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Family of Enneper Minimal Surfaces

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Abstract: We consider a family of higher degree Enneper minimal surface E_m for positive integers m in the three-dimensional Euclidean space $\mathbb{E}^3$. We compute algebraic equation, degree and integral free representation of Enneper minimal surface for $m = 1, 2, 3$. Finally, we give some results and relations for the family E_m.

Keywords: Enneper minimal surface family; Weierstrass representation; algebraic surface; degree; integral free representation

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1. Introduction

Minimal surfaces have an important role in the mathematics, physics, biology, architecture, etc. These kinds of surfaces have been studied over the centuries by many mathematicians and also geometers. A *minimal surface* in $\mathbb{E}^3$ is a regular surface for which the mean curvature vanishes identically.

There are many important classical works on minimal surfaces in the literature such as [1–10]. However, we only see a few notable works about algebraic minimal surfaces, including general results and the properties. They were given by Enneper [11,12], Henneberg [13,14] and Weierstrass [9,15].

One of them is the classical Enneper minimal surface that was given by Enneper. See [11,12] for details. About Enneper minimal surface, many nice papers were done such as [16–24] in the last few decades.

In this paper, we introduce a family of higher degree Enneper minimal surface E_m for positive integers m in the three-dimensional Euclidean space $\mathbb{E}^3$. In Section 2, we give the family of Enneper minimal surfaces E_m. We obtain the algebraic equation and degree of surface E_1 (resp., E_2, E_3). Using the integral free form of Weierstrass, we find some algebraic functions for E_m $(m \geq 1, m \in \mathbb{Z})$ in Section 3. Finally, we give some general findings for a family of higher degree Enneper minimal surface E_m with a table in the last section.

2. The Family of Enneper Minimal Surfaces E_m

We will often identify $\vec{x}$ and $\vec{x^t}$ without further comment. Let $\mathbb{E}^3$ be a three-dimensional Euclidean space with natural metric $\langle . , . \rangle = dx^2 + dy^2 + dz^2$.

Let $\mathcal{U}$ be an open subset of $\mathbb{C}$. A *minimal* (or *isotropic*) *curve* is an analytic function $\Psi : \mathcal{U} \to \mathbb{C}^n$ such that $\Psi'(\zeta) \cdot \Psi'(\zeta) = 0$, where $\zeta \in \mathcal{U}$, and $\Psi' := \frac{\partial \Psi}{\partial \zeta}$. In addition, if $\Psi' \cdot \overline{\Psi'} = |\Psi'|^2 \neq 0$, then Ψ is a *regular minimal curve.*

Thus, let see the following lemma for complex minimal curves.

Lemma 1. *Let $\Psi : \mathcal{U} \to \mathbb{C}^3$ be a minimal curve and write $\Psi' = (\varphi_1, \varphi_2, \varphi_3)$. Then,*

$$\mathcal{F} = \frac{\varphi_1 - i\varphi_2}{2} \quad \textit{and} \quad \mathcal{G} = \frac{\varphi_3}{\varphi_1 - i\varphi_2}$$

lead to the Weierstrass representation of Ψ. *That is,*

$$\Psi' = \left(\mathcal{F}\left(1-\mathcal{G}^2\right), i\mathcal{F}\left(1+\mathcal{G}^2\right), 2\mathcal{F}\mathcal{G}\right).$$

Therefore, we have minimal surfaces in the associated family of a minimal curve, as given by the following Weierstrass representation theorem [9] for minimal surfaces:

Theorem 1. *Let* $\mathcal{F}$ *and* $\mathcal{G}$ *be two holomorphic functions defined on a simply connected open subset* $\mathcal{U}$ *of* $\mathbb{C}$ *such that* F *does not vanish on* $\mathcal{U}$. *Then, the map*

$$\mathbf{x}(\zeta) = \mathrm{Re}\int^{\zeta}\begin{pmatrix}\mathcal{F}\left(1-\mathcal{G}^2\right)\\ i\,\mathcal{F}\left(1+\mathcal{G}^2\right)\\ 2\mathcal{F}\mathcal{G}\end{pmatrix}d\zeta$$

is a minimal, conformal immersion of $\mathcal{U}$ *into* $\mathbb{C}^3$, *and* $\mathbf{x}$ *is called the Weierstrass patch.*

We now consider the Enneper's curve of value m:

Lemma 2. *The Enneper's curve of value* m

$$E_m(\zeta) = \left(\zeta - \frac{\zeta^{2m+1}}{2m+1},\ i\left(\zeta + \frac{\zeta^{2m+1}}{2m+1}\right),\ 2\frac{\zeta^{m+1}}{m+1}\right) \tag{1}$$

is a minimal curve in $\mathbb{C}^3$, *where* $m \in \mathbb{R} - \{-1, -1/2\}$, $\zeta \in \mathbb{C}$, $i = \sqrt{-1}$.

