ASTROHELICOIDAL HYPERSURFACES IN 4-SPACE

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ABSTRACT. We consider an astrohelicoidal hypersurface which its profile curve has astroid curve in the four dimensional Euclidean space $\mathbb{E}^4$. We also calculate Gaussian curvature and the mean curvature, and Weingarten relation of the hypersurface. Moreover, projecting hypersurface into 3-spaces, we draw some figures.

1. INTRODUCTION

About (hyper)surfaces in the literature, we see some following papers: Arslan et al. [1], Ganchev and Milousheva [4], Güler et al. [6, 7], Güler and Turgay [8], and also some books: Eisenhart [2], Hacısalihoğlu [9], Nitsche [10], ect..

In this paper, we consider the astrohelicoidal hypersurface in Euclidean 4-space $\mathbb{E}^4$. We give some fundamental notions of three dimensional Euclidean geometry in section 2. In section 3, we define helicoidal hypersurface. We obtain astrohelicoidal hypersurface, and calculate its curvatures in the last section.

2. PRELIMINARIES

In the rest of this work, we shall identify a vector (a,b,c,d) with its transpose $(a,b,c,d)^t$. We will introduce the first and second fundamental forms, matrix of the shape operator $\mathbf{S}$, Gaussian curvature K, and the mean curvature H of hypersurface $\mathbf{M} = \mathbf{M}(u, v, w)$ in Euclidean 4-space $\mathbb{E}^4$. Let $\mathbf{M}$ be an isometric immersion of hypersurface M^3 in $\mathbb{E}^4$.

The inner product of vectors $\overrightarrow{x} = (x_1, x_2, x_3, x_4)$ and $\overrightarrow{y} = (y_1, y_2, y_3, y_4)$ on $\mathbb{E}^4$ is defined by as follows:

$$\overrightarrow{x} \cdot \overrightarrow{y} = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4.$$

The vector product of vectors $\overrightarrow{x} = (x_1, x_2, x_3, x_4)$, $\overrightarrow{y} = (y_1, y_2, y_3, y_4)$ and $\overrightarrow{z} = (z_1, z_2, z_3, z_4)$ on $\mathbb{E}^4$ is defined by as follows:

$$\overrightarrow{x} \times \overrightarrow{y} \times \overrightarrow{z} = \begin{pmatrix} x_2 y_3 z_4 - x_2 y_4 z_3 - x_3 y_2 z_4 + x_3 y_4 z_2 + x_4 y_2 z_3 - x_4 y_3 z_2 \\ -x_1 y_3 z_4 + x_1 y_4 z_3 + x_3 y_1 z_4 - x_3 z_1 y_4 - y_1 x_4 z_3 + x_4 y_3 z_1 \\ x_1 y_2 z_4 - x_1 y_4 z_2 - x_2 y_1 z_4 + x_2 z_1 y_4 + y_1 x_4 z_2 - x_4 y_2 z_1 \\ -x_1 y_2 z_3 + x_1 y_3 z_2 + x_2 y_1 z_3 - x_2 y_3 z_1 - x_3 y_1 z_2 + x_3 y_2 z_1 \end{pmatrix}.$$