanfurkan@hotmail.com


#### Abstract

In this manuscript, we view generalized Berinde-type contractions, which is known as generalized almost contractions in the literature, in the framework of partially ordered $G_{p}$-metric spaces to get some common fixed point results for self-mappings $f$ and $g$ and some fixed point results for a single mapping $f$. Presented theorems generalize several previously obtained classical results. We also state some examples which show the validity of our results. Keywords: Common Fixed Point, Partially Ordered Set, G_p-Metric Space, Weakly Increasing Maps, (c)-Comparison Function.


## References:

[1] M. R. A. Zand and A. D. Nezhad, "A generalization of partial metric spaces", J. Contemp. Appl. Math., 24, 86-93, (2011).
[2] H. Aydi, E. Karapınar and P. Salimi, "Some fixed point results in G_pmetric spaces", J. Appl. Math., 2012, 16 pages, (2012).
[3] M. A. Barakat and A. M. Zidan, "A common fixed point theorem for weak contractive maps in G_p-metric spaces", J. Egyptian Math. Soc., 23 (2015), 309-314.
[4] H. Aydi, S. H. Amor, E. Karapınar, "Berinde-Type generalized contractions on partial metric spaces", Abstract and Applied Analysis, 2013, 10 pages, (2013).
[5] F. Shaddad, M. S. M. Noorani, S. M. Alsulami, "Common fixed-point results for generalized Berinde-type contractions which involve altering distance function", Fixed Point Theory and Applications, 2014, 10 pages, (2014).

# On the Uniform Convergence of Spectral Expansions for a Spectral Problem Rationally Dependent on the Eigenparameter 

Nazim B. Kerimov, Sertac Goktas and Emir A. Maris<br>Department of Mathematics, Mersin University, 33343 Mersin,Turkey nazimkerimov@yahoo.com, srtcgoktas@gmail.com, e.ali.maris@gmail.com

Abstract: Consider the spectral problem

$$
\begin{gathered}
-y^{\prime \prime}+q(x) y=\lambda y, 0<x<1, \\
y(0) \cos \beta=y^{\prime}(0) \sin \beta, 0 \leq \beta<\pi ; \frac{y^{\prime}(1)}{y(1)}=h(\lambda),
\end{gathered}
$$

where $\lambda$ is a spectral parameter, $q(x)$ is real-valued continuous function on [0,1],

$$
h(\lambda)=a \lambda+b-\sum_{k=1}^{N} \frac{b_{k}}{\lambda-c_{k}},
$$

where all the coefficients are real and $a \geq 0, b_{k}>0, c_{1}<c_{2}<\ldots<c_{N}, N \geq 0$.
We investigate the uniform convergence of the spectral expansions of the continuous functions in the system of eigenfunctions of above spectral problem rationally dependent on the spectral parameter. To this end, we first obtain the sharpened asymptotic formulae for eigenvalues and eigenfunctions.
Note the papers [1]-[3] to which the present work is related.
Keywords: Differential Operator, Eigenvalues, Uniform Convergence of Spectral Expansion

## References:

[1] N.B. Kerimov and Y.N. Aliyev, "The basis property in $L_{p}$ of the boundary value problem rationally dependent on the eigenparameter", Studia Math., 174(2), (2006), 201-212.
[2] N. Yu. Kapustin and E.I. Moiseev, "Convergence of spectral expansions for functions of the Hölder class for two problems with spectral parameter in the boundary condition", Diff. Eq., 36, (2000), 1182-1188.
[3] N. B. Kerimov and E. A. Maris, "On the uniform convergence of the Fourier series for one spectral problem with a spectral parameter in a boundary condition", Math. Meth. Appl. Sci., 2015.

# Sink Mobility Under Extreme Conditions in Wireless Sensor Networks 

M. Emre Keskin<br>Department of Industrial Engineering, Ataturk University, Yakutiye, Erzurum, Turkey<br>m.emre.keskin@gmail.com


#### Abstract

The notion of sink mobility in Wireless Sensor Networks (WSN) is offered as a remedy for a well known phenomenon called differently in several sources such as `crowded center effect" (Popa et al. [1]), "energy hole problem" (Li and Mohapatra[2], Wu et al. [3]), and "sink neighborhood problem" (Basagni et al. [4]). Most of the mobile sink studies from the literature assume that the sink(s) has limitless energy and it instantaneously jumps from one point to another. Hence, sink travel times are usually taken as zero in the literature and the data collected during the sink travel times are also neglected. One of the rare studies that puts some limitations on the mobility of the sink is due to Keskin et al. [5] in which nonzero sink travel time for a single mobile sink is employed. Sink travel time is considered as a part of the network lifetime and the data accumulated during the sink travel time is also taken into account. This study extends the notion of nonzero sink travel times for the WSNs with multiple mobile sinks and it indicates that considering nonzero sink travel times is important if the sink speeds are slower than 1 km per hour, i.e., for the network under extreme conditions.


Keywords: Wireless Sensor Networks, Sink Mobility, Mixed Integer Linear Programming.

## References:

[1] Popa, Lucian, Afshin Rostamizadeh, Richard Karp, Christos Papadimitriou, and Ion Stoica, "Balancing traffic load in wireless networks with curveball routing.", In Proceedings of the 8th ACM international symposium on Mobile ad hoc networking and computing(2007), 170-179.
[2] Li, J. and Mohapatra, P, "Analytical modeling and mitigation techniques for the energy hole problem in sensor networks", Pervasive and Mobile Computing, 3.3(2007), 233-254.
[3] Wu, X., Chen, G. and Das, S.K, "Avoiding energy holes in wireless sensor networks with nonuniform node distribution", Parallel and Distributed Systems, IEEE Transactions on, 19.5(2008), 710-720.
[4] Basagni, S., Carosi, A., Melachrinoudis, E., Petrioli, C. and Wang, Z.M., "Controlled sink mobility for prolonging wireless sensor networks lifetime", Wireless Networks, 14.6(2008), 831-858.
[5] Keskin, M.E., Altınel, İ.K., Aras, N. and Ersoy, C, "Lifetime maximization in wireless sensor networks using a mobile sink with nonzero traveling time", The Computer Journal, 54.12(2011), 1987-1999.

# Car Insurance Customer Segmentation by Data Mining 

Sadi Khadidja, Kherchi Hanya, Lounici Nadjib<br>Laboratory of Applied Statistics<br>ENSSEA,Pöle universitaire Koléa<br>Algiers- Algeria<br>sadikh00@gmail.com, kherchi.hanya@gmail.com


#### Abstract

This paper concerns the study of customer segmentation in car insurance. Car insurance is an obligatory insurance, which encourages competition on this market. Indeed, each company seeks to attract a maximum number of insured to increase its turnover. Therefore, our main objective in this paper is to build a customer segmentation from five agencies of SALAMA (Algerian insurance company) with a data mining approach. For this, we have implemented a hybrid method that combines the K-means and multiple correspondence analysis (MCA). In addition, to identify the characteristics of each class discovered by the above-mentioned methods, we used the criterion of the V-test with Tanagra. Keywords: Car Insurance - Segmentation-Datamining - k-Means - V-test. References: [1] D.Henriet et J.C.Rochet :micro-économie de l'assurance ED. Economica, page 14, paris 1991. [2] Dominique Henriet, Jean Charles Rocher ; Micro- Economique de L'assurance ; Ed ; Economica, P24 ;Paris,2001 [3] Morineau, "Note sur la Caractérisation Statistique d'une Classe et les Valeurs-tests", Bulletin Technique Centre Statistique Informatique [4] Appliquées., Vol 2, n ${ }^{\circ} 1-2$, p 20-27, 1984 [5] Xindong Wu, Vipin Kumar, J. Ross Quinlan, Joydeep Ghosh, QiangTop : 10 algorithms in data mining. Knowledge and Information Systems,:1-37, Jan 2008.


# The Close Relationship Between Architecture and Mathematics 

Murat Kilic<br>Deparment of Interior Architechure \& Environment Design, KırıkkaleUniversity, Kırıkkale, Turkey<br>muratkilic@kku.edu.tr


#### Abstract

The unity of architecture and mathematics goes back to the very ancient times. The reason is not only the fact that architecture substantially makes use of mathematics, but that they both are in search of order and aesthetics. Mathematics aims at reaching these in nature and architecture in structures. Mathematics is one of the most important design factors which significantly influences aesthetic results. It is a known fact that mathematics is indispensible in terms of understanding the calculations and ideas in structures. Mathematics has been used for centuries as an element of creating a visual order and harmony and as a tool of reaching aesthetical beauty. While doing this, geometry and ratios in mathematics have been made use of. Architect has frequently used proportional systems and geometry throughout history to create certain forms or to limit the created forms. The purpose behind using such a system is to achieve harmony between the elements of structures and to create a sense of unity in general in structures with "the principle which beautifies the beautiful." "It is the measurement and the morphology brought on by the measurement which will create beauty or keep it safe" [1]. In almost all interhistorical and inter-cultural construction traditions, there is a mathematical system which determines the relationship between the elements of the design. In fact, the rational system is made up of very basic elements: these consist of geometrical shapes which can be created with whole number ratios or simple equipments such as rulers and ropes [2].


Keywords: Architecture, Mathematics, Fibonacci
References:
[1] T. Afsar, "Estetik", Insancil Yayinlari, Istanbul, (1993).
[2] URL1. http://www.banupekol.com/blog/2011/08/26/mimaride-oran-ve-geometri-kullaniminin-mimarlik-tarihi-icindeki-aktarimlari-ve-kentsel-baglamda-etkileri/

# An Inverse Result for the Periodic Boundary Conditions 

Alp Arslan Kirac<br>Department of Mathematics, Pamukkale University, Denizli, Turkey<br>aakirac@pau.edu.tr


#### Abstract

We obtain the classical Ambarzumyan's theorem for the SturmLiouville operatör $L$ with real-valued potential $q \in L_{1}[0,1]$ and periodic boundary conditions when the subset of the spectrum of $L$ and Fourier coefficients $c_{k}$ of the potential $q$ such that the condition


$$
\sum_{|k| \geq n_{0}}\left|k c_{k}\right|<\infty
$$

holds are given. The same result holds for the anti-periodic boundary conditions.
Keywords: Ambarzumyan Theorem, Inverse Spectral Theory, Hill Operator, Eigenvalue Asymptotics

## References:

[1] V. Ambarzumian, U* ber eine Frage der Eigenwerttheorie, Zeitschrift für Physik 53 (1929) 690-695.
[2] Y. H. Cheng, T. E.Wang, C. J.Wu, A note on eigenvalue asymptotics for Hill's equation., Appl. Math.Lett. 23 (9) (2010) 1013-1015.
[3] H. H. Chern, C. K. Lawb, H. J. Wang, Corrigendum to Extension of Ambarzumyan's theorem to
general boundary conditions, J. Math. Anal. Appl. 309 (2005) 764-768.
[4] H. H. Chern, C. L. Shen, On the n-dimensional Ambarzumyan's theorem, Inverse Problems 13 (1) (1997) 15-18.
[5] E. A. Coddington, N. Levinson, Theory of Ordinary Differential Equations, McGrawHill, New York, 1955.
[6] M. S. P. Eastham, The Spectral Theory of Periodic Differential Operators, Scottish Academic Press,Edinburgh, 1973.
[7] G. Freiling, V. A. Yurko, Inverse SturmLiouville Problems and Their Applications, NOVA Science Publishers, New York, 2001.
[8] W. Magnus, S. Winkler, Hill's Equations, Interscience Publishers, Wiley, 1969.
[9] A. A. Kırac Inverse problems associated with the Hill operator. Electron. J. Diff. Equ. 41, 1-12 (2016).
[10] A. A. Kırac,, On the asymptotic simplicity of periodic eigenvalues and Titchmarsh's formula, J. Math.Anal. Appl. 425 (1) (2015) 440 - 450.
[11] A. A. Kırac,, On the Ambarzumyan's theorem for the quasi-periodic problem, Analysis and Mathematical Physics, http://dx.doi.org/10.1007/s13324-015-0118-0, (2015) 1-4.
[12] F. M'oricz, Absolutely convergent fourier series and function classes, J. Math. Anal. Appl. 324 (2) (2006) 1168-1177.

# New Definitons About $A^{\prime}$ - Statistical Convergence with Respect to a Sequence of Modulus Functions and Lacunary Sequences 

Omer Kisi<br>Department of Mathematics, Faculty of Science and Literature<br>Bartın University, 74100, Bartın, Turkey<br>okisi@bartin.edu.tr<br>Hafize Gumus<br>Department of Mathematics, Faculty of Eregli Education, Necmettin Erbakan UniversityEregli, Konya, Turkey hgumus@konya.edu.tr Ekrem Savas<br>Department of Mathematics, Istanbul Ticaret University, Uskudar, Istanbul, Turkey ekremsavas@yahoo.com


#### Abstract

In this study, we introduce the notion of $\mathrm{A}^{\wedge}\{\mathrm{I}\}$-lacunary statistical convergence, strongly $\mathrm{A}^{\wedge}\{\mathrm{I}\}$-lacunary convergence with respect to a sequence of modulus functions. We study some collections between them. Also, we give some inclusion relations between $\mathrm{A}^{\wedge}\{\mathrm{I}\}$-lacunary statistical convergence, $\mathrm{A}^{\wedge}\{\mathrm{I}\}$ statistical convergence with respect to a sequence of modulus functions. Keywords: Lacunary Sequence, Ideal Convergence, Modulus Function.

\section*{References:} [1] H. Fast, "Sur la convergence statistique", Colloq. Math. 2(1951), 241-244. [2] I. J. Schoenberg, "The integrability methods", Amer. Math. Monthly, 66(1959), 361-375. [3] M. Gürdal, U. Yamancı,S. and Saltan, $\mathrm{A}^{\wedge}\{I\}$-statistical convergence with respect to a sequence of modulus functions, Contemporary Analysis and Applied Mathematics Vol.2, No.1, 136-145, 2014 [4] T. Salat, "On statistically convergent sequences of real numbers", Math Slovaca, 30.2 (1980), 139-150. [5] T. Bilgin, Lacunary strong A-convergence with respect to a modulus, Mathematica XLVI (4) (2001) 39-46.


# A Combinatorial Method for Characterizing the Linear Combinations of Finitely Many Diagonalizable Matrices That Mutually Commute 

Emre Kisi, Halim Ozdemir<br>Department of Mathematics, Sakarya University, Sakarya TR54187, Turkey<br>ekisi@sakarya.edu.tr, hozdemir@sakarya.edu.tr


#### Abstract

This paper provides a combinatorial method, which depends on solving systems of linear equations where the coefficients of the unknowns of these linear equations are chosen from the spectrums of the matrices considered, for characterizing the linear combinations of finitely many diagonalizable matrices that mutually commute. Moreover, the problem, which is one of the open problems given in [Linear Algebra Appl. 437 (2012) 2091-2109], of characterizing all situations wherein a linear combination of the form $c_{1} X_{1}+c_{2} X_{2}+c_{3} X_{3}$ is a tripotent matrix when $X_{1}$ is an involutive matrix and both $X_{2}$ and $X_{3}$ are tripotent matrices that mutually commute is considered by means of this method. The results obtained cover those established in the reference above.


Keywords: Diagonalizable Matrices, Commutativity, Spectrum, Systems of Linear Equations, Linear Combination.

## References:

[1] O.M. Baksalary, Idempotency of linear combinations of three idempotent matrices, two of which are disjoint, Linear Algebra Appl. 388 (2004) 67-78.
[2] J.K. Baksalary, O.M. Baksalary, Idempotency of linear combinations of two idempotent matrices, Linear Algebra Appl. 321 (2000) 3-7.
[3] J.K. Baksalary, O.M. Baksalary, On linear combinations of generalized projectors, Linear Algebra Appl. 388 (2004) 17-24.
[4] J.K. Baksalary, O.M. Baksalary, When is a linear combination of two idempotent matrices the group involutory matrix?, Linear and Multilinear Algebra 54(6) (2006) 429-435.
[5] J. Benitez, N. Thome, Idempotency of linear combinations of an idempotent matrix and a t-potent matrix that commute, Linear Algebra Appl. 403 (2005) 414-418.
[6] J. Benitez, N. Thome, Characterizations and linear combinations of kgeneralized projectors, Linear Algebra Appl. 410 (2005) 150-159.

# A New Approach to Canal Surface with Parallel Transport Frame 

Ilim Kisi, Gunay Ozturk<br>Department of Mathematics, Kocaeli University, Kocaeli, Turkey<br>ilim.ayvaz@kocaeli.edu.tr, ogunay@kocaeli.edu.tr


#### Abstract

In the present study, we attend to the canal surfaces with the spine curve $\gamma$ according to the parallel transport frame in Euclidean 4-space $\mathrm{IE}^{4}$. We give an example of these surfaces and obtain some results about curvature conditions in $\mathrm{IE}^{4}$. Moreover, we give the necessary and sufficient conditions for canal surfaces to become weak superconformal. Lastly, the visualizations of projections of canal surfaces are presented.


Keywords: Parallel Transport Frame, Normal Curvature, Curvature Ellipse. References:
[1] B. Bulca, "A characterization of surfaces in $\mathrm{IE}^{4}$, PhD., Uludag University, Bursa, Turkey, (2002).
[2] B. Bulca, K. Arslan, B. Bayram and G. Öztürk, "Canal surfaces in 4dimensional Euclidean space", Preprint.
[3] B. Y. Chen, "Geometry of submanifolds", Dekker, New York, (1973).
[4] M. Dajczer and R. Tojeiro, "All superconformal surfaces in IR ${ }^{4}$ in terms of minimal surfaces", Mathematische Zeitschrift, 261(2009), 869-890.
[5] F. Gökçelik, Z. Bozkurt, İ. Gök, F. N. Ekmekci and Y. Yaylı, "Parallel transport frame in 4-dimensional Euclidean space $\mathrm{IE}^{4} "$, Caspian J. of Math., 3(2014), 91-103.
[6] İ. Kişi, G. Öztürk and K. Arslan, "A new characterization of canal surfaces with parallel transport frame in $\mathrm{IE}^{4} "$, Preprint.
[7] L. F. Mello, "Mean directionally curved lines on surfaces immersed in $\mathrm{IR}^{4}$ ", Publ. Math., 47(2003), 415-440.
[8] D. K. H. Mochida, M. D. C. R. Fuster and M. A. S. Ruas, "The Geometry of surfaces in 4 -space from a contact viewpoint", Geometria Dedicata, 54(1995), 323-332.
[9] P. Wintgen, "Sur l'inegalite de Chen-Willmore", C. R. Acad. Sci. Paris, 288, 993-995.

# A Pseudo-Spectral Approach to the Multi-Pantograph Equation Systems 

Ayse Betul Koc<br>Department of Mathematics, Selcuk University, Konya, Turkey aysebetulkoc@selcuk.edu.tr


#### Abstract

The pseudo-spectral methods have been gained much attention in the last years to achieved good results for the problems [1-5]. In this presentation, we shall introduce a new solution procedure based on the pseudo-spectral approach for the multi-pantograph equation systems. The soltions of the problems are sought in the form of a linear expansion of so called trial basis functions. The efficiency and validity of the proposed scheme is tested on some numerical examples.


Keywords: Pseudo-Spectral Approach, Operational Matrices, Pantograph Equation System.