Then, we have $E'_m \cdot E'_m = 0$. Hence, Enneper's surface of value m in $\mathbb{E}^3$ is

$$E_m(\zeta) = \mathrm{Re}\int E'_m(\zeta)\,d\zeta. \tag{2}$$

Lemma 3. *The Weierstrass patch determined by the functions*

$$F(\zeta) = 1 \quad \textit{and} \quad \mathcal{G}(\zeta) = \zeta^m$$

is a representation of Enneper's higher degree surfaces E_m, *where* $m \geq 2$.

For $m = 1$, we get the classical Enneper's surface E_1 (see also [4,11,25] for details).

Remark 1. *Note that the catenoid and classical Enneper's surface are the only complete regular minimal surfaces in* $\mathbb{E}^3$ *with finite total curvature* -4π.

See [5] for details.

Gray, Abbena and Salamon [26] gave the complex forms of the Enneper's curve and surface of value m. Therefore, the associated family of minimal surfaces is described by

$$\begin{aligned} E(r,\theta;\alpha) &= \mathrm{Re}\int e^{-i\alpha}E'_m \\ &= \cos(\alpha)\,\mathrm{Re}\int E'_m + \sin(\alpha)\,\mathrm{Im}\int E'_m \\ &= \cos(\alpha)\ E_m(r,\theta) + \sin(\alpha)\ E^*_m(r,\theta). \end{aligned}$$

When $\alpha = 0$ (resp. $\alpha = \pi/2$), we have the Enneper's surface of value m (resp. the conjugate surface E^*_m).

The parametric equation of E_m, in polar coordinates, is

$$E_m(r,\theta) = \begin{pmatrix} r\cos(\theta) - \frac{r^{2m+1}}{2m+1}\cos\left[(2m+1)\theta\right] \\ -r\sin(\theta) - \frac{r^{2m+1}}{2m+1}\sin\left[(2m+1)\theta\right] \\ 2\frac{r^{m+1}}{m+1}\cos\left[(m+1)\theta\right] \end{pmatrix}. \tag{3}$$

Using the binomial formula, we obtain the following parametric equations of $E_m(u,v)$:

$$\begin{aligned} x(u,v) &= \mathrm{Re}\left\{u + iv - \frac{1}{2m+1}\left[\sum_{k=0}^{2m+1}\binom{2m+1}{k}u^{2m+1-k}(iv)^k\right]\right\}, \\ y(u,v) &= \mathrm{Re}\left\{-v + iu + \frac{i}{2m+1}\left[\sum_{k=0}^{2m+1}\binom{2m+1}{k}u^{2m+1-k}(iv)^k\right]\right\}, \\ z(u,v) &= \mathrm{Re}\left\{\frac{2}{m+1}\left[\sum_{k=0}^{m+1}\binom{m+1}{k}u^{m+1-k}(iv)^k\right]\right\}. \end{aligned} \tag{4}$$

Next, we will focus on the algebraic equation and degree of surface E_m.

With $\mathbb{R}^3 = \{(x,y,z) \mid x,y,z \in \mathbb{R}\}$, the set of roots of a polynomial $f(x,y,z) = 0$ gives an *algebraic surface*. An algebraic surface is said to be of *degree* n, when $n = \deg(f)$.

It is seen that $\deg(x) = 2m+1$, $\deg(y) = 2m+1$, $\deg(z) = m+1$ for $E_m(u,v)$ (see also Table 1 for details). Using polynomial eliminate methods, we calculate the algebraic equations and degrees of the surfaces E_1, E_2, E_3. For the surface E_1 (i.e., classical Enneper surface), it is known that the surface has degree 9. Thus, it is also an algebraic minimal surface. For expanded results of E_1, see [4].

2.1. *Algebraic Equation of Enneper Minimal Surface E_1*

The simplest Weierstrass representation $(\mathcal{F},\mathcal{G}) = (1,\zeta)$ gives classical Enneper minimal surface of value 1. In polar coordinates, the parametric equation of E_1 is

$$E_1(r,\theta) = \begin{pmatrix} r\cos(\theta) - \frac{r^3}{3}\cos(3\theta) \\ -r\sin(\theta) - \frac{r^3}{3}\sin(3\theta) \\ r^2\cos(2\theta) \end{pmatrix}, \tag{5}$$

where $r \in [-1,1]$, $\theta \in [0,\pi]$. The parametric form of the surface E_1, in (u,v) coordinates, is

$$E_1(u,v) = \begin{pmatrix} -\frac{1}{3}u^3 + uv^2 + u \\ -u^2v + \frac{1}{3}v^3 - v \\ u^2 - v^2 \end{pmatrix}, \tag{6}$$

where $u,v \in \mathbb{R}$.

