

# VII INTERNATIONAL MEETING ON LORENTZIAN GEOMETRY



IME - Instituto de  
Matemática e Estatística

Sao Paulo

July 22nd-26th, 2013

## Invited Speakers

Albert Fathi (University of Lyon, France)  
Bennet Palmer (Idaho State University, U.S.A.)  
Giovanni Calvaruso (University of Salento, Italy)  
Jorge Herbert de Lira (Federal University of Ceará, Brazil)  
José Luis Flores (University of Malaga, Spain)  
José M<sup>a</sup> Martín Senovilla (University of the Basque Country, Spain)  
Laszlo Beno Szabados (Hungarian Academy of Sciences, Hungary)  
Levi Lopes de Lima (Federal University of Ceará, Brazil)  
Miguel Brozos-Vázquez (University of A Coruña, Spain)  
Miyuki Koiso (Kyushu University, Japan)  
Nicolas Ginoux (Regensburg University, Germany)  
Piotr Chruściel (University of Vienna, Austria)  
Sergio Dain (National University of Cordoba, Argentina)  
Stefan Suhr (University of Hamburg, Germany)  
Neil Russell (Northern Michigan University, U.S.A.)  
James A. Isenberg (University of Oregon, U.S.A.)

## Scientific Committee

Paolo Piccione  
Miguel Sánchez  
Miguel Angel Javaloyes  
Erasmus Caponio  
Christian Bär  
Henri Anchiaux  
Marc Mars  
Greg Galloway  
Eduardo García-Río  
Roberto Giambò

## Courses

Levi Lopes de Lima (Federal University of Ceará)  
"An introduction to the Riemannian Penrose inequality"  
  
Nicolas Ginoux (Regensburg University)  
"Linear wave equations on spacetimes"

## Organizing Committee

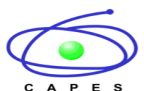
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# 7th International Meeting on Lorentzian Geometry

São Paulo

July 22nd--26th, 2013

Sponsored by:



GeLoSP2013 – 7th International Meeting on Lorentzian Geometry

Program: talks and posters

**Morning**

Chairman: P. Piccione	Chairman: M. A. Javaloyes	Chairman: H. Ancaux	Chairman: E. Garcia-Rio	Chairman: G. Galloway
Monday 22nd	Tuesday 23rd	Wednesday 24th	Thursday 25th	Friday 26th
Registration Coffee Opening 09:00—10:30	<u>Minicourse 1</u> <u>N. Ginoux</u> 9:00—10:00	<u>Minicourse 1</u> <u>N. Ginoux</u> 9:00—10:00	<u>Minicourse 1</u> <u>N. Ginoux</u> 9:00—10:00	<u>P. Chruściel</u> 9:00—10:00
<u>L. Szabados</u> 10:30—11:30	<u>S. Suhr</u> 10:00—11:00	<u>C. Aquino</u> 10:00--10:30	<u>M. Koiso</u> 10:00—11:00	<u>F. Mercuri</u> 10:00—11:00
<u>J. H. de Lira</u> 11:30--12:30	Coffee 11:00—11:30	Coffee 10:30—11:00	Coffee 11:00—11:30	Coffee 11:00—11:30
LUNCH 12:30—14:00	<u>I. Costa e Silva</u> 11:30—12:30	<u>J. Isenberg</u> 11:00—12:00	<u>M. Brozos-Vázquez</u> 11:30—12:30	<u>S. Dain</u> 11:30—12:30
	LUNCH 12:30—14:00	<u>Minicourse 2</u> <u>L. Lopes de Lima</u> 12:00—13:00		
		LUNCH	LUNCH 12:30—14:00	LUNCH 12:30—14:00

**Afternoon**

Chairman: M. Sánchez	Chairman: E. Caponio	Chairman: P. Piccione	Chairman: M. Mars
<u>J. M. M. Senovilla</u> 14:00—15:00	<u>Minicourse 2</u> <u>L. Lopes de Lima</u> 14:00—15:00	<u>Minicourse 2</u> <u>L. Lopes de Lima</u> 14:00—15:00	<u>A. Honda</u> 14:00—14:30
<u>M. Caballero</u> 15:00-15:30	<u>B. Palmer</u> 15:00—16:00	<u>N. Russell</u> 15:00—16:00	<u>J. J. Salamanca</u> 14:30-15:00
Coffee 15:30—16:00	Coffee 16:00—16:30	Coffee 16:00—16:30	<u>F. Cibotaru</u> 15:00—15:30
<u>Z. Dusek</u> 16:00—16:30	<u>G. Calvaruso</u> 16:30—17:30	<u>J. L. Flores</u> 16:30—17:30	<u>G. Ovando</u> 15:30—16:00
<u>R. Quiroga</u> 16:30—17:00	<u>E. Güler</u> 17:30—18:00	<u>H. de Lima</u> 17:30—18:00	Coffee 16:00—16:30
<u>V. Del Barco</u> 17:00—17:30			<u>Special Session</u> <u>XXI Century singularity</u> <u>theorems</u> J. M. M. Senovilla 16:30—17:30
<u>A. Albuje</u> 17:30—18:00		<u>OSESP Concert</u>	Closing