## References:

[1] E. H. Doha, D. Baleanu, A. H. Bhrawy, R. M. Hafez, "A pseudospectral algorithm for solving multipantograph delay systems on a semi-infinite interval using legendre rational functions," Abst. Appl. Analy., ArticleID: 816473, 2014.
[2] S. Yuzbasi, "An efficient algorithm for solving multi-pantograph equation systems", Comp. Math. Appl., 64(4), 589-603, 2012.
[3] A. B. Koc, M. Cakmak, A. Kurnaz, K. Uslu, "A new Fibonacci type collocation procedure for boundary value problems", Advances in Difference Equations, 262: 2013 (2013).
[4] A. B. Koc, M. Cakmak, A. Kurnaz, "A matrix method based on the Fibonacci polynomials to the generalized pantograph equations with functional arguments", Advances in Mathematical Physics Article ID: 694580 (2014).
[5] A. B. Koc, A. Kurnaz, "An efficient approach for solving telegraph equation", AIP Conf. Proc., 1648, 370006 (2015).

# Some Remarks Related to Orthogonality in the Space of pSummable Sequences 

${ }^{1}$ Sukran Konca and ${ }^{2}$ Mehmet Aslan<br>1,2 Department of Mathematics, Bitlis Eren University, 13000, Bitlis, Turkey skonca@beu.edu.tr; m.aslan027@hotmail.com


#### Abstract

In this paper, we aim to give some remarks on the space of psummable sequences $l^{p}$ and to formulate new definitions of orthogonality in it as a 2 -normed and n-normed space. Keywords: Orthogonality, 2-Normed Space, $n$-Normed Space, 2-Inner Product, $n$-Inner Product.

\section*{References:} [1] A. Misiak, "Orthogonality and orthonormality in n-inner product spaces", Math. Nachr. 143(1989), 249-261. [2] A. Misiak, "n-inner product spaces", Math. Nachr. 140(1989), 299-319. [3] H. Gunawan, Mashadi, S. Gemawati, Nursupuamin, I. Sihwaningrum, "Orthogonality in 2-normed spaces revisited", Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Math. 17(2006), 76-83. [4] H. Gunawan, "inner products on n-inner product spaces", Soochow J. Math. 28(2002), 289-298. [5] H. Gunawan, Mashadi, "On n-normed spaces", Int. J. Math. Math. Sci., 27(2001), 631-639.


# Double Weighted Lacunary Almost Statistical Convergence of Order $\alpha$ 

Sukran Konca ${ }^{1}$, Selman Ekin ${ }^{1}$, Ekrem Savas ${ }^{2}$<br>${ }^{1}$ Department of Mathematics, Bitlis Eren University,Bitlis,Turkey<br>${ }^{2}$ Department of Mathematics, Istanbul Commerce University, Istanbul, Turkey skonca@beu.edu.tr, selmanekin@hotmail.com, ekremsavas@yahoo.com


#### Abstract

In this work, we define and study double weighted lacunary almost statistical convergence of order $\alpha$. Further, some inclusion relations have been examined. Keywords: Weighted Lacunary Statistical Convergence, Double Statistical Convergence of Order $\alpha$, Almost Statistical Convergence of Double Sequences, Sequence Space, Double Sequence.

\section*{References:} [1] J.A. Fridy, "On statistical convergence", Analysis, 5(1985), 301-313. [2] R.Colak, "Statistical convergence of order $\alpha$ ", In. Modern Methods in Analysis and Its Applications, Anamaya Pub., New Delhi (2010), 121-129. [3] R.Colak, C. A. Bektas, " $\lambda$-statistical convergence of order $\alpha$ ", Acta Math. Sci. Ser. B., 31 (3), (2011), 953-959. [4] E.Savas, "Double almost statistical convergence of order $\alpha$ ", Adv. Difference Equ., 2013:62, (2013). [5] E.Savas, "Double almost lacunary statistical convergence of order $\alpha$ ", Adv. Difference Equ., 2013:254, (2013). [6] S.Konca and M.Basarir, "Riesz lacunary almost convergent double sequence spaces defined by Orlicz functions", Facta Univ., Ser. Math., 2015, preprint. [7] M.Basarir and S.Konca, "On some spaces of lacunary convergent sequences derived by Norlund-type mean and weighted lacunary statistical convergence", Arab J. Math. Sci., 20 (2) (2014), 250-263.




On Some Congruences

Sibel Koparal and Nese Omur<br>Department of Mathematics, Kocaeli University,<br>Kocaeli, Turkey<br>sibel.koparal@kocaeli.edu.tr \& neseomur@kocaeli.edu.tr

Abstract: In this study, we investigate some congruences involving the numbers
$B_{p, k-d}$ and harmonic numbers $H_{n}$, where $B_{p, k-d}=\frac{k-d}{p}\binom{2 p}{p-k+d}$ and $H_{n}=\sum_{k=1}^{n} \frac{1}{k}$.
Keywords: Congruences, Harmonic Numbers and Binomial Coefficients. References:
[1] L.W. Shapiro, "A Catalan triangle", Discrete Math., 14(1976), 83-90.
[2] N. Ömür and S. Koparal, "Some congruences involving numbers $\boldsymbol{B}_{p, k-d}$ ", Utilitas Mathematica, 95(2014), 307-317.
[3] V.J.W. Guo and J. Zeng, "Factors of binomial sums from the Catalan triangle", J. Number Theory, 130(2010), 172-186.

On Solutions to the Sylvester s-Conjugate Equations

$$
A\left({ }^{s} \bar{X}\right)-X B=C, s=1,2,3
$$

# Over Elliptic Quaternion Matrices 

Hidayet Huda Kosal and Murat Tosun<br>Department of Mathematics, Sakarya University,<br>Sakarya, Turkey<br>hhkosal@sakarya.edu.tr, tosun@sakarya.edu.tr

Abstract: In this study, the existence of solution to the elliptic quaternion matrix
equations $A\left({ }^{1} \bar{X}\right)-X B=C, A\left({ }^{2} \bar{X}\right)-X B=C$ and $A\left({ }^{3} \bar{X}\right)-X B=C$ is characterized and solutions of these matrix equations are derived by means of real representations of elliptic quaternion matrices.
Keywords: Elliptic Quaternion, Real Representation of Elliptic Quaternion Matrix.

## References:

[1] W.R. Hamilton, Lectures on quaternions, Hodges and Smith, Dublin, 1853.
[2] T.S. Jiang and M.S. Wei., On a solution of the quaternion matrix equation and its application, Acta Math. Sin., 21(2005): 483-490.
[3] T.S. Jiang and S. Ling, On a Solution of the quaternion matrix equation and its Appl., Adv. Appl. Clifford Algebras, 23(2013), 689-699.
[4] C. Segre, The real representations of complex elements and extension to bicomplex systems, Math. Ann. 40(1892), 413-467.
[5] F. Catoni, R. Cannata and P. Zampetti, An introduction to commutative quaternions, Adv. Appl. Clifford Algebras, 16(2005), 1-28.
[6] H.H. Kösal and Tosun M., Commutative quaternion matrices, Adv. Appl. Clifford Algebras, 24(2014), 769-779.
[7] H.H. Kösal, M. Akyiğit and M. Tosun, Consimilarity of commutative quaternion matrices, 16(2015), 965-977.

# Scale Laws of Prime Number Distributions by the Modified Chi-Square Function 

Daniele Lattanzi<br>former ENEA-FUS, Frascati Research Centre, Roma, Italy<br>Private address via La Spezia 81 - 00182 Roma, Italy<br>lattanzio.lattanzi@alice.it


#### Abstract

The statistical distribution of prime numbers represents an open problem in number theory still nowadays. The methodology of experimental mathematics [1] has not yet been attempted in this field, thus the present paper treats prime numbers as raw experimental data, with zero imprecision and zero inaccuracy, and as elements of larger and larger finite sequences $\left\{\mathrm{P}_{\mathrm{m}}\right\}$. The modified chi-square function $X^{2}(A, x / \mu)$ with the ad-hoc $A, k$ and $\mu=\mu(k)$ parameters is the best-fit function of the differential distribution functions of both prime finite sequences $\left\{\mathrm{P}_{\mathrm{m}}\right\}$ and progressions $\left\{\mathrm{n}^{\alpha}\right\}$ with $\alpha \in(1,2)$ so that an injective map can be set between the former and the latter through the parameter k of their common fit function $\mathrm{X}^{2}{ }_{k}(\mathrm{~A}, \mathrm{x} / \mu)$ showing that the property of scale invariance does not hold for prime distributions [2]. The histograms of prime gaps, best fitted by standard statistical distribution functions, show unexpected clustering effects and thus do the histograms of prime numbers themselves [3]. Keywords: Prime Distribution, Modified Chi-Square Function, Prime Gaps, Numeric Progressions.

\section*{References:} [1] D. H. Bailey and J.M. Borwein, "Exploratory Experimentation and Computation" August 14, 2010, Paper LBNL-3313E Lawrence Berkeley National Laboratory; Notices of the AMS 58(10), 1410-1419, Nov. 2011. [2] D. Lattanzi, "Distribution of prime numbers by the modified chi-square function" Notes on Number Theory and Discrete Mathematics ISSN 1310-5132, 21(1), 2015, 18-30, http://nntdm.net/volume-21-2015/number1/18-30/ [3] A. Granville, "Prime Number Patterns", The American Mathematical Monthly, 115(4) (Apr. 2008), 279-296, Mathematical Association of America.




# Some Properties of $\boldsymbol{k}$-Quasi Class $\boldsymbol{Q}$ Operators 

Shqipe Lohaj, Valdete Rexhëbeqaj Hamiti<br>Faculty of Electrical and Computer Engineering, University of Prishtina "Hasan Prishtina", Prishtinë, 10000, Kosova shqipe.lohaj@uni-pr.edu, valdete r@hotmail.com


#### Abstract

In this paper, we give some properties of $k$-quasi class $Q$. We proved that if $T$ is an invertible operator and $N$ is an operator such that $N$ commutes with $T^{*} T$, then $N$ is $k$-quasi-class $Q$ if and only if $T N T^{-1}$ is of $k$-quasi class $Q$ operator. Morever, $\tilde{T}$ is k-quasi-class $Q$ if and only if $\tilde{T}^{(*)}$ is k-quasi class $Q$, where the $\tilde{T}$ is Aluthge transformation and $\tilde{T}^{(*)}$ is $*-$ Aluthge transformation. We shown that if $k$-quasi class $Q$ operator $T$ commutes with an isometric operator $S$, then $T S$ is $k$-quasi class $Q$.


Keywords: $k$-Quasi Class $Q$, Aluthge Transformation.
References:
[1] V.R. Hamiti, "On $k$-quasi class $Q$ operators", Bulletin of Mathematical Analysis and Applications, Vol. 6 Issuse 3 (2014), pp. 31-37.
[2] Takeaki Yamazaki, "Parallelisms between Aluthge transformation and powers of operators", Acta Sci. Math. (Szeged) 67 (2001), pp. 809-820.
[3] B. P. Duggal, C. S. Kubrusly, and N. Levan, "Contractions of class $Q$ and invariant subspaces", Bull. Korean Math. Soc. 42(2005), No. 1, pp. 169-177.
[4] T. Furuta, "On The Class of Paranormal Operators", Proc. Jap. Acad. 43(1967), 594-598.
[5] D. Senthilkumar and T. Prasad, "Composition Operators of Class Q*", Int. Journal of Math. Analysis, Vol. 42010, no. 21, 1035-1040.
[6] S. C. Arora and J. K. Thukral, "On a class of operators", Glasnik Math. 21(41)(1986), 381-386.
[7] A. Devika, G. Suresh, "Some properties of quasi class Q operators", International Journal of Applied Mathematics and Statistical Sciences (IJAMSS), Vol. 2, Issue 1, Feb 2013, 63-68.
[8] Salah Mecheri, "Bishops property $\beta$ and Riesz idempotent for k-quasi-paranormal operators", Banach J. Math. Anal., 6(2012), No. 1, 147 154.

# Hele-Shaw Flow with a Time-Dependent Gap: The Schwarz Function Approach to the Interior Problem 

K. Malaikah, T.V. Savina, and A.A. Nepomnyashchy<br>Department of Mathematics, Taibah University, Madina, Sauid Arabia<br>Department of Mathematics, Ohio University, Athens, OH 45701, USA<br>Department of Mathematics, Technion-Israel Institute of Technology, Haifa, 32000, Israel<br>Km306206@ohio.edu


#### Abstract

A Hele-Shaw problem with a time-dependent width of the gap is considered. A governing equation in terms of the Schwarz function of the free boundary for a class of generalized Hele-Shaw flows, which includes the problem with a time-dependent gap as a special case, is derived. The exact solution is obtained for the droplet with initial elliptical shape when the surface tension is neglected, and the asymptotic solution is obtained when the surface tension is taken into account.


Keywords: Hele-Shaw Flow, Schwarz Function.

## References:

[1] O. Agam, Viscous fingering in volatile thin films, Phys. Rev. E 79 (2009).
[2] P. Constantin and M. Pugh, Global solutions for small data to the Hele-Shaw problem, Nonlinearity 6 (1993), 393-415.
[3] D. Crowdy, On a class of geometry-driven free boundary problems, SIAM J. Appl. Math. 62(2002), 945-964.
[4] D. Crowdy, Exact solutions to the unsteady two-phase Hele-Shaw problem, Quart. J. Mech.Appl. Math. 59 (2006), 475-485.
[5] D. Crowdy and H. Kang, Squeeze flow of multiply-connected fluid domains in a Hele-Shawcell, J. Nonlinear Sci. 11 (2001), 279-304.
[6] L.J. Cummings, S.D. Howison and J.R. King, Two-dimensional Stokes and Hele-Shaw flows with free surfaces, J. Appl. Math. 10 (1999), 635-680.
[7] Ph. Davis, The Schwarz Function and its Applications, Carus Mathematical Monographs,MAA, 1979.
[8] V.M. Entov and P. Etingof, On generalized two-fluid Hele-Shaw flow, European J. Appl.Math. 18 (2007), 103-128.
[9] A. Friedman and Y. Tao, Nonlinear stability of the Muskat problem with capillary pressure at the free boundary, Nonlinear Anal. 53 (2003), 45-80.
[10] B. Gustafsson and A. A. Vasil'ev, Conformal and Potential Analysis in Hele-Shaw Cells, Birkhauser, 2006.
[11] S.D. Howison, Complex variable methods in Hele-Shaw moving boundary problems, European J. Appl. Math. 3 (1992), no. 3, 209-224.
[12] S.D. Howison, A note on the two-phase Hele-Shaw problem, J. Fluid Mech. 409 (2000),243-249.

[13] D. Khavinson, Holomorphic Partial Differential Equations and Classical Potential Theory, Universidad de La Laguna, 1996.
[14] D. Khavinson, M. Mineev-Weinstein and M. Putinar, Planar elliptic growth, Complex Anal.Oper. Theory 3 (2009), no. 2, 425-451.
[15] A.A. Lacey, Moving boundary problems in the flow of liquid through porous media, J. Austral.Math. Soc. B24 (1982), 171-193.
[16] N.R. McDonald, Generalized Hele-Shaw flow: A Schwarz function approach, European J.Appl. Math. 22 (2011), 517-532.
[17] T. Savina, On non-local reflection for elliptic equations of the second order in R2(the Dirichlet condition), Trans. Amer. Math.Soc.364.5(2012),2443-2460.
[18] T.V. Savina and A.A. Nepomnyashchy, A dynamical mother body in a Hele-Shaw problem,Physica D 240 (2011), 1156-1163.
[19] H.S. Shapiro, The Schwarz Function and its Generalization to Higher Dimensions, John Wiley and Sons, Inc., 1992.
[20] M.J. Shelley, F.R. Tian and K. Wlodarski, Hele-Shaw flow and pattern formation in a time-dependent gap, Nonlinearity 10 (1997), 1471-1495.
[21] S. Tanveer, Evolution of Hele-Shaw interface for small surface tension, Philos. Trans. Roy.Soc. London, Ser. A 343 (1993), 155-204.
[22] J. Ye and S. Tanveer, Global solutions for a two-phase Hele-Shaw bubble for a near-circular initial shape, Complex Var. Elliptic Equ. 57.1 (2012), 23-61.

# Numerical Study of Heat Transfer and Entropy Generation in a Nanofluid Filled Two-Sided Lid-Driven Cavity 

Sumit Malik, A.K.Nayak<br>Department of Mathematics, Indian Institute of Technology Roorkee<br>Roorkee, Uttarakhand, India<br>ssumitmalik@gmail.com


#### Abstract

In this work, mixed convection flow and heat transfer inside a two sided lid-driven square enclosure filled with Cu -water nanofluid have been investigated numerically. The bottom and top walls are kept fixed while the left and right lids are moving in upward and downward directions. Two heat sources are placed equidistantly on the left and right part of the bottom wall maintaining a fixed distance from the mid section of bottom wall and side walls. A comparative study has been done based on the direction of moving walls. In Case-I , left lid is moving downwards and the right one is moving upwards and in Case-II, both the lids are moving in upward direction. The effect of various flow governing parameters such as Reynolds number, Richardson number and nanoparticle volume fraction are discussed. The thermodynamic optimization of the system is discussed by using entropy generation and Bejan number.


Keywords: Mixed Convection, Nanofluid, Entropy Generation, Bejan Number. References:
[1] H. C. Brinkman, "The viscosity of concentrated suspensions and solutions", Journal of Chemical Physics, 20.4(1952), 571-581.
[2] J. C. Maxwell, "A treatise on electricity and magnetism", second ed, Oxford University Press, Cambridge, (1904), 435-441
[3] H. Moumni, H. Welhezi, R. Djebali, E. Sediki, "Accurate finite volume investigation of nanofluid mixed convection in two-sided lid driven cavity including discrete heat sources", AMM,39.14.15(2015), 4164-4179.
[4] R.K. Nayak, S. Bhattacharyya, I. Pop, "Numerical study on mixed convection and entropy generation of a nanofluid in a lid-driven square enclosure". ASME. J. Heat Transfer, 138.1(2015), 012503-1-012503-11.

Modeling of Tomatoes Solar Drying Systm in Arid Areas in
Algeria

Saadeddine Manaa ${ }^{* 1,4}$, A. Beatriz ${ }^{2}$, D. Karlo $^{3}$ And N. Moummi ${ }^{4}$<br>${ }^{1}$ Hydrocarbon and Renewable Energy Department, Science and Technology Faculty, Ahmed Draia University - Adrar -Algeria.<br>manaasaadeddine@gmail.com<br>${ }^{2}$ National Food Institute, Technical University of Denmark, Kgs. Lyngby, Copenhagen, Denmark. emilie.beatriz@dtu.dk<br>${ }^{3}$ The Laboratory of Solar Energetic and Building Physics (LESBAT), High School of Engineering and Management, Western Switzerland, Switzerland.<br>(E-mail : daniel.karlo@heig-vd.ch )<br>${ }^{4}$ Mechanical Engineering Department, University of Biskra, Biskra-Algeria. dgm@univ-biskra.dz


#### Abstract

Good knowledge of the phenomenon of solar drying, can give us a great performance solar drying of food products more profitable, why knowledge starts with a good theoretical knowledge which will be followed by mathematical modeling and simulation and at the end will be confirmed by experimental applications. Drying the solid product based on the balance between the water contained in the product and water in gaseous form in air. This equilibrium can be promoted in a direction or the other according to the water content of the air and product. Our study is a theoretical contribution of the energy balance of a convective drying system of agro-food products in southern Algeria. In our work we focus on the energy aspects of a solar drying system. The aim of our work is to study the influence of various parameters at play in drying tomato by use of an indirect solar dryer.