Lemma 4. *A plane intersects an algebraic minimal surface in an algebraic curve* [13].

See also [4] for details. Considering the above lemma, we find the algebraic equation of the curve

$$E_1(u,0) = \gamma_1(u) = \left(u - \frac{u^3}{3}, 0, u^2\right)$$

on the xz-plane is as follows (see Figure 1, left):

$$z^3 - 6z^2 - 9x^2 + 9z = 0,$$

and its degree is $deg(\gamma_1) = 3$. Thus, xz-plane intersects the algebraic minimal surface E_1 in an algebraic curve $\gamma_1(u)$.

Using the polynomial eliminate method, we calculate the irreducible algebraic equation $E_1(x,y,z)=0$ of surface $E_1(u,v)$ by hand as follows (see Figure 1, right):

$$\begin{aligned}&-64z^9+432x^2z^6-432y^2z^6+1215x^4z^3+6318x^2y^2z^3+3888x^2z^5\\&+1215y^4z^3+3888y^2z^5+1152z^7+729x^6-2187x^4y^2+4374x^4z^2\\&+2187x^2y^4+6480x^2z^4-729y^6-4374y^4z^2-6480y^2z^4-729x^4z\\&+1458x^2y^2z-3888x^2z^3-729y^4z-3888y^2z^3-5184z^5=0.\end{aligned}$$

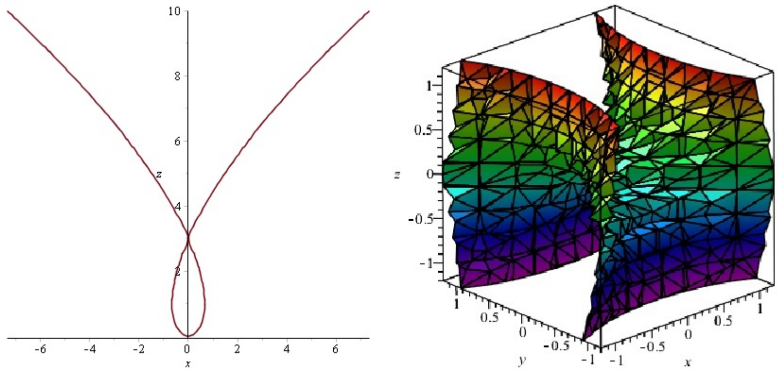

Figure 1. **left**: algebraic curve $\gamma_1(u)$; **right**: algebraic surface $E_1(x,y,z)=0$.

Its degree is $deg(E_1)=9$. Therefore, E_1 is an algebraic minimal surface. All of these results for classical Enneper surface E_1 were obtained first in [11] by Enneper.

Next, we study algebraic equations and degrees of the higher degree Enneper minimal surfaces for values $m=2$ and $m=3$.

2.2. Algebraic Equation of Enneper Minimal Surface E_2

In polar coordinates, the parametric equation of E_2 is

$$E_2(r,\theta)=\begin{pmatrix} r\cos(\theta)-\frac{r^5}{5}\cos(5\theta)\\ -r\sin(\theta)-\frac{r^5}{5}\sin(5\theta)\\ \frac{2}{3}r^3\cos(3\theta)\end{pmatrix}, \tag{7}$$

where $r\in[-1,1]$, $\theta\in[0,\pi]$. The parametric form of the surface E_2, in (u,v) coordinates, is

$$E_2(u,v)=\begin{pmatrix} u-\frac{1}{5}u^5+2u^3v^2-uv^4\\ -v-u^4v+2u^2v^3-\frac{1}{5}v^5\\ \frac{2}{3}u^3-2uv^2\end{pmatrix}, \tag{8}$$

where $u,v\in\mathbb{R}$.

Using the polynomial eliminate method, we find the algebraic equation of the curve

$$E_2(u,0)=\gamma_2(u)=\left(u-\frac{u^5}{5},0,\frac{2}{3}u^3\right)$$

on the xz-plane as follows (see Figure 2, left)

$$-243z^5-4000x^3-5400xz^2+6000z=0,$$

and its degree is $deg(\gamma_2) = 5$. Hence, xz-plane intersects the algebraic minimal surface E_2 in an algebraic curve $\gamma_2(u)$.

