ONE SIDED HENRY SMITH SURFACE

Erhan GÜLER[1] and Ömer KİŞİ[2]

[1,2]Bartın University, Faculty of Sciences, Department of Mathematics, Bartın, Turkey

eguler@bartin.edu.tr okisi@bartin.edu.tr

Abstract

We focus differential geometry of the Henry Smith surface in the three dimensional Euclidean space. We calculate the Gaussian and the mean curvatures of the surface, drawing its figure. In addition, we find algebraic equations in terms of running coordinates, and give degree and class of the surface

Keywords: 3-space, Henry Smith surface, Gaussian curvature, mean curvature.

1. Introduction

A surface is simply defined by a family of curves in the three dimensional Euclidean space. There are many classical, analytical and modern surfaces in the literature. For example plane and cylinder are two-sided surfaces, but Möbius strip, Klein bottle, Henneberg surface, cross cap, Roman surface, Boy surface are one-sided surfaces. One-sided surfaces have important role of complete space in differential geometry. These type surfaces have only one normal. See [1-6] for details.

In this paper, we focus one sided Henry Smith surface in Euclidean 3-space $\mathbb{E}^3$. We give some basic elements of Euclidean geometry in section 2. Showing Henry Smith surface is one sided, we calculate its curvatures in section 3. We find algebraic equations in terms of running coordinates, and finally give degree and class of the Henry Smith surface in the last section.

2. Preliminaries

We will identify a vector (a,b,c) with its transpose. We will introduce the first and second fundamental forms, matrix of the shape operator $\mathbf{S}$, Gaussian curvature K, and the mean curvature H of surface $\mathbf{M} = \mathbf{M}(u, v)$ in Euclidean 3-space $\mathbb{E}^3$.

Let $\mathbf{M}$ be an isometric immersion of surface M^2 in $\mathbb{E}^3$. The vector product of $\vec{x} = (x_1, x_2, x_3)$, $\vec{y} = (y_1, y_2, y_3)$ on $\mathbb{E}^3$ is defined as follows