Keywords: Solar Drying, Drying of Tomato, Modeling, Simulation.

# An Optimal Control Problem with Control in Coefficients 

Gabriela Marinoschi
Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy, Bucharest, Romania
gabriela.marinoschi@acad.ro


#### Abstract

We study the existence, uniqueness and approximation for the solution to an optimal control problem governed by a parabolic equation with control in the drift term. There is a large variety of physical problems which can be formulated in this setting, especially those referring to transport and diffusion processes. Examples arise from hydrology (pollutant propagation), image restoring ([1]) or movement of biological populations. The optimal control is represented by the velocity field in the transport term of the equation and its identification is done on the basis of some available observations upon the solution at certain times. Due to the singular aspects of the control problem, the optimality conditions are found by handling the Leray projector and by passing to a limit technique in an appropriate regularizing optimal control problem. Numerical simulations based on the control construction algorithm are provided. Keywords: Nonlinear Optimal Control, Inverse Problems, Parabolic Diffusion Partial Differential Equations, $m$-Accretive Operators, Leray Projector.


## References:

[1] V. Barbu, G. Marinoschi, An optimal control approach to the optical flow problem, Systems and Control Letters, 87 (2016), 1-9.


# Asymptotic Representations for Heavy-Tailed Distributions Under Random Censoring 

Djamel Meraghni<br>Abdelhakim Necir Mohamed khider University, Biskra, Algeria<br>djmeraghni@yahoo.com


#### Abstract

In the analysis of lifetime, reliability or insurance data, the observations are not always available: they are usually randomly censored. We model this situation by introducing a non-negative random variable (rv), called censoring rv, independent of the rv of interest. Then, we consider the minimum of the two rv's and an indicator rv which determines whether or not there has been censorship. In this work, we make use of the empirical process theory to provide Gaussian approximations to some useful statistics in the analysis of extremes under random censoring. As a consequence, we expand the estimator of the shape parameter of censored heavy-tailed data in terms of Gaussian processes leading to its asymptotic normality only under the second-order condition of regular variation very well-known in extreme value theory.


# Behavior of Cellular Beams with Various Opening Shapes 

Mostefa Mimoune, Saad Siouane and Fatima Z. Mimoune Civil engineering Department, Constantinel University, Algeria. mmimoune@gmail.com


#### Abstract

This paper develops a finite-element model using ANSYS for the buckling analysis of perforated beams and uses it to investigate the effects of slenderness on the moment-gradient factor of simply supported perforated web beams. Web post buckling is studied by using numerical model. An investigation on steel beams with various shapes and sizes is reported based on web post buckling. A parameter study was conducted based on web post width and the effect of various openings is described.


Key Words: Buckling Mode, Cellular Beams, Finite Element Modeling, Steel Structures.

## References:

[1] Tsavdaridis KD, D'Mello C. Web buckling study of the behaviour and strength of perforated steel beams with different novel web opening shapes. Journal of Constructional Steel Research 2011; 67:1605-20.
[2] Chen, W.F. and Lui, E.M. (1987). "Structural Stability: Theory and Implementation" New York: Elsevier.
[3] Chung K.F., Liu T.C.H., Ko. A.C.H. Investigation on Vierendeel mechanism in steel beams with circular web openings. Journal of Constructional Steel Research 57 (2001) 467-490.
[4] Ellobody E. Nonlinear analysis of cellular steel beams under combined buckling modes. Thin-Walled Structures 52 (2012) 66-79.
[5] ANSYS. User's manual, version 10.0.
[6] Mimoune M., Siouane S. and Mimoune M. Elastic buckling of web-post in cellular beams with various opening shapes. World Journal of Engineering. Oct. 2012, Vol.9, issue 5, pp.429-436.


# Magnetic Fields of Asymmetric Magnetic Recording Heads Using the Superposition of the Head Filed 

Ammar Edress Mohamed,<br>Exeter University<br>aie202@exeter.ac.uk


#### Abstract

Current magnetic recording heads, used in shingled magnetic recording and two-dimensional magnetic recording, often exhibit asymmetry in their structure. They consist of two semi-infinite poles separated by a gap (where the recording field is produced), with an inner gap faces inclined at an angle. Modelling of the fields from asymmetrical structures is complex, and no explicit solutions are currently available (only implicit conformal mapping solutions are available for rational inclination angles). An approximate solution for the fields from asymmetrical heads is derived in this paper. The derivation is based on the superposition of the surface fields from corner structures. The approximate expressions for the fields are compared to finite-element calculations revealing excellent agreement for inclination angles $\leq 140^{\circ}$.


# On Fractional Differential Inclusions in Banach Space 

Fatima Zohra Mostefai<br>Department of Mathematics, Dr. Moulay Tahar University, Saida, Algeria<br>fatymath@gmail.com


#### Abstract

We consider a class of boundary value problem in separable Banach space governed by fractional differential inclusion with boundary conditions, $$
\begin{gathered} D^{\alpha} u(t) \in F\left(t, u(t), D^{\alpha-1} u(t)\right), \quad t \in[0, T] \\ u(0)-u^{\prime}(0)=0, \quad u(T)+u^{\prime}(T)=0, \end{gathered}
$$ where $1<\alpha \leq 2, D^{\alpha}$ is the standard Caputo fractional derivative, and $F$ is a convex compact valued multifunction. Compactness property of the solutions set is presented.


Keywords: Boundary Value Problem, Fractional Derivative.

## References:

[1] A. Amrani, C. Castaing and M. Valadier, Convergence in Pettis norm under extreme point condition, Vietnam Journal of Mathematics,(4), 323-335 (1998)
[2] D. L. Azzam, C. Castaing and L. Thibault, Three boundary value problems for second order differential inclusions in Banach spaces, Control Cybernet, 31 (2001), 659-693.
[3] M.Benchohra, J. Graef and F-Z.Mostefai, Weak solutions for boundary-value problems with nonlinear fractional differential inclusions, Nonlinear Dynamics and System Theory, 11 (3) (2011), 227-237.

# Some Inequalities for the Multiplicative Zagreb Indices of Graph Operations 

Yasar Nacaroglu and A.Dilek Maden<br>Department of Mathematics, Selcuk University, Konya, Turkey<br>yasarnacar@hotmail.com<br>aysedilekmaden@selcuk.edu.tr


#### Abstract

Molecular descriptors have found a wide application in QSPR/QSAR studies [1]. Among them, topological indices have a prominent place. The Zagreb indices are among the oldest degree-based topological invariants, were introduces by Gutman et al [2]. The multiplicative Zagreb indices were introduced by Todeschini et al. in 2010 [3]. In this paper, we give some upper bounds for the multiplicative zagreb indices of various graph operations such as union, join, rooted product, corona product, tensor product, etc. Keywords: Degree, Upper Bounds, Multiplicative Zagreb Indices, Graph Operations.

\section*{References:}


[1] R. Todeschini, V. Consonni, Handbook of Molecular Descriptors, Wiley VCH, Weinheim, 2000.
[2] I. Gutman, N. Trinajstic, "Graph theory and molecular orbitals. III. Total Пelectron energy of alternant hydrocarbons", Chem. Phys. Lett., 17(1972), 535538.
[3] R. Todeschini, V. Consonni, "New local vertex invariants and molecular descriptors based on functions of the vertex degrees", MATCH Commun. Math. Chem. MATCH Commun. Math. 64(2010), 359-372.
[4] W. Imrich, S. Klavzăr, Product graphs: Structure and Recognition, Wiley, New York (2010)

# Statistics of the Extreme Values Under Random Truncation 

Abdelhakim Necir<br>Laboratory of Applied Mathematics, Mohamed Khider University, Biskra, Algeria<br>necirabdelhakim@yahoo.fr


#### Abstract

A weighted Gaussian approximation to tail product-limit process for Pareto-like distributions of randomly right-truncated data is provided and a new consistent and asymptotically normal estimator of the extreme value index is introduced. A simulation study is carried out to evaluate the finite sample behavior of the proposed estimator and compare it to that recently proposed by Gardes and Stupfler [2]. Also, a new approach of estimating extreme quantiles, under random right truncation, is derived and applied to a real dataset of lifetimes of automobile brake pads. All the given results are recently published in [1]. Keywords: Empirical Process, Extreme Value Index, Heavy-Tails; High Quantiles, Hill Estimator; Lynden-Bell Estimator, Random Truncation..

\section*{References:} [1] Benchaira, S., Meraghni, D., Necir, A., 2016. Tail product-limit process for truncated data with application to extreme value index estimation. Extremes, on line: http://link.springer.com/article/10.1007/s10687-016-0241-9. [2] Gardes, L., Stupfler, G., 2015. Estimating extreme quantiles under random truncation. TEST 24: 207-227.


# Vaccination for Preventing Future Rubella Epidemic in Japan 

Hiroshi Nishiura, Ryo Kinoshita<br>Graduate School of Medicine, Hokkaido University<br>Sapporo, Japan<br>nishiurah@gmail.com


#### Abstract

A large epidemic of rubella occurred from 2012-14 in Japan, which has been attributed to insufficient herd immunity level and previous vaccination policy shifting from junior high school female students to both gender at the age of 12-24 months and 5-7 years. The present study aimed to identify best vaccination strategy that could minimally help us achieve herd immunity by targeting specific age group and gender. The so-called next generation matrix was computed from the growth phase of rubella epidemic in 2013. It appears that the optimal target host depends on available vaccination stock, but targeting adult male appears to be more efficient than targeting female or both gender. Vaccinating adults at workplace is likely an efficient prevention strategy.


Keywords: Epidemiological Model, Rubella, Vaccination, Optimization, Age, Japan

## References:

[1] R. Kinoshita, and H. Nishiura, "Assessing herd immunity against rubella in Japan: a retrospective seroepidemiological analysis of age-dependent transmission dynamics", BMJ Open, 5 (2016), e009928.
[2] H. Nishiura, R. Kinoshita, Y. Miyamatsu, and K. Mizumoto, "Investigating the immunizing effect of the rubella epidemic in Japan, 2012-14.", Int. J. Infect. Dis., 38 (2015), 16-18.

# Application of Data Mining to Insurance Multi-Risk Habitation and Professional Branch 

Lounici Nora, Sadi Khadidja, Djemaa Hassiba<br>Laboratory of Applied Statistics<br>ENSSEA,Pöle universitaire Koléa, Algiers- Algeria<br>noralounici@yahoo.fr, sadikh00@gmail.com, djemaa_hassiba@yahoo.fr


#### Abstract

Data Mining is the set of techniques and methods used in a process for the extraction of knowledge from data which is applied in several areas, among others, the insurance sector. There are a variety of insurance products. In our study, we are interested in on insurance (fire, Accidents and various risk), such as multi-risk habitation and professionals to reduce their severities. The purpose of this paper is the extraction of the characteristics of multi-risk habitation and professional products, and on the other hand the study of the factors that make the peculiarity of each product, according to a approach of Data Mining. To do this, we started by treatment and analyzing data. Secondly, we proposed a general approach based on a number of techniques for the selection of the most informative variables. In this context and in order to solve our problem, we opted for the following methods: binary logistic regression, the STEPDISC procedure and the $v$-test of Tanagra.


Keywords: Data Mining - Multi-Risk Insurance - Binary Logistic Regression Stepdisc - Test of Value.

## References:

[1] Francis, Noël, La gestion des sinistres IRD incendies et risques divers, édition séfi, Canada 2014
[2] Jean-Jacques Boreux E" ric Parent Jacques Bernier ; Pratique du calcul bayesien Springer-Verlag France, Paris, 2010
[3] Mireille Bardos, Analyse discriminante «application au risque et scoring financier » édition Dunod, Paris 2011.
[4] Tuffery. S, Data mining et statistique décisionnelle : l'intelligence des données, édition TECHNIP, 4éme édition, Paris 2012.

# Exact Solutions of Some Nonlinear Evolution Equations 

Meryem Odabasi, Emine Misirli<br>Tire Kutsan Vocational School, Ege University, Tire, Izmir, Turkey meryemodabasi@gmail.com, Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>emine.misirl@ege.edu.tr


#### Abstract

In this study, exact solutions of the nonlinear partial differential equations have been examined. The Tzitzeica-Dodd-Bullough [1], the nonlinear telegraph [2] and the generalized Benjamin-Bona-Mahony [3] equations are studied by means of the complete discrimination system and the modified trial equation method [4]. These equations have various applications in communication theory, nonlinear optics, solid state physics, quantum field theory and fluid dynamics, hence obtaining exact traveling wave solutions of the equations can be useful to understand the phenomena they describe.


Keywords: Tzitzeica-Dodd-Bullough Equation, Nonlinear Telegraph Equation, Generalized Benjamin-Bona-Mahony Equation.

## References:

[1] A. M. Wazwaz, , "The tanh and the sine-cosine methods for a reliable treatment of the modified equal width equation and its variants", Communications in Nonlinear Science and Numerical Simulation, 11(2005), 148-160.
[2] M. Mirzazadeh and M. Eslami, "Exact solutions of the KudryashovSinelshchikov equation and nonlinear telegraph equation via the first integral method", Nonlinear Analysis: Modelling and Control, 17.4(2012), 481-488.
[3] W. Li and Y. M. Zhao, "Exact Solutions for a BBM(m,n) equation with generalized evolution", Applied Mathematical Sciences, 6.27(2012), 1325-1334.
[4] C. S. Liu, "Trial equation method to nonlinear evolution equations with rank inhomogeneous: Mathematical discussions and its applications", Communications in Theoretical Physics, 45(2006), 219-223.
[5] M. Odabasi and E. Misirli, "On the solutions of the nonlinear fractional differential equations via the modified trial equation method", Mathematical Methods in the Applied Sciences, (2015), Doi: 10.1002/mma. 3533.
[6] M. Odabasi and E. Misirli, "Application of the extended trial equation method to the nonlinear evolution equations", Mathematical Sciences and Applications E-Notes, 2.1 (2014), 28-33.


# Existence of Solutions for Dynamic Systems on an Infinite Time Scale 

Arzu Denk Oguz and S. Gulsan Topal<br>Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>arzu.denk@hotmail.com; f.serap.topal@ege.edu.tr


#### Abstract

In this study, we consider the systems of second order multipoint boundary value problems on infinite time scales. Using the monotone method, we give the existence result for these systems.


Keywords: Infinite Interval, Lower And Upper Solutions, Time Scales.
References:
[1] R.P. Agarwal, D.O'Regan, Nonlinear boundary value problems on the semiinfinite interval: An upper and lower solution approach, Mathematika 49 (2002) 129-140.
[2] F.M. Atici, A. Cabada, C.J. Chyan, B. Kaymakçalan, Nagumo type existence results for second-order nonlinear dynamic BVPS, 60 (2005) 209-220. [3] M. Bohner, A. Peterson, Advances in Dynamic Equations on Time Scales, ,Birkhauser, Boston, Cambridge, MA (2003).
[4] A. Denk, S.G. Topal, Existence of positive solutions for the second order semipositone m-point boundary value problem, Differ. Equ. Dyn. Syst., 22 (3) (2014) 265-280.
[5] H. Lian, P. Wang, W. Ge, Unbounded upper and lower solutions method for Sturn-Liouville boundary value problem on infinite intervals, Nonlinear Analysis, 70 (2009) 2627-2633.
[6] B.Q. Yan, D.O'Regan, R.P. Agarwal, Unbounded solutions for singular boundary value problems on the semi-infinite interval: Upper and lower solutions and multiplicity, J. Comput. Appl. Math., 197 (2006) 365-386.


# An Extension of a Valuation $v$ on a Field $K$ with rankv $=2$ to 

$K(x)$

Figen Oke<br>Department of Mathematics,Faculty of Science Trakya University,Edirne,Turkey figenoke@gmail.com.


#### Abstract

Let $v$ be a valuation on a field $K, G_{v}$ be the value of $v$ and $k_{v}$ be the residue field of $v$ and $w$ be an extension of $v$ to $K(x) . w$ is called a residual algebraic free extension of $v$ if $k_{w} / k_{v}$ is an algebraic extension and $G_{w} / G_{v}$ is not torsion group. In this study it is assumed that $v=v_{1} \circ v_{2}$ is a valuation on a field $K$ with rankv $=2$. In this case there exist three kind residual algebraic free extensions of $v$ to $K(x)$. Here the one of these extensions is described and its properties are investigated. Keywords: Extensions of Valuations, Residual Algebraic Free Extensions, Rank of Valuation.


## References:

[1] V. Alexandru - N. Popescu - A. Zaharescu, "A theorem of characterization of residual transcendental extension of a valuation", J. Math. Kyoto Univ., 28 (1988), 579-592.
[2] V. Alexandru - N. Popescu - A. Zaharescu, "Minimal pair of definition of a residual transcendental extension of a valuation", J. Math. Kyoto Univ. 30 (1990), no. 2, 207-225.
[3] V. Alexandru - N. Popescu - A. Zaharescu, All valuations on K(X), J. Math. Kyoto Univ. 30 (1990), no. 2, 281-296.
[4] N. Bourbaki," Algebre Commutative", Hermann, Paris (1964).
[5] O. Endler, "Valuation Theory", Springer,Berlin Heidelberg-NewYork (1972).
[6] N. Popescu, C. Vraciu," On the extension of valuations on a field $K$ to $K(x)$ I",Ren. Sem. Mat. Univ. Padova, 87 (1992), 151-168.
[7] N. Popescu, C. Vraciu, "On the extension of valuations on a field K to $\mathrm{K}(\mathrm{x})$ II",Ren. Sem. Mat. Univ. Padova, 96(1996), 1-14.

# On Tauberian Conditions for the $(\bar{N}, p)$ Summability of Integrals 

Muhammet Ali Okur*, Ibrahim Canak** and Umit Totur*<br>*Department of Mathematics, Adnan Menderes University, Aydin, Turkey<br>**Department of Mathematics, Ege University, İzmir, Turkey mali.okur2@gmail.com


#### Abstract

In this paper, we obtain some new Tauberian theorems for the weighted mean method of integrals by using the weighted general control modulo. Our results in this work are generalizations of some classical type Tauberian theorems.I Keywords: Tauberian Theorem, Tauberian Condition, Weighted General Control Modulo, ( $\bar{N}, p$ ) Summability of Integrals.


## References:

[1] G. H. Hardy, "Divergent series", Clarendon Press, Oxford, 1949.
[2] M. Dik, "Tauberian theorems for the sequences with moderately oscillatory control moduli", Math. Morav. 5(2001), 57-94.
[3] Ü. Totur and İ. Çanak, "Some general Tauberian conditions for the weighted mean summability method", Comput. Math. Appl. 63.5(2012), 999-1006.
[4] İ. Çanak and Ü. Totur, "Tauberian conditions for Cesáro summability of integrals", Appl. Math. Lett. 24.6(2011), 891-896.
[5] Ü. Totur and M. A. Okur, "Alternative proofs of some classical Tauberian theorems for the weighted mean of integrals", Filomat, 29.10(2015), 2281-2287.

# Eigenvalues and Spectral Singularities of Non-Selfadjoint Matrix Sturm-Liouville Operators with Eigenvalue-Dependent Boundary Conditions 

Murat Olgun<br>Department of Mathematics, Faculty of Science, Ankara University, Ankara, Turkey<br>olgun@ankara.edu.tr


#### Abstract

In this talk we investigate discrete spectrum of the non-selfadjoint matrix Sturm-Liouville operator $L$ generated in $L^{2}\left(\mathbb{R}_{+} ; S\right)$ by the differential expression $$
\ell(y)=y^{\prime \prime}+Q(x) y, \quad x \in \mathbb{R}_{+}:=[0, \infty)
$$


and the boundary condition

$$
y^{\prime}(0)-\left(\beta_{0}+\beta_{1} \lambda+\beta_{2} \lambda^{2}\right) y(0)=0
$$

where $Q$ is a non-selfadjoint matrix valued function. Also using the uniqueness theorem of analytic functions we prove that $L$ has a finite number of eigenvalues and spectral singularities with finite multiplicities.
Keywords: Eigenvalues, Spectral Singularities, Spectral Analysis, SturmLiouville Operator, Non-Selfadjoint Matrix Operator.