We calculate the irreducible algebraic equation $E_2(x,y,z) = 0$ of surface $E_2(u,v)$ by using Maple software (version 17, Waterloo Maple Inc., Waterloo, ON, Canada) as follows (see Figure 2, right)

$$\begin{aligned}&847288609443z^{25} + 4358480501250x^3z^{20} - 13075441503750xy^2z^{20}\\&-131157978046875x^6z^{15} - 474186536015625x^4y^2z^{15}\\&+107 \text{ other lower degree terms} = 0,\end{aligned}$$

and its degree is $deg(E_2) = 25$. Hence, E_2 is an algebraic minimal surface.

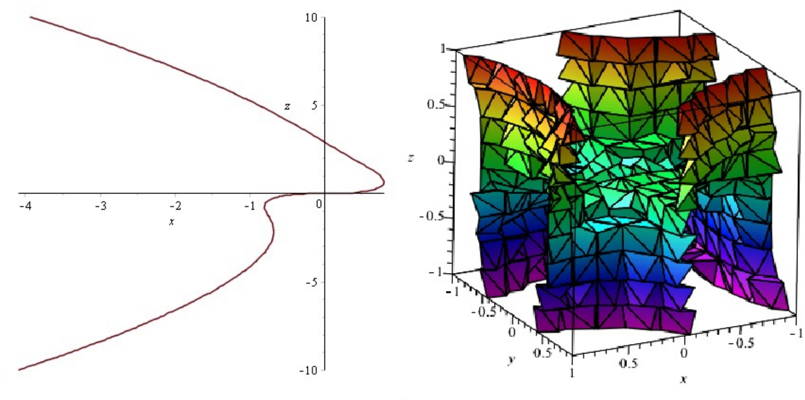

Figure 2. **left**: algebraic curve $\gamma_2(u)$; **right**: algebraic surface $E_2(x,y,z) = 0$.

2.3. *Algebraic Equation of Enneper Minimal Surface E_3*

The parametric equation of Enneper's minimal surface of value 3, in polar coordinates, is

$$E_3(r,\theta) = \begin{pmatrix} r\cos(\theta) - \frac{r^7}{7}\cos(7\theta) \\ -r\sin(\theta) - \frac{r^7}{7}\sin(7\theta) \\ \frac{1}{2}r^4\cos(4\theta) \end{pmatrix}, \tag{9}$$

where $r \in [-1,1]$, $\theta \in [0,\pi]$. In (u,v) coordinates, E_3 has the following form:

$$E_3(u,v) = \begin{pmatrix} u - \frac{1}{7}u^7 + 3u^5v^2 - 5u^3v^4 + uv^6 \\ -v - u^6v + 5u^4v^3 - 3u^2v^5 + \frac{1}{7}v^7 \\ \frac{1}{2}u^4 - 3u^2v^2 + \frac{1}{2}v^4 \end{pmatrix}, \tag{10}$$

where $u, v \in \mathbb{R}$.

We get the algebraic equation of the curve

$$E_3(u,0) = \gamma_3(u) = \left(u - \frac{u^7}{7}, 0, \frac{u^4}{2}\right)$$

on the xz-plane as follows:

$$128z^7 - 1568z^4 - 2401x^4 - 5488x^2z^2 + 4802z = 0.$$

Its degree is $deg(\gamma_3) = 7$. Then, we see that the xz-plane intersects the algebraic minimal surface E_3 in an algebraic curve $\gamma_3(u)$.

In Cartesian coordinates x, y, z, the algebraic equation $E_3(x, y, z) = 0$ of surface $E_3(u, v)$ by using Maple software is as follows:

$$
\begin{aligned}
&-24758800785707605497982484482z^{49} + 5079604062565768134821675008x^4z^{42} \\
&-30477624375394608808930050048x^2y^2z^{42} + 5079604062565768134821675008y^4z^{42} \\
&+63385035065421621776662449356 8x^8z^{35} + 406 \text{ other lower degree terms} = 0.
\end{aligned}
$$

Its degree is $deg(E_3) = 49$. Thus, E_3 is an algebraic minimal surface.