**Alma L. Albuje**  
**University of Córdoba, Spain**

**Monday 22nd, 17:30-18:00**

**Title:** *Complete spacelike hypersurfaces in a Robertson-Walker spacetime*

**Abstract:** In this talk we give some uniqueness results concerning to complete spacelike hypersurfaces with constant mean curvature immersed in a Robertson-Walker spacetime. These results are obtained as a suitable application of the well-known generalized maximum principle of Omori-Yau. We also present some consequences in the case of a static Robertson-Walker spacetime. Finally, we obtain a non-parametric versions of our results for entire vertical graphs. This contribution is a joint work with F. E. Camargo and H. F. de Lima, and is contained in [1].

**References**

[1] A. L. Albuje, F. E. C. Camargo and H. F. de Lima, *Complete spacelike hypersurfaces in a Robertson-Walker spacetime*. Math. Proc. Camb. Phil. Soc. 151 (2011), 271-282.

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**Cícero Aquino**  
**Federal University of Piauí, Brazil.**

**Wednesday 24<sup>th</sup>, 10:00—10:30**

**Title:** *Uniqueness of Complete Hypersurfaces with Bounded Higher Order Mean Curvatures in Semi-Riemannian Warped Products*

**Abstract:** In this talk, we deal with complete hypersurfaces immersed with bounded higher order mean curvatures in steady state-type spacetimes and in hyperbolic-type spaces. By applying a generalised maximum principle for the Yau's square operator [1], we obtain uniqueness results in each of these ambient spaces. These results are contained in [2].

**References**

[1] S. Y. Cheng and S. T. Yau, *Hypersurfaces with constant scalar curvature*, Math. Ann. 225 (1977), 195-204.

[2] C. P. Aquino and H. F. de Lima, *Uniqueness of Complete Hypersurfaces with Bounded Higher Order Mean Curvatures in Semi-Riemannian Warped Products*, Glasgow Math. J. 54 (2012) 201-212.

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**Miguel Brozos-Vázquez**  
**University of A Coruña, Spain**

**Thursday 25<sup>th</sup>, 11:30—12:30**

**Title:** *Lorentzian Gradient Ricci Solitons*

**Abstract:** A gradient Ricci soliton is a triple  $(M, g, f)$  where  $(M, g)$  is a pseudo-Riemannian manifold and  $f$  is a function satisfying the Ricci soliton equation

$$\text{Hess}(f) + \rho = \lambda g$$

where  $\text{Hess}(f)$  denotes the Hessian of  $f$ ,  $\rho$  denotes the Ricci tensor and  $\lambda$  is a real number. In this talk we will review some results on Lorentzian gradient Ricci solitons focussing on two classes of manifolds. First we will concentrate on locally conformally flat manifolds and afterwards on homogeneous manifolds. For locally conformally flat gradient Ricci solitons we will see that they are locally isometric to a Robertson Walker warped product, if the gradient of  $f$  is nonnull, and to a plane wave, if the gradient of  $f$  is null. For homogeneous gradient Ricci solitons we will provide some rigidity results and give evidence of important differences with the Riemannian setting.

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**Magdalena Caballero**  
**University of Córdoba, Spain**

**Monday 22<sup>nd</sup>, 15:00—15:30**

**Title:** *New Calabi-Bernstein results for some elliptic non-linear equations*

**Abstract:** In this work we deal with both the maximal surface equation and the constant mean curvature spacelike surface equation in three-dimensional Generalized Robertson Walker spacetimes satisfying certain energy conditions. We get uniqueness and non existence results of entire solutions for the case in which the fiber is complete. This is a joint work with Alfonso Romero, from the University of Granada, and Rafael M. Rubio, from the University of Córdoba, [1].

#### **References**

[1] M. Caballero, A. Romero and R. M, Rubio, *New Calabi-Bernstein results for some elliptic non-linear equations*. Analysis and Applications 11 (2013), no.1, 1350002.

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**Giovanni Calvaruso**  
**University of Salento, Italy**

**Tuesday 23<sup>rd</sup>, 16:30—17:30**

**Title:** *On the geometry of four-dimensional homogeneous Lorentzian Manifolds*

**Abstract:** We shall present an overview of several recent results concerning the geometry of four-dimensional homogeneous Lorentzian Manifolds. In particular, we shall discuss different approaches to the problem of classifying these spaces; we shall describe the geometry of non-reductive examples; we shall illustrate the examples with special geometric properties, like Einstein spaces, Ricci solitons, conformally flat spaces, Walker manifolds; we shall describe four-dimensional Lorentzian Lie groups, giving an explicit classification of the Einstein and Ricci-parallel examples.