## References:

[1] E. P. Dolzhenko, "Boundary value uniqueness theorems for analytic functions", Math. Notes 25 (1979), 437-442.
[2] V. A. Marchenko, "Sturm-Liouville Operators and Applications", Birkhauser Verlag, Basel,(1986).
[3] M. A. Naimark, "Investigation of the spectrum and the expansion in eigenfunctions of a non-selfadjoint operators of second order on a semi-axis", AMS Translations 2(16) 1960,103-193.
[4] E. Bairamov, and N. Yokus, "Spectral Singularities of Sturm-Liouville Problems with Eigenvalue-Dependent Boundary Conditions", Abstract and Applied Analysis, 2009, (2009) 1-8.
[5] M. Olgun, and C. Coskun, "Non-selfadjoint matrix Sturm-Liouville operators with spectral singularities", Applied Mathematics and Computations, 216 (8) 2010.
[6] A. M., Krall, E. Bairamov, and O. Cakar, "Spectral analysis of a nonselfadjoint discreteSchrödinger operators with spectral singularities", Math. Nachr. 231 (2001), 89-104.

# A New Type of Convergence for a Sequence of Rays 

Oznur Olmez and Salih Aytar<br>Department of Mathematics,Suleyman Demirel University, Isparta, Turkey

oznur_olmez@hotmail.com , salihaytar@sdu.edu.tr


#### Abstract

In this work, we introduce the concept of rough convergence of a sequence of rays. We give an alternative definition of the rough convergence for a sequence of rays. We also investigate the relations between the rough limit sets of a sequence of rays and its subsequence. Later, we prove the monotonicity of the rough limit set of a sequence of rays. We say that the rough limit set of such a sequence does not convex. In addition, we prove that if $\left(x^{\nu}\right) \xrightarrow{r}(x)$ and $\left(y^{v}\right) \xrightarrow{r}(y)$, then we have $\left(x^{v}+y^{v}\right) \xrightarrow{2 r}(x+y)$. Keywords: Rough Convergence, Sequence of Rays, Rough Limit Set.

\section*{References:} [1] W. Fenchel, (1953), "Convex Cones, Sets and Functions", Mimeographed notes by D.W.Blackett, Princeton Univ. Press Princeton, N.J. [2] H.X. Phu, "Rough convergence in normed linear spaces", Numer. Funct. Anal. and Optimiz., 22(2001), 201-224. [3] H.X. Phu, "Rough convergence in infinite dimensional normed spaces", Numer. Funct. Anal. and Optimiz., 24(2003), 285-301. [4] R.A. Wijsman, "Convergence of sequences of convex sets, cones and functions II", Trans. Amer. Math. Soc., 123(1966), 32-45.


# Some Commutativity Theorems for Rings with Involution 

Lahcen Oukhtite<br>Department of Mathematics, Faculty of Science and Technology, University Moulay<br>Ismail, Errachidia, Morocco oukhtitel@hotmail.com


#### Abstract

The purpose of this work is to study commutativity of certain rings with involution equipped with derivations (generalized derivations) satisfying particular identities. Some well-known results characterizing commutativity of prime rings have been generalized. Moreover, examples proving the necessity of our hypotheses are given.


Keywords: Commutativity, Derivations, Prime Rings, Involution.

## References:

[1] Ali, Shakir; Dar, Nadeem Ahmad; Khan, Abdul Nadim. On strong commutativity preserving like maps in rings with involution.
Miskolc Math. Notes 16 (2015), no. 1, 17-24.
[2] Ashraf, Mohammad; Siddeeque, Mohammad Aslam. On certain differential identities in prime rings with involution. Miskolc Math. Notes 16 (2015), no. 1, 33-44.
[3] De Filippis, Vincenzo; Mamouni, Abdellah; Oukhtite, Lahcen, Generalized Jordan semiderivations in prime rings. Canad. Math. Bull. 58 (2015), no. 2, 263-270.
[4] Bell, H. E.; Boua, A.; Oukhtite, L. Semigroup ideals and commutativity in 3prime near rings. Comm. Algebra 43 (2015), no. 5, 1757-1770.
[5] Oukhtite, Lahcen; Mamouni, Abdellah. Generalized derivations centralizing on Jordan ideals of rings with involution. Turkish J. Math. 38 (2014), no. 2, 225232.

# A Nonstandard Numerical Scheme for a Predator-Prey Model with Allee Effect 

Nihal Ozdogan<br>Isparta Vocational School,Suleyman Demirel University, Isparta, Turkey Mevlude Yakit Ongun<br>Department of Mathematics, Suleyman Demirel University, Isparta, Turkey mevludeyakit@sdu.edu.tr


#### Abstract

Predator-prey dynamics play an important role in mathematical biology. In this talk, we present a Lotka-Volterra predator-prey model with Allee effect. In this system with general functional response has an Allee effect on Prey population. $\alpha(x):=\frac{x}{\beta+x}, \beta>0$ term is called "Allee effect constant". A nonstandard finite difference scheme is constructed to simulate for the model. We use the Jury test which deal with coefficients of the characteristic polynomial for determining the stability of discrete time system.


Keywords: Allee Effect, Stability Analysis, Nonstandard Numerical Scheme, Schur-Cohn Criteria.

## References:

[1] R.E. Mickens, 'Nonstandard finite difference models of differential equations, World Scientific Publishing Co., Inc., River Edge, NJ, (1994), 981-02-1458-8.
[2] R.E. Mickens, "Difference equations. Theory, applications and advanced topics. Second edition. Monographs and Research Notes in Mathematics,(1990),978-1-4822-3078-9.
[3] D.T. Dimitrov, H.V. Kojouharov, "Nonstandard finite difference methods for predator-prey models with general functional response" Technical Report, 10, 2007.
[4] H. Merdan, "Stability Analysis of a Lotka-Volterra Type Predator-Prey System involving Allee effects. Anziam J, 52, (2010), 139-145.
[5] S.M. Moghadas, M.E. Alexander, B.D. Corbett, "A non-standard numerical scheme for a generalized Gause-type predator-prey model", Physica D, 188(2004)134-151.

On the Stability of a Neural Field Model

Berrak Ozgur and Ali Demir<br>Department of Mathematics Kocaeli University, Izmit, Kocaeli, Turkey<br>berrak.ozgur@kocaeli.edu.tr<br>ademir@kocaeli.edu.tr


#### Abstract

In this work, we present some conditions for the stability of a neural field model and the asymptotic stability region is determined in the parameter space. Moreover we determine the number of unstable characteristic roots in the regions where the asymptotic stability fails. Keywords: Neural Field Model, Stability Analysis. References: [1] Wilson, H., Cowan, J., A Mathematical theory of the functional dynamics of cortical and thalamic nervous tissue. Biol. Cybern. 13(2), 55-80 (1973) [2] Amari, S.I., Dynamics of pattern formation in lateral-inhibition type neural fields. Biol. Cybern. 27(2), 77-87 (1977) [3] Veltz, R., Faugeras, O., Stability of the stationary solutions of neural field equations with propagation delay. Journal of Mathematical Neuroscience 1:1 (2011) [4] Veltz, R., Interplay between synaptic delays and propagation delays in neural field equations. Siam Journal of Applied Dynamical Systems, 12(3), 1566-1612 (2013) [5] Insperger, T., Stépán, G., Semi-discretization for time-delay systems, Stability and engineering applications, Springer New York, (2011) [6] Veltz, R., Faugeras, O.,A center manifold result for delayed neural fields equations, Siam Journal on Mathematical Analysis 45(3), 1527-1562 (2013) [7] Atay, F.M., Hutt, A., Stability and bifurcations in neural fields with finite propagation speed and general connectivity, Siam Journal on Mathematical Analysis 5(4), 670-698 (2006) [8] Coombes, S., Waves, bumps, and patterns in neural field theories, Biological Cybernetics, Volume 93, Issue 2, pp 91-108, (2005) [9] Coombes, S., Venkov, N.A., Shiau, L., Bojak, L., Liley, D.T.J., Laing, C.R., Modeling electrocortical activity through improved local approximations of integral neural field equations, Phys. Rev. E 76, 051901, (2007) [10] Grimbert F., Mesoscopic models of cortical structures, Ph.D. Thesis, (2008) [11] Veltz, R., Nonlinear analysis methods in neural field models, Ph.D. Thesis, (2011) [12] Stépán, G., Retarded dynamical systems: stability and characteristic functions, Longman Scientific \& Technical, England, (1989) [13] Huang, C., Vandewalle, S., An analysis of delay dependent stability for ordinary and partial differential equations with fixed and distributed delays, SIAM Journal on Scientific Computing, 25(5):1608--1632, (2004)


# Value Groups and Residue Fields Writing via Distinguished Chains 

Burcu Ozturk, Figen Oke<br>Department of Mathematics, Trakya University, Edirne, Turkey<br>burcuozturk@trakya.edu.tr, figenoke@gmail.com


#### Abstract

Let $v$ be a valuation of a field $K$ and $\bar{v}$ be the fixed extension of $v$ to the algebraic clousure $\bar{K}$ of $K$. Let $v_{a}$ be the restriction of $\bar{v}$ to the field $K(a)$ where $a \in \bar{K} \backslash K$. In this study a distinguished chain of an element $a \in \bar{K} \backslash K$ is considered and the value group and residue field of the valuation $v_{a}$ is written by using the $\delta_{K}$ constants and minimal polynomials of the elements in the chain. Also the value groups and residue fields of the valuations $v_{a_{i} b_{i}}$ of the fields $K\left(a_{i}, b_{i}\right)$ is compared where $a=a_{0}, a_{1}, \ldots, a_{n}$ is a distinguished chain of $a \in$ $\bar{K} \backslash K$ and $b=b_{0}, b_{1}, \ldots, b_{n}$ is a distinguished chain of $b \in \bar{K} \backslash K$.


Keywords: Valuations, Distinguished Chain, Algebraic Extension.
References:
[1] K. Aghigh, S.K. Khanduja, On Chains Associated with Elements Algebraic over a Henselian Valued Field, Algebra Colloquium 12:4, 607-616, (2005)
[2] N. Popescu, A. Zaharescu, On The Structure of Irreducible Polynomials over Local Fields, J. Number Theory, 52, 98-118, (1995)

# Fixed Point Results for General Type Contractions in Modular Spaces 

Mahpeyker Ozturk<br>Department of Mathematics, Sakarya University, Sakarya, Turkey<br>mahpeykero@sakarya.edu.tr


#### Abstract

In the context of modular space, by using the existing findings we introduce some general type contractions and investigate the existence of fixed points and coincidence points for the mappings satisfying such contractions.


Keywords: Modular space, Coincidence Point, Common Fixed Point.
References:
[1] G.V.R. Babu, P. D. Sailaja, "Fixed point results on ordered metric spaces via generalized altering distance functions in four variables", J. Advanced Research in Pure Mathematics, 5.1 (2013), 143-158.
[2] F. Zabihi, A. Razani, "Fixed point theorems for hybrid rational Geraghty contractive mappings in ordered $b$-metric spaces", Journal of Applied Mathematics, 2014 (2014), 9 pages.
[3] C. Mongkolkeha, P. Kumam, "Some fixed point results for generalized weak contraction mappings in modular spaces", International Journal of Analysis, 2013(2013), 6 pages.
[4] C. Mongkolkeha, P. Kumam, "Fixed point and common fixed point theorems for generalized weak contraction mappings of integral type in modular spaces", International Journal of Mathematics and Mathematical Sciences, 2011(2011), 12 pages.

# Common Best Proximity Points of Generalized Almost ( $\alpha, \boldsymbol{\beta}$ )$(\psi, \phi)$-Geraghty Contractive Mappings in Metric Spaces 

Mahpeyker Ozturk, Neslihan Kaplan<br>Department of Mathematics, Sakarya University, Sakarya, Turkey<br>neslihankaplan.nk@gmail.com, mahpeykero@sakarya.edu.tr


#### Abstract

The main purpose of this study is to introduce the notions of cyclic $(\alpha, \beta)$ - proximal admissible pair and almost generalized $(\alpha, \beta)-(\psi, \phi)$-Geraghty contractive nonself mappings in metric spaces. Also, we demonstrate the existence and uniqueness of common best proximity point of such contractions in the context of metric space. Some concrete example is furnished which shows validity of our results.


Keywords: Common Best Proximity Point, Metric Space, Generalized Almost $(\alpha, \beta)-(\psi, \phi)$-Geraghty Contractions.

## References:

[1] W. Sintunavarat, M. Öztürk and E. Girgin, "Common fixed points of almost generalized $(\alpha, \beta)-(\psi, \phi)$-weakly contractive mappings in modular spaces", submitted.
[2] H. Aydi, A. Felhi and E. Karapınar, "On common best proximity points for generalized $\alpha$ - $\psi$-proximal contractions", J. Nonlinear Sci. Appl., 9(2016), 24582466.
[3] M. Öztürk and N. Kaplan, "Best proximity point results for F- contractions satisfying rational expressions in complex valued metric spaces", submitted.
[4] E. Karapınar, "On best proximity point of $\psi$ - Geraghty contractions", Fixed Point Theory and Applications, 1(2013), 1-9.
[5] D. Doric, "Common fixed point for generalized ( $\psi, \varphi)$ - weak contractions", Appl. Math Lett., 22(2009), 1896-1900.

# Relations Between Darboux and Bishop Frames on a Regular Curve in Minkowski Space 

Emin Ozyllmaz, Amine Yılmaz<br>Faculty of Science, Department of Mathematics, Ege University, Bornova, İzmir, Turkey<br>emin.ozyilmaz@ege.edu.tr, amineyilmaz2020@hotmail.com


#### Abstract

In this work, we give relationships between Darboux and Type-2 Bishop frames in Minkowski space. We find the geodesic curvature , normal curvature and geodesic torsion according to Type-2 Bishop curvatures of a spacelike unit speed curve on a timelike surface. Also, we give transition matrix between the Darboux and Type-2 Bishop frames. Moreover, we give some relations between Darboux and Type-2 Bishop frames of the spherical images of the apparatus of type-2 Bishop frames. Finally, we obtain some interesting relations and illustrates of the examples by the aid of Maple programe.


Keywords: Bishop Frame, Darboux Frame, Regular Curve, Minkowski Space.

## References:

[1] E.Özyılmaz, S.Yılmaz, M.Turgut, ' 'Relationships among Darboux and Bishop Frames" In Gediz University,1st International Symposium on Computing in Science \& Engineering,(2010).
[2] J.Walrave, "Curves and Surfaces in Minkowski space", Ph.D.Thesis,K.U.Leuven, Leuven, Belgium,(1995).
[3] O’Neill B " Elemantary Differential Geometry. Academic Press Inc. New York,(1966).
[4] Özdemir M, Ergin AA "Parallel Frames of Non-Lightlike Curves." Missouri J. Math. Sci., 20(2): 1-10,(2008).
[5] Yılmaz S., Bishop spherical images of a spacelike curve in Minkowski 3Space, Int. Jour. Phys. Scien., 5 (6) (2010) 898-905.
[6] Lopez R "Differential Geometry of Curves and Surfaces in LorentzMinkowski Space." Preprint 2008: arxiv:0810.3351v1 [math.DG],(2008).
[7] M.Do Carmo,".Differential Geometry of Curves and Surfaces,New Jersey:Prentice-Hall Inc,1976.
[8] Uğurlu H.H.(1997)."On the Geometry of Time-Like Surfaces", Commund.Fac.Sci.Univ.Ank.Series A1V.46.pp.211-223, (1997).

# Results on Hadamard Codes and Codes Over Rings 

Mustafa Ozkan and Figen Oke<br>Department of Mathematics,Faculty of Science Trakya University,Edirne,Turkey mustafaozkan@trakya.edu.tr figenoke@gmail.com.


#### Abstract

In this paper, the results obtained by authors M. Özkan, F. Öke [1] are extended for codes over the ring $\mathrm{F}_{2}+u \mathrm{~F}_{2}+u^{2} \mathrm{~F}_{2}$ where $u^{3}=0$. Using special matrices over $\mathrm{F}_{2}+u \mathrm{~F}_{2}+u^{2} \mathrm{~F}_{2}$ where $u^{3}=0, C^{\alpha, \beta}$ codes are defined. It is shown that Gray images of the codes $C^{\alpha, \beta}$ are Hadamard codes over binary field. In addition to these codes, new codes are defined over the ring $\mathrm{F}_{2}+u \mathrm{~F}_{2}+u^{2} \mathrm{~F}_{2}$. Moreover, a relation between new codes and Hadamard codes is established.


Keywords: Codes Over Rings, Hadamard Codes, Linear Codes.

## References:

[1] M. Özkan, F. Öke, "A relation between Hadamard codes and some special codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}$ " App.Mathematics and Inf Sci. (accepted).
[2] J.F.Qian, L.N.Zhang, S.X Zhu, " (1-u) - constacyclic and cyclic codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}{ }^{\text {" }}$, Applied Mathematics Letters 19,(2006)820-823.
[3] J.F.Qian, L.N.Zhang, Shixin Zhu, "Constacyclic and cyclic codes over $\mathrm{F}_{2}+u \mathrm{~F}_{2}+u^{2} \mathrm{~F}_{2} \quad$ ",IEICE Trans.Fundamentals, E89-a, no 6(,2006),1863-1865.
[4] Vermani, L. R., "Elements of Algebraic Coding Theory", Chapman Hall , India.,(1996).
[5] Krotov, D. S. "Z4-linear perfect codes" ,Diskretn. Anal. Issled. Oper. Ser.1.Vol. 7, 4. (2000)78-90.
[6] Krotov,D. S. "Z4-linear Hadamard and extended perfect codes", Procs. of the International Workshop on Coding and Cryptography,Paris,(2001)329-334.
[7] A. Bonnecaze and P. Udaya,"Cyclic codes and self dual codes $\mathrm{F}_{2}+u \mathrm{~F}_{2}$ ", IEEE Trans. Inf. Theory, 45, (1999),1250-1255.
[8] S. Zhu, Y. Wang, M. Shi, "Some Result On Cylic Codes Over $\mathrm{F}_{2}+v \mathrm{~F}_{2}$ "
IEEE Trans. Inf. Theory, 56,no 4 (2010),1680-1684.
[9] W. C. Huffman and Vera Pless, "Fundamentals of Error Correcting Codes, Cambridge" (2003).

