HELICOIDAL SURFACE OF LOGARITMIC SPIRAL TYPE IN 3-SPACE

Erhan GÜLER[1] and Ömer KİŞİ[2]

[1,2]Bartın University, Faculty of Sciences, Department of Mathematics, Bartın, Turkey

eguler@bartin.edu.tr okisi@bartin.edu.tr

Abstract

In this paper, we study the logaritmic spiral-type helicoidal surface in Euclidean 3-space. We obtain logaritmic spiral-type helicoidal surface with some figures, and calculate its curvatures.

Keywords: 3-space, logaritmic spiral-type helicoidal surface, Gaussian curvature, mean curvature.

1. Introduction

The spirals were discovered by the studies of the ancient Greeks. They have seen in DNA double helix, sunflowers, the path of draining water, weather patterns (including hurricanes), vine tendrils, phyllotaxis (the arrangement of leaves on a plant stem), galaxies, the horns of various animals, mollusc shells, the nautilus shell, snail shells, whirlpools, ferns and algae. They were studied by many mathematicians including Leonardo Fibonacci, who tried to understand order in nature. See also [5] for spirals.

Logarithmic spiral, which was discovered by Rene Descartes, is a special type of spirals. Expanded results of it was given by Jacob Bernoulli. See [6] for details.

Focusing the ruled (helicoid) and rotational characters, we meet Bour's theorem in [1]. Do Carmo and Dajczer [2] proved that there exists a two-parameter family of helicoidal surfaces isometric to a given helicoidal surface using a result of Bour [1]. See also [3] for details. Some relations among the Laplace-Beltrami operator and curvatures of the helicoidal surfaces were shown by Güler, Yaylı and Hacısalihoğlu [4].

In this paper, we study the logaritmic spiral-type helicoidal surface in Euclidean 3-space $\mathbb{E}^3$. We give some basic elements of three dimensional Euclidean geometry in section 2. In section 3, we define helicoidal surface. In section 4, we define spiral surface. Finally, we obtain logaritmic spiral-type helicoidal surface, and calculate its curvatures in the last section.

2. Preliminaries

In the rest of this paper, we shall identify a vector (a,b,c) with its transpose. Let **M** be an isometric immersion of surface M^2 in $\mathbb{E}^3$**.** The vector product of $\vec{x} = (x_1, x_2, x_3)$, $\vec{y} = (y_1, y_2, y_3)$ on $\mathbb{E}^3$ is defined as follows