ASTROHELICOIDAL SURFACES

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Abstract. We construct an astrohelicoidal surface which its profile curve has astroid curve in the three dimensional Euclidean space $\mathbb{E}^3$. We also find Gaussian curvature and the mean curvature, and Weingarten relation of the surface with some figures.

1. Introduction

The surface theory has been worked by many geometers with nice papers. We meet nice books for the theory, such as Eisenhart [1], Forsyth [2], Gray et al. [3], Hacısalihoğlu [4], Nitsche [5], Spivak [6].

In this paper, we consider the astrohelicoidal surface in Euclidean 3-space $\mathbb{E}^3$. We give some basic notions of three dimensional Euclidean geometry in section 2. In section 3, we define helicoidal surface. We obtain astrohelicoidal surface, and calculate its curvatures in the last section.

2. Preliminaries

In the rest of this talk, we shall identify a vector (a,b,c) with its transpose $(a,b,c)^t$.

We will introduce the first and second fundamental forms, matrix of the shape operator $\mathbf{S}$, Gaussian curvature K, and the mean curvature H of surface $\mathbf{M} = \mathbf{M}(u, v)$ in Euclidean 3-space $\mathbb{E}^3$.

Let $\mathbf{M}$ be an isometric immersion of surface M^2 in $\mathbb{E}^3$. The vector product of $\overrightarrow{x} = (x_1, x_2, x_3)$, $\overrightarrow{y} = (y_1, y_2, y_3)$ on $\mathbb{E}^3$ is defined as follows

$$\overrightarrow{x} \times \overrightarrow{y} = \det \begin{pmatrix} e_1 & e_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{pmatrix}.$$

For a surface $\mathbf{M}$ in $\mathbb{E}^3$ we have

$$\det I = \det \begin{pmatrix} E & F \\ F & G \end{pmatrix} = EG - F^2,$$

and

$$\det II = \det \begin{pmatrix} L & M \\ M & N \end{pmatrix} = LN - M^2,$$

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