# Perturbation-Iteration Method for Singular Problems 

Mehmet Pakdemirli<br>Applied Mathematics and Computation Center, Celal Bayar University, Muradiye, Yunusemre, Manisa, Turkey<br>mpak@cbu.edu.tr


#### Abstract

Perturbation-iteration method of solving differential equations is first proposed by Pakdemirli et al. [1] for the first order ordinary differential equations. The method is called as "the perturbation iteration method" to distinguish it from the past literature on the so-called "iteration-perturbation methods" [2-5] which are not systematic approaches and does not produce general algorithms valid for various types of differential equations. The new perturbation-iteration method is directly applicable in a systematic algorithmic way, does not require special transformations or ad hoc assumptions. Second order differential equations were treated by the same method by Aksoy and Pakdemirli [6] for Bratu type equations. Aksoy et al. [7] further solved some nonlinear heat transfer equations. Dolapci et al. [8] applied the method to the Fredholm and Volterra integral equations. Şenol et al. [9] treated the first order differential equation systems using the method. Pakdemirli [10] reviewed the mentioned work. In a very recent work, Pakdemirli [11] successfully applied the method to problems with secularities also. Non-singular perturbation problems were investigated in all of the previous mentioned work. In this study, the perturbation-iteration method is applied to singular perturbation problems (Boundary layer type equations). The approximate solutions are contrasted with the available exact or numerical solutions. It is found that the new method produces compatible solutions with the exact or numerical solutions.


Keywords: Perturbation-Iteration Method, Singular Perturbation Problems, Numerical Computations.

## References:

[1] M. Pakdemirli, Y. Aksoy and H. Boyacı, "A New Perturbation-Iteration Approach for First Order Differential Equations", Mathematical and Computational Applications, 16(2011), 890-899.
[2] J. H. He, "Iteration Perturbation Method for Strongly Nonlinear Oscillators", Journal of Vibration and Control, 7(2001), 631-642.

[3] R. E. Mickens, "Iteration Procedure for Determining Approximate Solutions to Nonlinear Oscillator Equations", Journal of Sound and Vibration, 116(1987), 185-187.
[4] R. E. Mickens, "A Generalized Iteration Procedure for Calculating Approximations to Periodic Solutions of 'Truly Nonlinear Oscillators' ", Journal of Sound and Vibration, 287(2005), 1045-1052.
[5] R. E. Mickens, "Iteration Method Solutions for Conservative and Limit-cycle Force Oscillators", Journal of Sound and Vibration, 292(2006), 964-968.
[6] Y. Aksoy and M. Pakdemirli, "New Perturbation-Iteration Solutions for Bratu-type Equations", Computers \& Mathematics with Application, 59(2010), 2802-2808.
[7] Y. Aksoy, M. Pakdemirli, S. Abbasbandy and H. Boyacı, "New PerturbationIteration Solutions for Nonlinear Heat Transfer Equations", International Journal of Numerical Methods for Heat \& Fluid Flow, 22(2012), 814-828.
[8] İ. T. Dolapci, M. Şenol and M. Pakdemirli, "New Perturbation Iteration Solutions for Fredholm and Volterra Integral Equations", Journal of Applied Mathematics, (2013), Article ID 682537, http://dx.doi.org/10.1155/2013/682537. [9] M. Şenol, İ. T. Dolapci, Y. Aksoy and M. Pakdemirli, "Perturbation-Iteration Method for First-Order Differential Equations and Systems" Abstract and Applied Analysis (2013), Article ID 704137, http://dx.doi.org/ 10.1155/2013/704137.
[10] M. Pakdemirli, "Review of the new perturbation iteration method", Mathematical and Computational Applications, 18(2013), 139-151.
[11] M. Pakdemirli, "Perturbation-iteration method for strongly nonlinear vibrations", Journal of Vibration and Control, (article in press), doi:10.1177/1077546315586647.

# Transportation Situations and Related Games with Interval Uncertainty 

Osman Palanci*, Sirma Zeynep Alparslan Gok**, Hamid Alzaki, Sule Nizamoglu<br>*Department of Mathematics, Suleyman Demirel University, Isparta, Turkey<br>osmanpalanci@sdu.edu.tr<br>**Department of Mathematics, Suleyman Demirel University, Isparta, Turkey<br>sirmagok@sdu.edu.tr


#### Abstract

Cooperative interval games have broad applicability in Operations Research (OR) such as sequencing, minimum cost spanning tree situations, in economy and also on modern finance. On the other hand, uncertainty is present in almost every real-world situation, it is influencing and questioning our decisions. In this study, we analyze an OR game arising from an OR situation under uncertainty namely transportation interval games. Hence, we use the theory of cooperative interval games. Firstly, we introduce the interval Shapley value of the game. Then, we obtain a one-point solution using the one-stage producere depending on the proportional rule (PROP), the constrained equal awards rule (CEA) and the constrained equal losses (CEL) rule. On the other hand, the nonemptiness of the interval core for the transportation interval games, and some results on the relationship between the interval core and the dual interval optimal solutions of the underlying transportation situations are provided.


Keywords: Operations Research, Transportation Situations, Cooperative Interval Games, The Proportional Rule, The Constrained Equal Awards Rule, The Constrained Equal Losses Rule.


# Solutions of Some Diophantine Equations 

Bilge Peker<br>Department of Mathematics Education, Necmettin Erbakan University,<br>Meram, Konya, Turkey<br>bilge.peker@yahoo.com


#### Abstract

One of the most complicated equations are Diophantine equations and solutions of such equations attract many mathematicians. In this study, our aim is to obtain the general solutions of some Diophantine equations in terms of some special sequences.


Keywords: Diophantine Equations, Pell Equations, Integer Sequences.

## References:

[1] W.J. LeVeque, Topics in Number Theory, Volume I and II, Dover Publications, New York, 2002.
[2] J.P. Robertson, Solving the generalized Pell equation $x^{2}-D y^{2}=N$, www.jpr2718.org/pell.pdf, 2004.
[3] T. Koshy, Fibonacci and Lucas numbers with applications, John Wiley and Sons, New York, Toronto, 2001.

Numerical Solution for a Free Convection Flow

Haldun Alpaslan Peker<br>Department of Mathematics, Selcuk University,<br>Campus, Konya, Turkey<br>pekera@gmail.com


#### Abstract

Heat transfer is an important process in engineering applications and physics and for this reason, advances in heat transfer improve the efficiency of many processes. The aim of this study is to present a semi-analytic solution of a free convection flow. Keywords: Analytic Solution, Convection, Heat Transfer.

\section*{References:} [1] E. Schmidt und W. Beckmann, "Das Temperatur-und Geschwindigkeitsfeld vor einer Wärme abgebenden senkrechter Platte bei natürlicher Konvektion", Tech. Mech. U. Thermodynamik, 1.10 (Okt. 1930), 341-349; cont., 1.11 (Nov. 1930), 391-406. [2] G.A. Baker, Essentials of Padé Approximants, Academic Press, London, 1975. [3] B.L. Kuo, "Application of the differential transformation method to the solutions of the free convection flow", Applied Mathematics and Computation, 165.1 (2005), 63-79.


# A Method for Decision Making Problems by Using Graph Representation of Soft Set Relations 

Nazan Cakmak Polat, Bekir Tanay<br>Department of Mathematics, Muğla Sitkı Koçman University, Muğla, Turkey<br>ncakmak@mu.edu.tr


#### Abstract

Soft set theory defined by Molodtsov in [8], has a rich potential for applications in several directions in life. One of successful application of soft set theory is about constructing new methods which are effective for Decision Making problems. In this study we introduced a method by using graph representation of soft set ralations, for solving Decision Making problems. Then we gave examples as an application of this method.


Keywords: Soft Set, Decision Making Problems, Soft Set Relations.

## References:

[1] H. Aktas, \& N. Çağman, Soft sets and soft groups, Information Sciences, 117(2007), 2726-2735.
[2] K. V. Babitha and J. J.Sunil, Soft set relations and functions, Comput. Math. Appl., 60(2010), 1840-1849.
[3] Babitha, K. V. and J. J. Sunil, Transitive closures and ordering on soft sets, Comput. Math. Appl. 62(2011), 2235-2239.
[4] N. Çağman and S. Enginoglu, Soft matrix theory and its decision making, Computers and Mathematics with Applications 59(2010.), 3308-3314.
[5] M. K. Dauda, I. Aliyu and A. M. Ibrahim, Partial Ordering in Soft Set Context, Mathematical Theory and Modeling, 3(2013), No.8.
[6] A. M. Ibrahim, M. K. Dauda and D. Singh, Composition of soft set relations and construction of transitive closure, Mathematical Theory and Modeling, 2(2012), No. 7 .
[7] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications, 45(2003), 555-562.
[8] D. Molodtsov, Soft Set Theory-First Result, Computers and Mathematics with Applications, 37(1999), 19-31.
[9] J. H. Park, O.H. Kim and Y.C. Kwun, Some properties of equivalence soft set relations, Computers \& Mathematics with Applications, 63(2012), (6) 10791088.
[10] H. Yang and Z. Guo, Kernels and closures of soft set relations, and soft set relation mappings, Comput. Math. Appl. 61(2011), 651-662.
[11] R. P. Grimaldi, Discrete and Combinatorial Mathematics (an Applied Introduction), Addison-Wesley, Fifth Edition.
[12] Xiaohong Zhang, On Interval Soft Sets with Applications, International Journal of Computational Intelligence Systems, 7(2014),1, 186-196.

# The Best Quadratic Approximation of Hyperbola with Order Four 

Abedallah Rababah<br>Department of Mathematics and Statistics, Jordan University of Science and Technology, 22110 Irbid, Jordan<br>rababah@just.edu.jo


#### Abstract

The best parametric polynomial approximation of degree 2 to the hyperbola is given. This approximation has order 4. The associated error function vanishes 4 times and equioscillates 5 times. For an arc of the hyperbola of length 4.16708 , the error is bounded by 0.06 .. Details of the derivation are presented to show how to apply the method. The method is simple; this encourages and motivates people working in CG and CAD to apply it in their works.


Keywords: Bezier Curves, Uniform Approximation, Hyperbola, High Order of Approximation, Chebyshev Polynomials.

## References:

[1] A. Rababah, "Taylor Theorem for Planar Curves", Proc. Amer Math. Soc. 119(3) (1993), 803-810.
[2] A. Rababah, "High Accuracy Hermite Approximation for Space Curves in R^d", J. Math. Anal. And Appl. 325(2) (2007), 920-931.
[3] A. Rababah, "The Best Uniform Quadratic Approximation of Circular Arcs with High Accuracy", Open Math. Soc. (2016).

# Existence Results for Weak Solution of Multi-Term Fractional Differential Equations in Non-Reflexive Banach Spaces via Riemann Pettis Integral 

Ghaus Ur Rahman<br>Department of Mathematics \& Statistics, University of Swat, KPK, Pakistan<br>dr.ghaus@uswat.edu.pk


#### Abstract

Fractional differential equations have fascinated scientists by virtue of its various applications in many branches of sciences e.g. physics, chemistry, mathematical biology [1]. Usually we deal with those differential equations [2], which contain one differential operator; nevertheless, in certain situation we need to solve a phenomina while using multi-term fractional differential operator [3]. The present paper explorer sufficient conditions for the existence of weak solution of the multi-term fractional differential equations in non-reflexive Banach spaces. Here differential operators are weak fractional Caputo derivatives while the solution is weakly absolutely continuous function. Weak measure of non-compactness and O'Regan fixed point theorem is used to show existence results of the solution of differential equation.


Keywords: Weak Measure of Non-Compactness, Non-Reflexive Banach Spaces, Fractional Differenational Equations.

## References:

[1] K. Diethelem, The Analysis of Fractional Differential Equations, Springer Heidelberg Dordrecht London New York, 2004
[2] A. M. Gomaa, Weak and strong solutions for differential equations in Banach spaces, Chaos, Solutions and Fractional 18(2003) 687-692.
[3] T. Edwards1, Neville J. Ford;, A. Charles Simpson The numerical solution of linear multi-term fractional differential equations: Journal of Computational and Applied Mathematics 148 (2002) 401 - 418

# An M/M/2 Retrial Queue with Breakdowns and Repairs 

Lila Raiah and Nadia Oukid<br>Saad Dahlab University<br>Department of Mathematics<br>Blida, Algeria<br>raiah.lila@yahoo.fr; oukidnad@yahoo.fr


#### Abstract

Retrial queues have been widely used to model many problems arising in telephone switching systems, telecommunication networks, computer networks and computer systems, etc. It is of basic importance to study reliability of retrial queues with server breakdowns and repairs because of limited ability of repairs and heavy influence of the breakdowns on the performance measure of the system. For a detailed review of the main results and the literature on this topic, the reader is referred to the monographs of Falin and Templeton [1], Artalejo and Gomez-Corral [2]. Multi-server retrial queueing systems, in general, are difficult to analyze from a mathematical standpoint see [3] and [4]. Exact results for the steady-state probabilities of reliable systems are given only for the single and two-server cases. In the unreliable model, there are no exact solutions when the number of servers exceeds one [5]. In this paper, we consider an unreliable $\mathrm{M} / \mathrm{M} / 2$ retrial queueing system for which both servers are subject to active and idle breakdowns. Arriving customers who find both servers busy or failed are given the choice to abandon their service request or enter a retrial orbit. Should a failure occur while a customer is in service, the customer is given the option to depart the system or proceed to the retrial orbit. We also assume that preempted customers, once able to regain access to a server, repeat their service requests. We assume that each server has its own dedicated repair person, and repairs begin immediately following a failure. Inter-failure times, repair times and times between retrials are exponentially distributed, and all processes are assumed to be mutually independent. Our main purpose is to derive approximate expressions for several congestion and delay measures. Next, we give approximate analysis for reliability of such a retrial queues. We obtain approximations of some main reliability indexes such as the availability, failure frequency and reliability function of the servers. Extensive numerical illustrations are provided.




Keywords: Reliability, Retrial Queues, Approximations, Breakdowns.

## References:

[1] G. Falin and J. G. C. Templeton, "Retrial Queues", Chapman \& Hall, London. (1997).
[2] J.R. Artalejo and A. Gomez-Corral, "Retrial Queueing Systems: A Computational Approach, ystems with customer retrials ", Springer, Berlin, Germany, (2008).
[3] J.R. Artalejo and M.J. Lopez-Herrero, "Analysis of a multiserver queue with setup times". Queueing Systems 52 (2005), 53-76.
[4] V. M. Abramov, "Analysis of multiserver retrial queueing system: A martingale approach and an algorithm of solution". Annals of Operations Research, 141(2006), 19-50.
[5] J. Wang, J. Cao, and Q. Li, "Reliability analysis of the retrial queue with server breakdowns and repairs". Queueing Systems, 38 (4) (2001), 363-380.


# Isothermic Hopf Cylinders and Slant Helices 

Cagla Ramis and Yusuf Yayli<br>Department of Mathematics, Ankara University, Tandogan, Istanbul, Turkey cramis@ankara.edu.tr yayli@science.ankara.edu.tr


#### Abstract

Hopf cylinder which is the inverse image of spherical curves in 2sphere by means of the Hopf fibration $\pi: S^{3} \rightarrow S^{2}$ is introduced by Pinkall [4]. Besides, the isothermic surface characterization for Hopf cylinders is obtained in terms of the geodesic curvature of the generator curve of Hopf cylinder in [2]. This paper aims that the search of the isothermic condition for Hopf cylinders determined by spherical slant helices with their new characterization endowed by Menninger [1]. Moreover, we give an efficient geometric interpretation of curves of constant precession introduced by Scofield [3] with the isothermic theory. Keywords: Hopf Map, Hopf Cylinder, Isothermic Surface, Slant Helix, Curve of Constant Precession.

\section*{References:} [1] A. Menninger, "Characterization of the slant helix as a successor curve of the slant helix", Int. Electron. J. Geom. 7(2014), 84-91. [2] G. Preissler, "Isothermic surfaces and Hopf cylinders", Beitr Algebra and Geom. 44(2003), 1-8. [3] P. D. Scofield, "Curves of constant precession", Amer. Math. Monthly, 102(1995), 531-537. [4] U. Pinkall, "Hopf tori in S3", Invent. Math. 81(1985), 379-386.




# On the M-Power Class (N) 

M.H.M.Rashid<br>Dept. of Mathematics \& Statistics<br>Faculty of Science P.O.Box(7)<br>Mu'tah University-Jordan<br>malik_okasha@yahoo.com

Abstract: A Hilbert space $T \in B(H)$ is said to $M$-power class $(N)$ if
there is a real number $M>0$ such that $\left\|(T-\lambda)^{n} x\right\|^{2} \leq\left\|(T-\lambda)^{2 n} x\right\| \mid\|x\|$
for all $\lambda>0$ and all $x \in H$. In this paper we prove the following assertions:
(i) $T$ is $M$ - power class ( $N$ ) if and only if

$$
M^{2}(T-\lambda)^{* 2 n}(T-\lambda)^{2 n}-2 r(T-\lambda)^{* n}(T-\lambda)^{n}+r^{2} I \geq 0
$$

for all $\lambda>0$ and $r>0$.
(ii) If $T$ is invertible $M$-power class ( $N$ ), then $T^{-1}$ is also $M$-power class ( $N$ ).
(iii) If $T$ is partial isometry $M$-power class $(N)$ satisfies $\|T-\lambda\| \leq \frac{1}{M}$, then $T$ is subnormal.
(iv) If $T$ is $M$-power class ( $N$ ), then it is an isoloid.

## References:

[1] S.C. Arora, Ramesh Kumar, M-Paranormal operators, Publ. Inst. Math., Nouvelle serie 29 (43) (1981), 5-13.
[2] P.R. Halmos, A Hilbert Space Problem Book, Van Nostrand, Princeton, 1967.
[3] V. Istratescu, Some result on M-hyponormal operators, Math. Seminar Notes 6 (1978), 77-86.
[4] K. B. Laursen, Operators with _nite ascent, Pacific J. Math. 152 (1992), 323336.
[5] J. G. Stampi, Hyponormal operators and spectrum density, Trans. Amer. Math. Soc. 117 (1965), 469-476.
[6] C.Schmoeger, On totally paranormal operators, Bull. Austral. Math. Soc. 66 (2002), 425-441.
[7] K. Tanahashi and A. Uchiyama, A note on *-paranormal operators and related classes of operators, Bull. Korean Math. Soc. 51(2014), no.2, 357-371.

# Generalized Topological Vector Spaces 

Salti Samarah<br>Department of Mathematics, Jordan University of Science and Technology<br>Jordan<br>samarah@just.edu.jo


#### Abstract

Let $X$ be a non-empty set and $\exp X$ the power set of $X$.We call a class $\mu \subseteq \exp X$ a generalized topology [5] (briefly, GT) if $\phi \in \mu$ and the arbitrary union of elements of $\mu$ belongs to $\mu$. A set $X$ with a GT $\mu$ on it is called a generalized topological space (briefly, GTS) and is denoted by $(X, \mu)$.


For a GTS $(X, \mu)$, the elements of $\mu$ are called $\mu$-open sets and the complements of $\mu$-open sets are called $\mu$-closed sets
A space $\left(X_{\mu}, \mathrm{F}_{\eta}\right)$ is said to be a generalized topological vector space over the filed F if the following two conditions are satisfied: 1. for each $x, y \in X$ and each $\mu$-open neighbourhood $W$ of $x+y$ in $X$, there exist $\mu$-open neighbourhood $U$ and $V$ in $X$ of $x$ and $y$ respectively, such that $U+V \subseteq W$.
2. for each $x \in X, \lambda \in \mathrm{~F}$ and for each $\mu$-open neighbourhood $W$ of $\lambda x$ in $X$, there exist $\eta$-open neighbourhood $U$ of $\lambda$ in F and $V$ of $x$ in $X$ such that $U \bullet V \subseteq W$.
In this paper, we introduce and study the new classes of generalized topological vector spaces are investigated and several new facts concerning generalized topological vector spaces are established.
Keywords: Generalized Topology, Topological Vector Space, $(\mu, \lambda)$ Continuous, $(\mu, \lambda)$ - Homeomorphism.

