4th International Conference on Mathematics*
"An Istanbul Meeting for World Mathematicians"

ICOM 2020 Conference Proceedings Book

ISBN: 978-605-67964-6-3

Editor Kenan Yıldırım

***Online conference due to pandemic**

ICOM 2020
ISTANBUL / TURKEY

On Fourth Fundamental Form of the Translation Hypersurface

Erhan Güler[1], Ömer Kişi[2]

[1,2]Faculty of Sciences, Departmant of Mathematics, Bartın University, Turkey
E-mail(s): eguler@bartin.edu.tr okisi@bartin.edu.tr

Abstract

We examine the fourth fundamental form of the translation hypersurface in the four dimensional Euclidean space. We also discuss I, II, III and IV fundamental forms of a translation hypersurface.

1. Introduction

A *translation surface* is a surface that is generated by translations: for two space curves α, β with a common point P, the curve α is shifted such that point P is moving on β. Then the curve α generates a translation surface $\mathbf{x}(u, v) = \alpha(u) + \beta(v)$.

Translation surface examples: The *elliptic paraboloid* $z = x^2 + y^2$ can be generated by parabolas α: $(x, 0, x^2)$ and β: $(0, y, y^2)$. *Right circular cylinder*: α is a circle (or another cross section) and β is a line. The *hyperbolic paraboloid* $z = x^2 - y^2$ can be generated by parabolas α: $(x, 0, x^2)$ and β: $(0, y, -y^2)$.

A *translation hypersurface* (TH) in $\mathbb{E}^4$ is a hypersurface that is generated by translations: for three space curves α, β, γ with a common point P, the curve α is shifted such that point P is moving on β and γ, respectively. So, the curve α generates a translation hypersurface in $\mathbb{E}^4$.

Parametrization of the translation hypersurface is given by

$$\mathbf{x}(u, v, w) = \big(u, 0,0, f(u)\big) + \big(0, v, 0, g(v)\big) + \big(0,0, w, h(w)\big) = \alpha(u) + \beta(v) + \gamma(w),$$

where $f(u), g(v), h(w)$ are differentiable functions for all $u, v, w \in I \subset \mathbb{R}$. More clear form of it as follows

$$\mathbf{x}(u, v, w) = \big(u, v, w, f(u) + g(v) + h(w)\big). \tag{1.1}$$

Arslan et al [1] studied translation surfaces in 4-dimensional Euclidean space. Chen, Sun, and Tang [2] introduced translation hypersurfaces with constant mean curvature (CMC) in $(n+1)$-dimensional spaces. Dillen, Verstraelen, and Zafindratafa [3] worked a generalization of the translation surfaces of Scherk. Inoguchi, Lopez, and Munteanu [4] studied minimal translation surfaces in the Heisenberg group Nil_3. Lima, Santos, and Sousa [5] gave translation hypersurfaces with constant scalar curvature into the Euclidean space. Lima, Santos, and Sousa [6] considered generalized translation hypersurfaces in Euclidean space. Liu [7] introduced translation surfaces with CMC in 3-dimensional spaces. Lopez [8] studied minimal translation surfaces in hyperbolic space. Lopez and Moruz [9] obtained translation and homothetical surfaces with constant curvature in Euclidean space. Lopez and Munteanu [10] worked minimal translation surfaces in Sol_3. Moruz and Munteanu [11] considered hypersurfaces in the Euclidean space $\mathbb{E}^4$ defined as the sum of a curve and a surface whose mean curvature vanishes. Munteanu, Palmas, and Ruiz-Hernandez [12] studied minimal translation hypersurfaces in Euclidean space. Scherk [13] gave his classical minimal translation surface. Seo [14] worked translation hypersurfaces with constant curvature