Corollary 1. *The family of higher degree (also classical) Enneper minimal surfaces $E_m(u, v)$ are algebraic minimal surfaces, where $m \in \mathbb{Z}$, $m \geq 1$ (see Table 1).*

Next, we obtain the general algebraic equation for the curve γ_m:

Corollary 2. *We consider the curve*

$$E_m(u, 0) = \gamma_m(u) = \left(u - \frac{u^{2m+1}}{2m+1}, 0, \frac{2u^{m+1}}{m+1}\right)$$

on the xz-plane. By using Mathematica (version 8, Wolfram Research Inc., Champaign, IL, USA; Oxfordshire, UK; Tokyo, Japan; Boston, MA, USA), we get the following algebraic equation:

$$(2m+1)(x - 2^{-\frac{1}{m+1}}[(m+1)z]^{\frac{1}{m+1}})^{m+1} + (2^{-1}(m+1)z])^{2m+1} = 0, \tag{11}$$

where $m + 1 \neq 0$, $2m + 1 \neq 0$, and its degree is $deg(\gamma_m) = 2m + 1$.

3. Integral Free Form

Integral free form of the Weierstrass representation (see [15]) is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \operatorname{Re} \begin{pmatrix} (1 - w^2)\,\phi''(w) + 2w\phi'(w) - 2\phi(w) \\ i\left[(1 + w^2)\,\phi''(w) - 2w\phi'(w) + 2\phi(w)\right] \\ 2\left[w\phi''(w) - \phi'(w)\right] \end{pmatrix} \equiv \operatorname{Re} \begin{pmatrix} f_1(w) \\ f_2(w) \\ f_3(w) \end{pmatrix}, \tag{12}$$

where algebraic function $\phi(w)$ and the functions $f_i(w)$ are connected by the relation

$$\phi(w) = \frac{1}{4}\left(w^2 - 1\right) f_1(w) - \frac{i}{4}\left(w^2 + 1\right) f_2(w) - \frac{1}{2} w f_3(w) \tag{13}$$

for $w \in \mathbb{C}$. Integral free form is suitable for algebraic minimal surfaces. For instance, $\phi(w) = \frac{1}{6}w^3$ gives rise to classical Enneper minimal surface E_1 (see [4] for details).

After some calculations by using the last two equations above, we get following corollary:

Corollary 3. *We obtain algebraic functions $\phi(w)$, and then get the function $\phi(w) = \frac{w^3}{2} - \frac{w^4}{3} + \frac{w^5}{10}$, which leads to Enneper minimal surface E_2. We also find $\phi(w) = \frac{w^3}{2} - \frac{w^5}{4} + \frac{w^7}{14}$ for E_3, $\phi(w) = \frac{w^3}{2} - \frac{w^6}{5} + \frac{w^9}{18}$ for E_4, and so on.*

Hence, we have following lemma:

Lemma 5. *The algebraic function in the integral free form for a higher degree (also classical) Enneper minimal surfaces E_m is as follows:*

$$\phi_{E_m}(w) = \frac{w^3}{2} - \frac{w^{m+2}}{m+1} + \frac{w^{2m+1}}{2(2m+1)}, \tag{14}$$

where $m \geq 1, m \in \mathbb{Z}$.

4. Conclusions

Briefly, we give all findings, calculated in Sections 2 and 3 for the Enneper surface family, in Table 1 as follows.

Table 1. Algebraic Enneper minimal surfaces E_m, $m \geq 1, m \in \mathbb{Z}$.

Surface	**deg**(x, y, z)	**deg** (E_m)	**deg** (γ_m)	*Algebraic Function*
E_1 (*classical*)	$(3, 3, 2)$	9	3	$\frac{1}{6}w^3$
E_2	$(5, 5, 3)$	25	5	$\frac{1}{2}w^3 - \frac{1}{3}w^4 + \frac{1}{10}w^5$
E_3	$(7, 7, 4)$	49	7	$\frac{1}{2}w^3 - \frac{1}{4}w^5 + \frac{1}{14}w^7$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
E_m	$(2m+1, 2m+1, m+1)$	$(2m+1)^2$	$2m+1$	$\frac{1}{2}w^3 - \frac{1}{m+1}w^{m+2} + \frac{1}{2(2m+1)}w^{2m+1}$

Looking at the table above, we also have the following results:

Corollary 4. *We find the following relation between degree of algebraic function* $\phi_{E_m}(w)$ *in the integral free form and curve* γ_m *of surface* E_m *:*

$$\deg(\gamma_m) = 2m + 1 = \deg(\phi_{E_m})$$

and

$$\deg(E_m) = (2m+1)^2 = (\deg(\gamma_m))^2 = (\deg(\phi_{E_m}))^2,$$

where integers $m \geq 1$.

Remark 2. *For integers* $m \geq 4$*, algebraic equations and also degrees of Enneper minimal surfaces* E_m *can be calculated. However, calculation is a time problem for software programmes.*

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