## References:

[1] A. Al-Omari and T. Noiri, A unified theory of contra- $(\mu, \lambda)$-continuous functions in generalized topological spaces, Acta Math. Hungar., 135 (1-2) (2012), 31-41.
[2] A. Al-Omari and T. Noiri, A Unified Theory of Weakly Contra- $(\mu, \lambda)$ continuous Functions in Generalized Topological Spaces, Stud. Univ. Babe S Bolyai Math. 58 (2013), (1), 107-117

[3] Á. Császár, Generalized open sets, Acta Math. Hungar. 75 (1997), 65-87.
[4] Á. Császár, $\gamma$-compact spaces, Acta Math. Hungar. 87 (2000), 99-107.
[5] Á. Császár, Generalized topology, generalized continuity, Acta Math. Hungar. 96 (2002), 351-357.
[6] Ả. Császár, $\gamma$-connected sets, Acta Math. Hungar. 101 (2003), 273-279.
[7] Á. Császár, Generalized open sets in generalized topologies, Acta Math.Hungar. 106 (2005), 53-66.
[8] Á. Császár, Further remarks on the formula for $\gamma$-interior, Acta Math. Hungar. 113 (2006), 325-332.
[9] Á. Császár, Products of generalized topologies, Acta Math. Hungar. 123 (2009), 127-132.
[10] A. Grothendiec, Topological vector spaces Gordon and Breach Science Publishers, New York, (1973).
[11] Y. Q. Chen, Fixed Points for convex Continuous Mappings in Topological Vector Space, American Mathematical Society, 129, 2157-2162.


# On $S_{\lambda}^{L}(I)^{\alpha}$ Asymptotically Statistical Equivalent of Functions 

Rabia Savas, Mahpeyker Ozturk<br>Department of Mathematics, Sakarya University, Sakarya, Turkey<br>rabiasavass@hotmail.com, mahpeykero@sakarya.edu.tr


#### Abstract

This paper uses $x(t)$ and $y(t)$ two nonnegative real-valued Lebesque measurable functions in the interval $(1, \infty)$ instead of sequences to introduce new definitions which are interrelated to notions asymptotically $I_{\lambda}$-statistical equivalent of order $\alpha$ to multiple L and strongly $I_{\lambda}$-asymptotically equivalent of order $\alpha$ to multiple L. In addition, we shall also present some of inclusion theorems. Keywords: Ideal, Filter, I-Statistical Convergence, Real Valued Function, Asymptotical Equivalent.

\section*{References:} [1] S. Bhunia, Pratulananda Das, S. Pal, Restricting statistical convergenge, Acta Math. Hungar, 134 (1-2) (2012), 153-161. [2] H. Cakalli, A study on statistical convergence, Funct. Anal. Approx. Comput., 1(2), (2009), 19-24. [3] R. Colak, Statistical convergence of order $\alpha$, Modern methods in Analysis and its Applications, New Delhi, India, Anamaya Pub., (2010), 121-129. [4] R. Colak, and C. A. Bektas, $\lambda$-statistical convergence of order $\alpha$, Acta Math. Scientia, 31B (3) (2011), 953-959.


# On Asymptotically I-Lacunary Statistical Equivalent Functions of Order $\alpha$ 

Rahmet Savas<br>Department of Mathematics, Istanbul Medeniyet University, Uskudar, Istanbul, Turkey<br>rahmet.savas@medeniyet.edu.tr


#### Abstract

In this paper we introduce new definitions which are interrelated to notions asymptotically $I_{\theta}$-statistical equivalent of order $\alpha$ to multiple L and strongly $I_{\theta}$-asymptotically equivalent of order $\alpha$ to multiple L by using $x(t)$ and $y(t)$ two nonnegative real-valued Lebesque measurable functions in the interval ( $1, \infty$ ) instead of sequences to. Futhermore some of inclusion theorems are presented.


Keywords: Ideal, filter, I-statistical convergence, real valued function, asymptotical equivalent,

## References:

[1] S. Bhunia, Pratulananda Das, S. Pal, Restricting statistical convergenge, Acta Math. Hungar, 134 (1-2) (2012), 153-161.
[2] H. Cakalli, A study on statistical convergence, Funct. Anal. Approx. Comput., 1(2), (2009), 19-24.
[3] R. Colak, Statistical convergence of order \alpha, Modern methods in
Analysis and its Applications, New Delhi, India, Anamaya Pub., (2010), 121129.

# A New Numerical Method Based on Hybrid Taylor and Lucas Polynomials for Solving a Class of Linear Volterra-Type Functional Integto-Differential Equations with Proportional and Variable Delays 

${ }^{l}$ Nurcan Baykus Savasaneril and ${ }^{2}$ Mehmet Sezer<br>${ }^{1}$ Izmir Vocational School, Dokuz Eylül University, Izmir, Turkey<br>${ }^{2}$ Celal Bayar University, Faculty of Science, Department of Mathematics, 45040, Manisa,<br>${ }^{1}$ nurcan.savasaneril@deu.edu.tr<br>${ }^{2}$ mehmet.sezer@cbu.edu.tr


#### Abstract

In this paper, a numerical matrix method is developed for numerically solving high-order linear Volterra-type functional integro- differential equations with mixed mixed proportional and variable delays under initial conditions. These type problems often appear in mathematical physics, mechanics, electronics, geophysics and other branches of natural sciences. The technique we have used is essentially based on Taylor and Lucas polynomials together with the standard or Chebyshev-Lobatto collocation points, and then the solution of problem is reduced to the solution of a system of algebraic equations. Also, to demonstrate the accuracy and efficiency of the method, an error analysis is performed based on residual functions.


Keywords: Volterra-Type Functional Equations, Integro Differential Equations, Matrix Method; Collocation Points, Taylor and Lucas Polynomials, Residual Functions.

## References:

[1] Çetin, M., Sezer, M., Güler, C., Lucas polynomial approach for system of high-order linear differential equations and residual error estimation, Mathematical Problems in Engineering, Volume 2015 (2015), Article ID 625984, 14 pages.
[2] J. Biazar et al, Numerical solution of functional integral equations by the variational
iteration method, Journal of Comput. and Appl. Math. 235 (2011) 2581-2585.
[3] M.T. Rashed, Numerical solution of functional differential, integral and integrodifferential equations, Appl. Math. Comput. 156 (2004) 485-492.
[4] N. Kurt, M.Sezer, Polynomial solution of high-order linear Fredholm integrodifferential equations with constant coefficients, J. Franklin Inst. 345 (2008) 839850.
[5] N. Kurt, M. Çevik, Polynomial solution of the single degree of freedom system by Taylor matrix method, Mech. Res. Commun. 35 (2008) 530-536.

# Existence Result for Nonlinear Fractional Differential Equations with Nonlocal Fractional IntegroDifferential Boundary Conditions in Banach Spaces 

Djamila Seba<br>Department of Mathematics, Boumerdes University,<br>Boumerds, Algeria<br>seba@univ-boumerdes.dz


#### Abstract

The topic of fractional differential equations has been of great interest for many researchers in view of its theoretical development and widespread applications in various fields of science and engineering such as physics, biophysics, chemistry, statistics, economics, blood flow, phenomena, control theory, porous media, electromagnetic, and other fields. Boundary value problems with integral boundary conditions constitute an important class of problems and arise in the mathematical modeling of various phenomena such as heat conduction, wave propagation, gravitation, chemical engineering, underground water flow, thermoelasticity, and plasma physics. They include two-point, three-point, multipoint, and nonlocal boundary value problems. In this paper, we consider the following boundary value problem of fractional differential equation with nonlocal fractional integro-Differential Boundary Conditions of the form $$
\left\{\begin{array}{c} { }^{c} D^{r} x(t)=f(x, x(t)), t \in[0,1], 1<r \leq 2 \\ \alpha_{1} x(0)+\beta_{1} \quad{ }^{c} D^{q} x(0)=\gamma_{1} \int_{0}^{\vartheta} \frac{(\vartheta-s)^{r-2}}{\Gamma(r-1)} x(s) d s, \quad 0<q<1 \\ \alpha_{2} x(1)+\beta_{2} \end{array}{ }^{c} D^{q} x(1)=\gamma_{2} \int_{0}^{\sigma} \frac{(\sigma-s)^{r-2}}{\Gamma(r-1)} x(s) d s, 0<\vartheta, \quad \sigma<1 .\right.
$$

Our investigation relies upon the method associated with the technique of measures of noncompactness and the fixed point theorem of $\mathrm{M} \backslash$ "onch type. Keywords: Caputo Fractional Derivative, Riemann Liouville Integral, Measure of Noncompactness, Fixed Point, Banach Space.




# On Statistical Convergence of $\operatorname{Order}(\boldsymbol{\alpha}, \boldsymbol{\beta})$ 

Hacer Sengul<br>Department of Mathematics, Siirt University, Siirt, Turkey<br>hacer.sengul@hotmail.com


#### Abstract

The concept of statistical convergence was introduced by Fast[1] and later reintroduced by Schoenberg[2] independently. Later on, it was further investigated from the sequence space point of view and linked with summability theorem by Salát[3] and Fridy[4]. The order of statistical convergence of a sequence of numbers was given by Gadjiev and Orhan[5]. In this paper, we shall study statistical convergence of order $(\alpha, \beta)$ and strongly p -Cesaro summability of order $(\alpha, \beta)$ for sequences of complex (or real) numbers. Also, some inclusion relations between the statistical convergence of order $(\alpha, \beta)$ and strongly p Cesaro summability of order $(\alpha, \beta)$ are given.


Keywords: Sequences, Statistical Convergence, Cesaro Summability. References:
[1] H. Fast, "Sur la convergence statistique", Colloq. Math. 2(1951), 241-244.
[2] I. J. Schoenberg, "The integrability methods", Amer. Math. Monthly, 66(1959), 361-375.
[3] T. Salat, "On statistically convergent sequences of real numbers", Math Slovaca, 30.2(1980), 139-150.
[4] J. A. Fridy, "On statistical convergence", Analysis, 5(1985), 301-313.
[5] A. D. Gadjiev and C. Orhan "Some approximation theorems via statistical convergence", Rocky Mountain J. Math, 2002, 32(1): 129-138.

# Reduced Differential Transform Method with Fixed Grid Size for Solving Klein Gordon Equations 

Sema Servi ${ }^{1}$, Yildiray Keskin ${ }^{2}$, Galip Oturanc ${ }^{2}$<br>1.Vocational School of Technical Sciences, Selcuk University<br>2.Department of Mathematics, Science Faculty, Selcuk University, Selcuklu,, Konya, Turkey<br>semaservi@ selcuk.edu.tr<br>goturanc@selcuk.edu.tr<br>ykeskin@selcuk.edu.tr


#### Abstract

In this study, a method was developed by applying a new algorithm to the reduced transformation method[4] that is an effective method for solving partial differentiable equasions in literature and RDTM with fixed grid size was formed.[1,3]The sampling of the method was made on the equation of Klein gordon in the application part[2]. Finally, efficiency of the method was indicated by comparing the results obtained through exact solution and varational iteration method.


Keywords: Reduced Differential Transform Method, Variational Iterationa Method, Klein Gordon Equations.

## References:

[1] S. Servi,Phd Thesis, Selcuk University, 2014 (in Turkish), Konya (to appear)
[2] Keskin, Yıldıray, Sema Servi, and Galip Oturanç. "Reduced Differential Transform Method for Solving Klein Gordon Equations." Proceedings of the World Congress on Engineering. Vol. 1. 2011.
[3] M. Jang, C. Chen, Y. Liy, On solving the initial-value problems using the differential transformation method, Applied Mathematics and Computation, 115 (2000) 145-160.
[4] Keskin Y.Oturanç G., "Reduced differential transform method for partial differential equations",International Journal of Nonlinear Sciences and Numerical Simulation, 10,6, 741-750,2009

# Tauberian Conditions Under Which Ordinary Convergence Follows From Logarithmic Type Summability Methods 

Sefa Anıl Sezer***, Ibrahim Canak**<br>*Department of Mathematics, İstanbul Medeniyet University, İstanbul, Turkey<br>**Department of Mathematics, Ege University, İzmir, Turkey<br>sefaanil.sezer@medeniyet.edu.tr


#### Abstract

The notion of general control modulo of the oscillatory behavior of a real sequence $\left(s_{n}\right)$ was first presented by $\operatorname{Dik}[1]$ for $(C, 1)$ summability method. In this work, inspired by $\operatorname{Dik}[1]$ we introduce the general control modulo for logarithmic means, that leads new Tauberian conditions and consequently new Tauberian theorems for logarithmic power series method ( $L$ ). Also we define generalized logarithmic power series method ( $L, m$ ) and establish Tauberian conditions of the slowly decreasing type to obtain ordinary convergence of $\left(s_{n}\right)$ from its ( $L, m$ ) summability. The results of this study extend and improve some of the well-known Tauberian theorems in the literature. Keywords: Tauberian Theorems, Logarithmic Means, General Control Modulo. References: [1] M. Dik, "Tauberian theorems for sequences with moderately oscillatory control modulo", Math. Morav., 5(2001), 57-94. [2] K. Ishiguro, "Tauberian theorems concerning the summability methods of logarithmic type", Proc. Japan Acad., 39(1963), 156-159. [3] B. Kwee, "Some Tauberian theorems for the logarithmic method of summability", Canad. J. Math., 20(1968), 1324-1331. [4] F. Moricz, "Theorems relating to statistical harmonic summability and ordinary convergence of slowly decreasing or oscillating sequences", Analysis (Munich), 24.2(2004), 127-145. [5] F. Moricz, "Necessary and sufficient Tauberian conditions for the logarithmic summability of functions and sequences", Studia Math., 219.2(2013), 109-121.


# BEM Solution of MHD Flow in a Semi-Infinite Channel with Variable Wall Conductivity 

M. Tezer-Sezgin and Canan Bozkaya<br>Department of Mathematics, Middle East Technical University, Ankara, Turkey<br>munt@metu.edu.tr


#### Abstract

In this study, we consider the magnetohydrodynamic (MHD) flow that is laminar and steady of a viscous, incompressible and electrically conducting fluid in a semi-infinte channel under an externally applied magnetic field. The flow is driven by the current produced by a pressure gradient. The external magnetic field is applied either perpendicular or parallel to the semi-infinte walls which have constant conductivities opposite in sign or perfectly conducting. The wall that connects the two semi-infinite walls has variable conductivity in terms of a mixed boundary condition. A boundary element method (BEM) solution is obtained using a fundamental solution that enables to treat the MHD equations in coupled form with general wall conductivities. The inhomogeneity of the equations due to pressure gradient is tackled obtaining a particular solution. Constant elements are used for the discretization of the boundaries restricting semi-infinte walls to finite lengths due to the reqularity conditions as $y \rightarrow \infty$. The complete solution is obtained in terms of the velocity and the induced magnetic field together with the convergence analysis of infinite integrals using boundedness conditions of the velocity and the induced magnetic field, and the properties of Bessel functions that appear in the fundamental solution for large arguments. The results are obtained for several values of Hartmann number ( $M$ ), boundary conductivity parameters ( $\lambda, k$ ) discussing the effect of these parameters on the solution. It is found that when the connecting wall is perfectly conducting and semi-infinite walls have constant conductivities $\pm k$, an increase in $M$ causes boundary layer formations near the side walls and the fluid becomes stagnant at the center of the channel. For the varying conductivities on all the walls, fluid concentrates in front of the connecting wall and the induced magnetic filed obeys the conductivity variation. BEM gives the solution at a small computational expense due to its boundary only nature.


Keywords: MHD, BEM, Semi-Infinite Channel.
References:
[1] C. Bozkaya and M. Tezer-Sezgin, "Fundamental solution for coupled magnetohydrodynamic flow equations", Journal of Computational and Applied Mathematics 203(2007), 125-144.
[2] C. Bozkaya and M. Tezer-Sezgin, "BEM solution to magnetohydrodynamic flow in a semi-infinite duct", International Journal for Numerical Methods in Fluids 70(2012), 300-312.

# A General Solution to a Dynamical Problem of Continuous Media Having Cubic Nonlinearities 

B.Gültekin Sinır<br>Department of Civil Engineering, Manisa Celal Bayar University, Muradiye, Manisa, Turkey<br>gultekin.sinir@cbu.edu.tr<br>Sümeyye Sintr<br>Department of Mechanical Engineering, Bilecik Şeyh Edebali University, Bilecik, Turkey<br>sumeyve.sinir@cbu.edu.tr


#### Abstract

In this study, a general model having cubic nonlinearities on dynamic problems is represented. The general model shows the presence of a non-linear damping, inertia and geometry. The general model is modified to be able to solve ordinary differential equations. As a solution technique, direct application of the multiple time scale method that is a powerful technique in dynamics analysis is chosen. The general solution procedure is applied some mathematical models. The solutions obtained by using general model are the same as the solution in the literature. It is seen that, the presence of non-linear inertia and the geometric terms make the non-linear natural frequencies to be dependent on constant amplitude of vibration. But, when damping nonlinearities are present, it is seen that the amplitude is exponentially time-dependent, and so, the nonlinear natural frequencies will be logarithmically time-dependent.


Keywords: General Model, Multiple Time Scales, Cubic Nonlinearities, NonLinear Inertia, Non-Linear Damping, Non-Linear Frequencies.

# Kinematic Surfaces with Constant Scalar Curvature in Euclidean 5-Space 

Emad Solouma<br>Department of Mathematics \& Statistics, College of Science, Al Imam Mohammad Ibn Saud Islamic University, Riyahd, Kingdom of Saudi Arabia<br>emadms74@gmail.com


#### Abstract

An equiform transformation in the n-dimensional an Euclidean space $\mathrm{E}^{n}$ is an affine transformation whose linear part is composed by an orthogonal transformation and homothetical transformation [2-6]. Such an equiform transformation maps points $x \in \mathrm{E}^{n}$ according to the rule $x \mapsto s A x+d, A \in S O(n), s \in R^{+}, d \in R^{n}$


The number $s$ is called the scaling factor. An equiform motion is defined if the parameters of (1), including $s$, are given as functions of a time parameter $t$. Then a smooth one-parameter equiform motion moves a point $x$ via
$x(t)=s(t) A(t) x(t)+d(t)$. The kinematic corresponding to this ransformation group is called similarity kinematics. See [1]. In this paper we study the scalar curvature $S$ of kinematic surfaces foliated by a similarity kinematics of an eggpear curve in the five dimensional Euclidean space $E^{5}$. We prove that the if scalar curvature $S$ is constant, then $S=0$. We describe the equations that govern such the surfaces. Finally examples of kinematic surfaces with zero scalar curvature are given.
Keywords: Kinematic Surfaces, Egg-Pear Curve, Scalar Curvature, Similarity Kinematics.

## References:

[1] O. Bottema and B. Roth, "Theoretical kinematic", Dover publications Inc., New York, 1990.
[2] E. M. Solouma et al., "Three Dimensional Surfaces Foliated by Two Dimensional Spheres", J. Egypt. Math. Soc. 15.1(2007), 101-110.
[3] E. M. Solouma,"Local study of scalar curvature of two-dimensional surfaces obtained by the motion of circle", J. of Applied Math. and computation 219 (2012), 3385-3394.
[4] F. M. Hamdoon and A. T. Ali, "Constant scalar curvature of three dimensional surfaces obtained by the equiform motion of a sphere", Inte. Elect. J. of Geometry 6.1(2013), 68-78.

# Fuzzy Complex System Reliability Analysis Using Depth-First Search 

Halil Ibrahim Sahin, Melek Eris<br>Department of Statistic and Computer Sciences, Karadeniz Technical University, Trabzon, Turkey<br>hisahin@ktu.edu.tr, melekeris@ktu.edu.tr


#### Abstract

An Engineering system can be modeled as a serial, parallel, serialparallel, redundancy, k-out-of-n or complex and analysis its reliability according to these models [1]. In some situations component/system reliability is not crisp [2-3]. In this study fuzzy complex reliability is analysed. Tie-Set method is choosen for analyzing system reliability. In order to find all paths from input to output, we apply depth-first search in a graph [4]. Of particular interest are the minimal tie-sets. We eliminate some paths are contained within minimal tie-sets and then use rules of union of sets for finding system reliability.Fuzzy approach is used for evaluating the component and system reliability. Reliability of each component is represented with fuzzy triangle membership function. Transition from component reliability to system reliability is obtained by implementation of fuzzy arithmetic operations. To get estimate reliability of the complex system in the fuzzy sense, defuzzify is used.


Keywords: Depth-First Search, Fuzzy, Reliability.

## References:

[1] E. A. Elsayed, ''Reliability Engineering’", Addison Wesley Longman Inc., USA, 1996.
[2] I. M. Aliev and Z. Kara, "Fuzzy System Reliability Analysis Using Dependent Fuzzy Set'" Control and Cybernetics, 33, 653-662, 2004.
[3] J. Yao, J. Su and T. Shih, "Fuzzy System Reliability Analysis Using Triangular Fuzzy Numbers Based on Statistical Data", Journal of Information Science and Engineering, 24, 1521-1535, 2008.
[4] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein,' 'Introduction to Algorithms,The MIT Press, Massachusetts Institute of Technology',,USA, 2009.

# Interpretation of Syllogisms on Boolean Algebras 

Ibrahim Sentürk, Tahsin Oner, Urfat Nuriyev Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>ibrahim.senturk@ege.edu.tr, tahsin.oner@ege.edu.tr, urfat.nurivev@ege.edu.tr


#### Abstract

The first systematic solution for syllogisms was introduced by the Greek philosopher Aristotle[1]. In the 19th. and 20th. centuries, George Boole[2], Lewis Caroll[3] and Jan Łukasiewicz[4] made a significant contribution in the realm of analysis of categorical syllogisms. In this study, we shall scrutinize interpretation of syllogisms on Boolean Algebras. Accordingly, we firstly construct categorical syllogisms together with a representation of syllogistic arguments by using sets. Thereafter, we compare this interpretation method with the George Boole's algebraic approach to Syllogistic Logic in The Laws of Thought in 1854.


Keywords: Categorical Syllogisms, Boolean Algebras, Carrolls' Diagram Method

## References:

[1] Aristotle, "Prior Analytics" translation and commentary by Robin Smith, Hackett Publishing, (1989).
[2] G. Boole, "The Laws of Thought", Walton \& Maberly(1854), Reprinted, Dover 1958.
[3] L. Carroll, "Symbolic Logic", Clarkson N. Potter, (1896).
[4] J. Łukasiewicz, "Aristotle's Syllogistic From the Standpoint of Modern Formal Logic", (1957), Oxford University Press.
[5] A. E. Kulinkovich, "Algorithmization of Reasoning in Solving Geological Problems", in Proceedings of the Methodology of Geographical Sciences, Naukova Dumka, (1979), 145-161.

# Self-Organizing Topological Maps for Classification of the Hercynian Granitoids From Their Geochemical Characteristics: Case of the Aouli Pluton (High Moulouya, Morocco) 

Abdelghani Talhaoui ${ }^{1}$, Imad Manssouri ${ }^{2}$, Abdellah El Hmaidi ${ }^{1}$, Mohamed Berrada ${ }^{3}$, Tajeddine Manssouri ${ }^{4}$<br>${ }^{1}$ Team of Sciences of Water and Environmental Engineering, Faculty of sciences, University Moulay Ismail, B.P. 11201, Zitoune, Meknès- Morocco. talhaouiabdelghani25@gmail.com ; elhmaidi@yahoo.fr;<br>${ }^{2}$ Laboratory of Mechanics, Mechatronics and Command, ENSAM Engineering high school, University Moulay Ismail, BP 4042, 50000, Meknès, Morocco. imade mansouri@yahoo.fr;<br>${ }^{3}$ Laboratory of Mathematical Modelling and Computing, ENSAM Engineering high school, University Moulay Ismail, BP 4042, 50000, Meknès, Morocco. berrada.mohamed@gmail.com;<br>${ }^{4}$ Laboratory of Geo-Engineering and Environment, Faculty of Science, University Moulay Ismail, B.P. 11201, Zitoune, Meknès- Morocco. t.manssouri@yahoo.fr;


#### Abstract

The study of rare earth, trace elements and major elements allows distinguishing of the various entities of a plutonic complex. The present work focus on the application of the model of unsupervised classification based on the self-organizing topological maps (SOM). The database contains 167 samples of Hercynian granitoid rocks of the Aouli pluton (High Moulouya, Morocco). The data correspond to the contents of major elements, trace elements and rare earths, and are composed respectively of 11 samples of granodiorite, 81 samples of gray granite, 70 samples of pink granite and 5 samples of muscovite granite. The map size was taken $15 \times 11$ neurons which allow minimizing the quantization error (QE) and the topological error (TE). Hierarchical clustering on the SOM has allowed to highlight four classes. The 11 samples of the granodiorite have been well classified in class 1 with $100 \%$. Those of gray granite were distributed in class 2 with $32 \%$, class 3 with $19.7 \%$ and class 4 with $48.1 \%$. For the pink granite, the corresponding samples were mostly classified in class 3 with $91.4 \%$ and in class 2 and 3 with $4.2 \%$ each one. For the muscovite granite, $80 \%$ of corresponding samples were classified in class 4 and $20 \%$ in class 3 . This study highlighted the classification capability of the self-organizing maps (SOM) on all the Hercynian granitoid rocks of the Aouli pluton. Keywords: Unsupervised classification, Self Organizing Maps Hercynian granitoids, Aouli Pluton, High Moulouya.


Demonic Operational and Denotational Semantics Commute

Fairouz Tchier<br>Mathematics department, King Saud University, P.O.Box 22452 Riyadh 11495, Saudi Arabia<br>ftchier@ksu.edu.sa


#### Abstract

Semantics is the study of meaning of languages. Operational semantics describes the effect of each statement on the state-usually giving the post-state as a function of the pre-state. Denotational semantics defines a translation into some (partial) function space usually defined in set/category theory [2]. The meaning of a program is a (partial) function/relation in that space. The angelic semantics is the input/output relation obtained by considering the best execution of the program if there is a possibility to terminate it will terminate. It corresponds to the choice that termination is guaranteed as long as termination is possible. The demonic semantics is the input/output relation by considering the worst execution of the program; if there is a possibility for the program not to terminate normally, then it will not terminate normally [1]. Our aim is to show that relational demonic operational semantics of nondeterministic program and its demonic denotational semantics commutes.


Keywords: Demonic Operational Semantics, Demonic Denotational Semantics Relational Semantics.

## References:

[1] F.Tchier. Demonic Semantics: using monotypes and residuals. International Journal of Mathematics and Mathematical Sciences, IJMMS, 2004:3, 135-160, 2004.
[2] J. Tiuryn and M. Wand. Untyped Lambda-Calculus with Input-Output. In H. Kirchner, editor, Trees in Algebra and Programming:
CAAP'96, Proc. 21st International Colloquium, volume 1059 of Lecture Notes in Computer Science, pages 317--329, Berlin, Heidelberg, and New York, April 1996.


# On a Fractional Model Arising in Spintronics 

Mouhcine Tilioua<br>M2I Laboratory, MAMCS Group<br>FST Errachidia<br>P.O Box 509 Boutalamine<br>52000 Errachidia<br>Morocco<br>m.tilioua@fste.umi.ac.ma


#### Abstract

We study the existence of weak solutions to a fractional model arising in spintronics. The model consists on fractional Landau-Lifshitz-Gilbert equation coupled to a fractional diffusion equation modelling spin transfer in ferromagnetic sample. We apply the Galerkin method to get an approximate solutions and we use a fractional calculus inequality will play a critical role in the convergence of the nonlinear terms. Keywords: Ferromagnetism, Spintronics, Fractional Derivatives, LLG Equation. References: [1] B. Guo and X. Pu. Global smooth solutions of the spin polarized transport equation. Electron. J. Differential Equations, 63:1-15, 2008. [2] X. Pu, B. Guo and J. Zhang. Global weak solutions to the 1-D Fractional Landau-Lifshitz Equation, Discrete Contin. Dyn. Syst. Ser. B, 14(1):199-207, 2010. [3] X. Pu and B. Guo. Global smooth solutions for the one-dimensional spinpolarized transport equation. Nonlinear Anal., 3-4(72):1481-1487, 2010.


# Existence and Multiplicity of Positive Solutions for Fractional Boundary Value Problems 

Fatma Serap Topal<br>Department of Mathematics, Ege University,<br>Bornova, Izmir, Turkey<br>f.serap.topal@ege.edu.tr


#### Abstract

In this paper, by introducing a new operator, improving and generating a p-Laplace operator for some $\mathrm{p}>1$, we discuss the existence and multiplicity of positive solutions to the four point boundary value problems of nonlinear fractional differential equations. Our results extend some recent works in the literature.


Keywords: Positive Solutions, Fractional Differential Equations, Fixed Point Theorems.

## References:

[1] B. Ahmada and G. Wang, A study of an impulsive four-point nonlocal boundary value problem of nonlinear fractional differential equations, Comput. Math. Appl., 62(2011), 1341-1349.
[2] B. Ahmad and S. Sivasundaram, On four-point nonlocal boundary value problems of nonlinear integro-differential equations of fractional order, Appl. Math. Comput., 217(2010), 480-487.
[3] A. Babakhani and V.D. Gejji, Existence of positive solutions of nonlinear fractional differential equations, J. Math. Anal. Appl., 278(2003), 434-442.
[4] Z.B. Bai and H.S. L, Positive solutions of boundary value problems of nonlinear fractional differential equation, J. Math. Anal. Appl., 311(2005), 495505.
[5] D. Delbosco, Fractional calculus and function spaces, J. Fract. Calc., 6(1994), 45-53.
[6] D. Delbosco and L. Rodino, Existence and uniqueness for a nonlinear fractional differential equation, J. Math. Anal. Appl., 204(1996), 609-625.
[7] C. Goodrich, Existence of a positive solution to a class of fractional di_erential equations, Appl. Math. Lett., 23(2010), 1050-1055.
[8] J. Graef and B. Yang, Positive solutions of a nonlinear fourth order boundary value problem, Commun. Appl. Nonlinear Anal., 14(2007), 61-73.
[9] D. Ji and W. Ge, Positive solution for four-point nonlocal boundary value problems of fractional order, Math. Meth. Appl. Sci., 37(2014), 1232-1239.
[10] R.W. Legget and L.R. Williams, Multiple positive fixed points of nonlinear operators on ordered Banach spaces, Indiana Univ. Math. J.: 28(1979), 673-688.


# Applying Graph Coloring to Schedule Doctors' Work in Hospital 

Fardous Toufic and Khouloud Al-Gahtani<br>ftoufic@ksu.edu.sa<br>4322010960@ksu.student.sa


#### Abstract

In this paper; we present an application of graph theory to schedule doctors 'or nurses' work. We choose an algorithm and modify it to find a solution for that problem. We will present a comparaison of our work to previous given algorithms to solve this problem. Keywords: Graph Theory, Coloring Graphs, And Algorithms.




# A-Compactly Uniform Integrability of Sequences of Random Elements 

Mehmet Unver<br>Department of Mathematics, Faculty of Science, Ankara University, Ankara, Turkey<br>munver@ankara.edu.tr


#### Abstract

In this talk we introduce a new type of uniform integrability for sequences of Banach valued measurable functions (random elements) by using Bochner integral so that we generalize the concept of A-compactly uniform integrability. Furthermore, we study the concepts of A-strong convergence and A-statistical convergence, which are some of the main concepts of the summability theory, for sequences of random elements and we investigate the relationship among these concepts by using this new type of uniform integrability. Keywords: Sequence of Random Element, Uniform Integrability, A-Statistical Convergence, A-Strong Convergence.


Solution to General Weakly Non-Linear Dynamic Problem

Leyla Usta<br>Department of Mathematics, Manisa Celal Bayar University, Muradiye, Manisa, Turkey<br>leylausta.3461 @gmail.com<br>B.Gültekin Sinir<br>Department of Civil Engineer, Manisa Celal Bayar University, Muradiye, Manisa, Turkey<br>gultekin.sinir@cbu.edu.tr


#### Abstract

In this study, we consider a dynamic system having a single degree of freedom under the influence of general forces, $f(u, \dot{u}, \ddot{u})$. We do not restrict $f$ to be an analytic function of deflection $(u)$, velocity $(\dot{u})$ and inertia $(\ddot{u})$. Specifically, we consider the equation $$
\begin{equation*} \ddot{u}+\omega_{0}^{2} u=\varepsilon f(u, \dot{u}, \ddot{u}) \tag{1} \end{equation*}
$$


where $\mathcal{E}$ is a small dimensionless parameter. The function $f$ is general but piecewise continuous. The equation

$$
\begin{equation*}
\ddot{u}+u=\varepsilon f(u, \dot{u}) \tag{2}
\end{equation*}
$$

is solved in the literature [1]. The effect of quadratic term is disappeared on the solution [1]. In our study, we added inertia term into $f$ and we developed a new solution procedure to show the effect quadratic non-linearity on the solution. A perturbation technique called the multiple time scale method is used to solve the equation. Some special cases are presented. The solution procedure is generalized for higher order nonlinearities.
Keywords: Multiple Time Scale, Nonlinear Vibration, General Solution. Fourier Transform.

## References:

[1] A. H. Nayfeh, "Introduction to perturbation techniques", John Wiley\&Sons, Newyork, 1980


# Self-Dual Codes and the Steenrod Algebra 

Tane Vergili and Ismet Karaca<br>Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>tane.vergili@ege.edu.tr ismet.karaca@ege.edu.tr


#### Abstract

All stable cohomology operations with coefficient group $\mathrm{Z}_{2}$ constitute an algebra, called the mod 2 Steenrod algebra A [2]. Each of its finite sub Hopf algebras $A(n)$ determined by a fixed profile function for $n \geq 1$ [4] has a Frobenius ring structure so they allow us to construct codes over themselves. Codes over $\mathrm{A}(\mathrm{n})$ for $\mathrm{n} \geq 1$ was first studied in [6]. In this work, we construct the Euclidean and the Hermitian self-dual codes over A(n). This work shows that Euclidean and Hermitian self-dual codes exist in all length over A(n) and unlike the mod 2 Steenrod algebra, the sub algebra $A(n)$ is not prime for all $n \geq 1$. Keywords: Steenrod Algebra, Frobenius Ring, Algebraic Coding Theory. References: [1] J. Milnor, "The Steenrod algebra and its dual", Annals of Mathematics, 67(1958), 150-171. [2] N.E. Steenrod and D.B.A. Epstein, Cohomology operations, Annals of Mathematics Studies, Princeton University Press 50, 1962. [3] A.M. Gallant, "Excess and conjugation in the Steenrod algebra", Proceedings of the American Mathematical Society, 76(1979), 161-166. [4] H.R. Margolis, Spectra and the Steenrod algebra, North-Holland Mathematical Library, Elsevier, 29, 1983. [5] I. Kashkarev, "The primality of the Steenrod algebra", Communications in Algebra, 37(2009), 1182-1185. [6] J.A. Wood, "Anti-isomorphism, character modules and self-dual codes over non-commutative rings", International Journal of Information and Coding Theory, 1(2010), 429-444. [7] S.T. Dougherty and T. Vergili, "Codes and the Steenrod algebra", to appear in JACODESMATH (2016).


# Integral Equation Methods for the Planar Exterior Robin Boundary Value Problem of the Laplacian 

Olha Ivanyshyn Yaman, Gazi Ozdemir<br>Department of Mathematics, Izmir Institute of Technology, Izmir, Turkey gaziozdemir@iyte.edu.tr


#### Abstract

The Laplace equation arises in many areas in physics and mathematics, such as electromagnetism, fluid mechanics, heat conduction, geometry etc. The Robin or impedance condition models the situation when the boundary absorbs some part of the energy, heat, mass, which is transmitted through it. It is well-known that the problem has a unique solution, [1]. We propose a method for the numerical solution of the problem which is based on a boundary integral equation. Since the fundamental solution to the Laplace equation is logarithmic in 2D, we apply a modification which keeps it bounded in the unbounded domain. Representing the solution by a modified single-layer potential we reduce the differential problem in an unbounded domain to the Fredholm integral equation of the second kind over the boundary. Investigating the properties of the integral operators and employing the Fredholm alternative we show that the obtained boundary integral equation has a unique solution. Numerically, the integral equation is solved by the Nyström method based on weighted trigonometric quadratures on an equidistant mesh, [2]. This approach guarantees exponential convergence for analytic boundaries and boundary data. The feasibility of the numerical method and convergence order is verified by numerical examples.


Keywords: Laplacian, Robin Boundary Condition, Layer Potentials, Nyström Method, Fredholm Alternative, Boundary İntegral Equations.

## References:

[1] C. Constanda, "Direct and indirect boundary integral equation methods", Chapman \& Hall/CRC, (2000).
[2] R. Kress, "Linear integral equations", 3 ed., Springer, New York, (2014).

# On Numerical Radius and Berezin Number Inequalities for Reproducing Kernel Hilbert Space 

Ulas Yamanci and Mehmet Gurdal<br>Department of Mathematics, Suleyman Demirel University, ulasyamanci@sdu.edu.tr, gurdalmehmet@sdu.edu.tr


#### Abstract

The fundamental inequality $\mathrm{w}\left(\mathrm{A}^{\mathrm{n}}\right) \leq \mathrm{w}^{\mathrm{n}}(\mathrm{A})$, ( $\mathrm{n}=1,2, \ldots$ ) for numerical radius is studied in the literature. But, the inverse inequalities for numerical radius is not well known in the literature. For this reason, by using Hardy-Hilbert type inequalities, we give inverse numerical radius inequality for Reproducing Kernel Hilbert Space. Also, we get inverse power inequality for Berezin number of operators


## References:

[1] G. Hardy, J.E. Littlewood, G. Polya, "Inequalities", 2 nd ed. Cambridge University Press, Cambridge, 1967.
[2] M.T. Karaev, "Reproducing Kernels and Berezin symbols Techniques in Various Questions of Operator Theory", Complex Anal. and Oper. Theory, 7(2013), 983-1018.
[3] M. Kian, "Hardy-Hilbert type inequalities for Hilbert space operators", Ann. Funct. Anal., 3(2)(2012), 128-134.
[4] E. Nordgren, P. Rosenthal, "Boundary values of Berezin symbols", Oper. Theory Adv. Appl., 73 (1994), 362-368.

# A Note on Power Central Values of Generalized Skew Derivations with Annihilating Conditions 

Nihan Baydar Yarbil<br>Department of Mathematics, Ege University, Bornova, Izmir, Turkey<br>nihan.baydar.yarbil@ege.edu.tr


#### Abstract

In recent years a number of authors have been treating the additive mappings of a ring and examining the behaviour of these mappings. In most of the cases the results provide useful informations about the structure of the ring and the map. In this line we handled a problem in which a generalized skew derivation of a ring constructing a power central identity. Let $G$ be a generalized skew derivation of a prime ring $R$ with center $Z(R)$ and extended centroid $C, a \in R$ and $n$ is a fixed positive integer. If $a G(x)^{n} \in Z(R)$ for all $x \in R$ then $a G(x)=0$ unless $\operatorname{dim}_{C} R C=4$. We conclude with a generalization of this result to noncommutative Lie ideals of $R$. This is a joint work with Nurcan Argaç.


Keywords: Prime Ring, (Generalized) Derivation, (Generalized) Skew Derivation, Lie Ideal.

## References:

[1] I. N. Herstein, "Derivations of prime rings having power central values", Contemp. Math., Amer. Math. Soc., Providence, R. I., 13(1982), 193-171.
[2] M. Bresar, "A note on derivations", Math. J. Okayama Uni., 32(1990), 83-88
[3] J. C. Chang, "Right Generalized $(\alpha, \beta)$-derivations having power central values", Taiwanese J. Math., 13(4) (2009), 1111-1120.
[4] J. C. Chang, " Annihilators of power values of a right generalized $(\alpha, \beta)$ derivation", Bull. Inst. Math. Acad. Sin. (N.S), 4(1) (2009), 67-73.
[5] J. C. Chang, " Generalized skew derivations with nilpotent values on Lie ideals", Monatsh Math., 161(2010), 155-160.

# New Analytic Solutions of the Space-Time Fractional CahnHilliard Equations 

Handan Cerdik Yaslan<br>Department of Mathematics, Pamukkale University, Denizli, Turkey<br>hcerdik@pau.edu.tr


#### Abstract

In the present study, new analytical solutions for the space-time fractional Cahn-Hilliard equation with the Jumarie's modified Riemann-Liouville derivative of order $\alpha$ and $\beta$ are obtained using tanh method with the fractional complex transform. The accuracy and efficiency of the method is shown by numerical examples.


Keywords: The Space-Time Fractional Cahn-Hilliard Equation, Fractional Complex Transform, Modified Riemann-Liouville Derivative, Tanh Method.

## References:

[1] S.M. Choo, S. K. Chung, Y.J. Lee, A conservative difference scheme for the viscous Cahn-Hilliard with a nonconstant gradient energy coefficient, Appl. Numb. Math. 51 (2004) 207-219.
[2] E. Gurtin Morton, Generalized Ginzburg-Landau and Cahn-Hilliard equations based on a microforce balance, Phys. D. 92 (1996) 178-192.
[3] S. Liu, F. Wang, H. Zhao, Global existence and asymptotics of solutions of the CahnHilliard equation, J. Differential Equations 238 (2) (2007) 426-469.
[4] . Bekir, . Gner, A. C. Cevikel, Fractional Complex Transform and expFunction Methods for Fractional Differential Equations, Abstr. Appl. Anal. 2013 (2013) Article ID 426462, 8 pages.
[5] Elliott Charles M., French Donald A., Numerical studies of the CahnHilliard equation for phase separation IMA J. Appl. Math. 38 (2) (1987) 97-128.
[6] A. Bouhassoun, M. Hamdi Cherif, Homotopy Perturbation Method for Solving the Fractional Cahn-Hilliard Equation, 18 (2015) J. Interdiscip. Math. 513-524.
[7] M.S. Mohamed, Kh.S. Mekheimer, Analytical Approximate Solution for Nonlinear Space-Time Fractional Cahn-Hilliard Equation, Int. Electron. J. Pure Appl. Math. 7 (2014) 145-149.
[8] Z. Dahmani, M. Benbachir, Solutions of the Cahn-Hilliard Equation with Time-and Space-Fractional Derivatives, 8 (2009) Int. J. Nonlinear Sci. 19- 26.
[9] S. T. Demiray, Y. Pandir, H. Bulut, Generalized Kudryashov Method for TimeFractional Differential Equations, Abstr. Appl. Anal. 2014 (2014) Article ID 901540, 13 pages.
[10] H. Jafari, H. Tajadodi, N. Kadkhoda, D. Baleanu, Fractional Subequation Method for Cahn-Hilliard and Klein-Gordon Equations, Abstr. Appl. Anal. 2013 (2013) Article ID 587179, 5 pages.
[11] W. Li, H. Yang, B. He, Exact Solutions of Fractional Burgers and Cahn-Hilliard Equations Using Extended Fractional Riccati Expansion Method, Math. Probl. Eng. 2014 (2014) Article ID 104069, 9 pages.

# Multi-point Boundary Value Problems on an Unbounded Time Scale 

Ismail Yaslan and Mustafa Gunendi<br>Department of Mathematics, Pamukkale University, Denizli, Turkey<br>iyaslan@pau.edu.tr, _mustafa_87875@hotmail.com


#### Abstract

In this paper, we establish the criteria for the existence of at least one and three positive solutions for nonlinear second order multi-point time scale boundary value problem on infinite interval by using the Leray-Schauder fixed point theorem and the five functionals fixed point theorem, respectively.


Keywords: Boundary value problems, Cone, Fixed Point Theorems, Positive Solutions, Time Scales.

## References:

[1] R.P. Agarwal, M. Bohner and D. O'Regan, "Time scale boundary value problems on infinite intervals", J. Math. Comput. Appl. Math. 141 (2002), 27-34.
[2] R. I. Avery, "A generalization of the Legget-Williams fixed point theorem", Math. Sci. Res. Hotline 2 (1998), 9-14.
[3] M. Bohner and A. Peterson, "Dynamic Equations on Time Scales: An Introduction with Applications", Birkhauser, Boston, 2001.
[4] M. Bohner and A. Peterson (editors), "Advances in Dynamic Equations on Time Scales", Birkhauser, Boston, 2003.
[5] S. Hilger, "Analysis on measure chains-A unified approach to continuous and discrete calculus", Results Math., 18 (1990), 18--56.
[6] I. Y. Karaca and F. Tokmak, "Existence of three positive solutions for mpoint time scale boundary value problems on infinite intervals", Dynam. Systems Appl., 20 (2011), 355--368.
[7] X. Zhao and W. Ge, "Unbounded positive solutions for m-point time-scale boundary value problems on infinite intervals", J. Appl. Math. Comput., 33 (2010), 103--123.

# Results on Soft Continuous Functions in the Soft Topological Spaces Equipped with Soft Scott Topology 

Gozde Yaylall, Bekir Tanay<br>Mjuğla Sıtkı Koçman University, Muğla, Turkey<br>gozdeyaylali@mu.edu.tr


#### Abstract

In this study, some properties of Soft Scott Topology are examined; and some relations between Soft Scott Topology and Way Below Soft Set Relations are shown. Also the notion of Soft Scott continuous function on Soft Topological Spaces, which is equipped with Soft Scott topology, is defined by focusing on the structure of the continuity of soft function and some examples are illustrated.


Keywords: Soft Set, Soft Topology, Soft Continuous Function, Soft Scott Topology.

## References:

[1] D. Molodtsov, Soft Set Theory-First Results, Comput. Math. Appl. 37 (1999) 19-31.
[2] K. V. Babitha, J. J. Sunil, Soft Set Relations and Functions, Comput. Math.Appl. 60 (2010) 1840-1849
[3] K. V. Babitha, J. J. Sunil, Transitive Closures and Ordering on Soft Sets Comput. Math.Appl. 62 (2011) 2235-2239.
[4] Gierz G. , Hofmann K.H., Keimel K., Lawson J. D. , Mislove M. and Scott D. S. , "Continuous Lattices and Domains", Encyclopedia of Mathematics and its Applications 93
[5] Maji P.K., Biswas R., Roy A.R., "Soft set theory", Comput. Math. Appl. 45 555-562 (2003)
[6] Çağman N., Karataş S., Enginoglu S., "Soft topology", Comput. Math. Appl. 62 351258 (2011)
[7] Roy S. , Samanta T.K. , "An Introduction of a Soft Topological Spaces" Proceeding of UGC sponsored National seminar on Recent trends in Fuzzy set theory, Rough set theory and Soft set theory at Uluberia College on 23rd and 24th September, 2011 ISBN 978-81-922305-5-9, 9-12, 2011.
[8] Tanay B. and Yaylalı G. , "New structures On Partially Ordered Soft Sets and Soft Scott Topology", Ann. Fuzz. Math. Inform., 7 89-97 (2014).
[9] Tanay B. and Yaylalı G. , "Soft Way Below Relation" Internatioanl Conference on Recent Advanced in Pure and Applied Mathematics 2014, Antalya Turkey.
[10] Wardorwski D., "On a soft mappings and its fixed points", Fix Point Theory and Applications, 182 (2013)
[11] G. Yaylalı and B. Tanay, Some New Results on Orderings on Soft Sets, Journla of Technology of Dumlupınar University 34 (2015)
[12] G. Yaylali, B. Tanay, Intervals Soft Ordered Topology and Some Results, 27. National Mathematics Symposium UMS 2014

# Idempotent and Nilpotent Subsemimodules of Semimodules 

Gulsah Yesilkurt<br>Department Of Mathematics, Yıldız Technical University, MSc Student<br>Davutpaşa /Güngören, Istanbul, Turkey<br>gulsahyesilkurt@gmail.com


#### Abstract

In this work we investigate some properties of semirings and semimodules. First we give some characterization of Noetherian semirings and show that the set of nonnegative integers is a Noetherian semiring by showing the characterization of all ideals of nonnegative integers. Second we give the definition of idempotent and nilpotent subsemimodules of semimodules. Further we give some properties of its.


Keywords: Semiring, Semimodule, Idempotent and Nilpotent Subsemimodule, Ideals, $k$-ideals.

## References:

[1] Çallıalp, F. ve Tekir, Ü. (2009). Değişmeli Halkalar ve Modüller, Birsen Yayınevi, İstanbul.
[2] Hebisch, U. and Weinert, H. J. (1998). Algebraic Theory and Applications in Computer Science, World Scientific, Singapore
[3] El-Bast, Z. A. and Smith, P. F. (1988). Multiplication Modules, Commun. Algebra, 755-779.
[4] Dubey M. K. and Sarohe P. (2013), On 2- absorbing semimodules, Quasigroups and Related Systems, 175-184
[5] Deore, R.P. (1994),On associated primes and primary subsemimodules , Indian J. Pure Appl. Math. 25, 647-654
[6] Smith, P. F. (1988), Some remarks on multiplication modules, Arch. Math. 50, 223-235.
[7] Gupta, V. and Chaudhari J. N. (2011), Prime İdeals in Semirings, Bull. Malays. Math. Sci. Soc. 417-421
[8] Yeşilot, G. (2010), On Prime and Maximal k-Subsemimodules of Semimodules, Hacettepe Journal of Mathematics and Statistics, 305-312
[9] Yeşilot, G. (2010), On Prime Subsemimodules of Semimodules, International Journal of Algebra, 53-60

# Convolution and Approximation in Weighted Lorentz Spaces 

Yunus Emre Yildirir<br>Department of Mathematics, Ballkesir University, Necatibey Education Faculty, Balkesir, Turkey<br>yildirir@balikesir.edu.tr


#### Abstract

The convolution type transforms play an important role in the many areas of the theoretical and applied mathematics. Particularly, these transforms are very useful in the approximation theory for the constructions of the approximating polynomials. Therefore, we need to study the relations between these transforms and the best approximation numbers. Such problems were investigated by some authors in Lebesgue, Orlicz and weighted Orlicz spaces [1, 2, 3]. In this paper, we obtain that the similar results are valid in weighted Lorentz spaces. For this, we prove the multiplier theorem and Littlewood-Paley theorem in weighted Lorentz spaces with Muckenhoupt weights. Keywords: Convolution Type Transform, Weighted Lorentz Space, Best Approximation, Muckenhoupt Weight.

\section*{References:} [1] M. F. Timan, "Best approximation of periodic functions by trigonometric polynomials and transformations of convolution type" Dokl. Akad. Nauk SSSR, 198, 776-778, (1971). [2] V. G. Ponomarenko, M. F. Timan, "On the behaviour of convolution type transforms in the Orlicz spaces", Proceedings of the Inst. Appl. Math. Mech., Donetks, 187-191, (1998). [3] Yildirir, Y. E., Isralov, D. M. "The properties of convolution type transforms in weighted Orlicz spaces", Glasnik Matematicki, 45 (2010), 461-474.


# k-Chinese Postman Problem Approach for Snow Plowing Operations: A Case Study 

Mustafa Yilmaz, Merve Kayaci Codur<br>Department of Industrial Engineering, Atatürk University, Erzurum, Turkey<br>mustafay@atauni.edu.tr,mkayaci@atauni.edu.tr


#### Abstract

Snow plowing activity involves the removal of snow from roadways and sidewalks, loading snow into vehicles and transporting snow to disposal sites in modern socities during winter. These operations in a region is accomplished with a set of vehicles or trucks with specialized equipment. The routing of vehicles that plow through travelling all roads is critically important in terms of cost and distance. In current practice, determine the route of these vehicles is largely manual and primarily relies on the knowledge and judgment of drivers. In this study is focused on k-Chinese Postman Problem (k-CPP) is a multiple vehicle variant of the CPP, which has many real-world applications. The k-CPP model approach is based on mathematical optimization for the finding a set of vehicle routes that cover all roads at least once with a minimum cost. The different variants of the k-CPP model under different assumptions are applied to a snow plowing operations conducted on the network of University of Atatürk at campus and the results are compared.


Keywords: Arc Routing, Chinese Postman Problem, k- Chinese Postman Problem

# On Soft RIC-Continuous Functions 

Yunus Yumak and Aynur Keskin Kaymakci Department of Mathematics, Selcuk University, Campus, Konya, Turkey yunusyumak@yahoo.com akeskin@selcuk.edu.tr


#### Abstract

Firstly, we introduce and study a new the notion in soft ideal topological space, called soft $f_{l}$-set. Secondly, we obtain that the notion of soft $f_{l}$ -set is stronger than the notion of soft regular-I-closed set and weaker than the notion of soft semi-I-open set. To give a decomposition of soft continuous function for soft regular-I-closed sets we define the concepts soft $R_{I} C$ continuous, soft $f_{I}$-continuous and soft contra*-continuous functions. Finally, we show that a function $f_{p u}:(X, \tau, E, I) \rightarrow(Y, \phi, V)$ is soft $R_{I} C$-continuous if and only if it is soft $f_{1}$-continuous and soft contra*-continuous functions.


Keywords: Soft Regular-I-Closed, Soft Fi-Set, Soft Regular-I-Continuity, Soft Ideal Topology.

## References:

[1] B. Ahmad, A. Kharal, Mappings on soft classes, New Math.Nat. Comput., 7 (2011), 471-481.
[2] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Comp. \& Math. with Applications, 57 (2009), 1547-1553.
[3] I. Arockiarani, A.A. Lancy, Generalized soft g $\beta$-closed sets and soft g s $\beta$ closed sets in soft topological spaces, Int. Jour. of Mathe. Archive, 4 (2013), 1-7.
[4] A. Aygünoglu and H. Aygün, Some notes on soft topological spaces, Neural Comput. Applic., 21 (2012), 113-119.
[5] S. Bayramov, C.G. Aras, Soft local compact and soft paracompact spaces, Journal of Mathematics and System Science, 3 (2013), 122-130.
[6] N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, European Journal of Operational Research, 207 (2010), 848-855.
[7] B. Chen, Soft semi-open sets and related properties in soft topological spaces, Applied Mathematics and Information Sciences, 7 (2013), 287-294.
[8] S.A. El-Sheikh, Decompositions of some types of soft sets and soft continuity via soft ideals, Gen. Math. Notes, 24 (2014), 103-124.
[9] F. Feng, Y.B. Jun, X. Zhao, Soft semirings, Computers and Mathematics with Applications, 56 (2008), 2621-2628.
[10] S. Hussain, B. Ahmad, Some properties of soft topological spaces, Computers and Mathematics with Applications, 62 (2011), 4058-4067.
[11] A. Kandil, O.A.E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft ideal theory, soft local function and generated soft topological spaces, Appl. Math. Inf. Sci., 8(4)(2014), 1595-1603.

[12] P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, Computers and Mathematics with Applications, 45 (2003), 555-562.
[13] D. Molodtsov, Soft set theory-First results, Computers and Mathematics with Applications, 37 (1999), 19-31.
[14] S. Nazmul and S.K. Samanta, Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform., 6 (2013), 1-15.
[15] R. Sahin and A. Kucuk, Soft filters and their convergence properties, Ann. Fuzzy Math. Inform., 6 (2013), 529-543.
[16] M. Shabir, M. Naz, On soft topological spaces, Computers and Mathematics with Applications, 61 (2011), 1786-1799.
[17] Y. Yumak, A. K. Kaymakci, Soft $\beta$-open sets and their applications, Journal of New Theory, 4 (2015), 80-89.
[18] Y. Yumak, A. K. Kaymakci, Soft idealization of a decomposition theorem, Filomat (accepted).
[19] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform., 3(2)(2012), 171-185.

# A Bound for the Number of Symmetric Colorings of a Finite Group 

Yuliya Zelenyuk<br>School of Mathematics, University of the Witwatersrand, Johannesburg, South Africa<br>yuliya.zelenyuk@wits.ac.za


#### Abstract

Let $G$ be a finite group and let $r$ be a natural number. An r-coloring of G is any mapping $\chi: \mathrm{G} \rightarrow\{1, \ldots \mathrm{r}\}$. A coloring of G is symmetric if there exists an element g in G such that $\chi\left(\mathrm{gx}^{-1} \mathrm{~g}\right)=\chi(\mathrm{x})$ for every x from $\mathrm{G}[1,2]$. We show that if $G$ is not Boolean, then the number of symmetric r-colorings of $G$ does not exceed $n r^{7 n / 8}$, where $n=|G|$, and consequently, if $r>1$, the probability that an $r$ coloring of G is symmetric approaches to 0 as n tends to infinity [3]. Keywords: Symmetric Coloring, Finite Group, Boolean Group.

\section*{References:} [1] Y. Gryshko (Yu. Zelenyuk) and I. Protasov, "Symmetric colorings of finite Abelian groups", Dopov. Akad. Nauk Ukr., No.1(2000), 32-33. [2] Y. Gryshko (Yu. Zelenyuk), "Symmetric colorings of regular polygons", Ars. Combinatorica 78(2006), 277-281. [3] Yu. Zelenyuk, "A bound for the number of symmetric colorings of a finite group", submitted.


# One by One Embedding the Crossed Hypercube into Pancake Graph 

Mohamed Faouzi Zerarka, Smain Femmam<br>Polytechnic Engineers School Labs EPF, 3 bis rue Lakanal, 92330 Sceaux, France. faouzi.zerarka@gmail.com


#### Abstract

Let $G$ and $H$ be two simple undirected graphs. An embedding of the graph $G$ into the graph H is an injective mapping $f$ from vertices of $G$ to the vertices of $H$. The dilation of embedding is the maximum distance between $f(u)_{,} f(v)$ taken over edges $(u, v)$ of $G$. The Pancake graph is one as viable interconnection scheme for parallel computers, which has been examined by a number of researchers. The Pancake was proposed as alternatives to the hypercube for interconnecting processors in parallel computer. Some good attractive properties of this interconnection network include: vertex symmetry, small degree, a sub-logarithmic diameter, extendability, and high connectivity (robustness), easy routing and regularity of topology, fault tolerance, extensibility and embeddability of others topologies. In this paper, we give a construction of one by one embedding of dilation 5 of crossed hypercube into Pancake graph.