**References**

- [1] G. Calvaruso and A. Fino, *Ricci solitons and geometry of four-dimensional non- reductive homogeneous spaces*, Canadian J. Math., 64 (2012), 778-804.
- [2] G. Calvaruso and A. Zaeim, *Four-dimensional homogeneous Lorentzian manifolds*, submitted.
- [3] G. Calvaruso and A. Zaeim, *Geometric structures over non-reductive homogeneous 4-spaces*, Adv. Geom., to appear.
- [4] G. Calvaruso and A. Zaeim, *Conformally flat homogeneous pseudo-Riemannian four-manifolds*, Tohoku Math. J., to appear.
- [5] G. Calvaruso, A. Fino, A. Zaeim, *Homogeneous geodesics of non-reductive homogeneous pseudo-Riemannian 4-manifolds*, submitted
- [6] G. Calvaruso and A. Zaeim, *Four-dimensional Lorentzian Lie groups*, Diff. Geom. Appl. 31 (2013), 496--509.

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**Piotr Chruściel**  
**University of Vienna, Austria**

**Friday 26<sup>th</sup>, 09:00—10:00**

**Title:** *On Lorentzian causality with continuous metrics*

**Abstract:** I will discuss the problems which arise when attempting to do Lorentzian causality theory with metrics which are merely continuous, and present an application of such studies to the general relativistic Cauchy problem.

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**Florentiu Cibotaru**  
**Federal University of Ceará, Brazil**

**Friday 26<sup>th</sup>, 15:00—15:30**

**Title:** *Area and Co-area Formulas in a Finslerian Context*

**Abstract:** We will present the classical area and coarea formulas for maps between Finsler manifolds for any given definitions of volume. The jacobian and cojacobian are described by very simple expressions very much in the spirit of Riemannian geometry. The applications we will give are an "euclidean" proof of the anisotropic Sobolev inequality and an anisotropic tube formula for hypersurfaces.

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**Ivan Costa e Silva**

**Federal University of Santa Catarina, Brazil**

**Tuesday 23<sup>rd</sup>, 11:30—12:30**

**Title:** *On the geodesic incompleteness of spacetimes containing marginally (outer) trapped surfaces*

**Abstract:** In a recent paper [1], Eichmair, Galloway and Pollack have proved a Gannon-Lee-type singularity theorem based on the existence of marginally outer trapped surfaces (MOTS) on noncompact initial data sets for globally hyperbolic spacetimes. A natural question is whether the corresponding incomplete geodesics could still be complete in a possible non-globally hyperbolic extension of space-time. In this paper, some variants of their result are given with weaker causality assumptions, thus suggesting that the answer is generically negative, at least if the putative extension has no closed timelike curves. We consider first marginally trapped surfaces (MTS) in chronological spacetimes, introducing the natural notion of a generic MTS, a notion also applicable to MOTS. In particular, a Hawking-Penrose-type singularity theorem is proven in chronological spacetimes with dimension  $n \geq 3$  containing a generic MTS. Such surfaces naturally arise as cross-sections of quasi-local generalizations of black hole horizons, such as dynamical and trapping horizons, and we discuss some natural conditions which ensure the existence of MTS in initial data sets. Nevertheless, much of the more recent literature has focused on marginally outer trapped surfaces (MOTS) rather than MTS as quasi-local substitutes for the description of black holes, as they are arguably more natural and easier to handle in a number of situations. It is therefore pertinent to ask to what

extent one can deduce the existence of singularities in the presence of MOTS alone. We address this issue and show that singularities indeed arise in the presence of generic MOTS, but under slightly stronger causal conditions than those in the case of MTS (specifically, for causally simple spacetimes). On the other hand, we show that with additional conditions on the MOTS itself, namely that it is either the boundary of a compact spatial region, or strictly stable in a suitable sense, then a Penrose-Hawking-type singularity theorem can still be established for chronological spacetimes containing generic MOTS.

## References

[1] M. Eichmair, G.J. Galloway and D. Pollack, *Topological censorship from the initial data point of view*, arXiv:1204.0278v1 (2012).

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**Sergio Dain**

**Universidad Nacional de Córdoba, Argentina**

**Friday 26th, 11:30—12:30**

**Title:** *Geometric inequalities for black holes*

**Abstract:** A geometric inequality in General Relativity relates quantities that have both a physical interpretation and a geometrical definition. It is well known that the parameters that characterize a stationary black hole (angular momentum, mass and horizon area) satisfy several important geometric inequalities. Remarkably enough, some of these inequalities also hold for dynamical black holes. This kind of inequalities, which are valid in the dynamical and strong field regime, play an important role in the characterization of the gravitational collapse. They are closely related with the cosmic censorship conjecture. In this talk I will review recent results in this subject.

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**Viviana Del Barco**  
**National University of Rosario, Argentina**

**Monday 22<sup>nd</sup>, 17:00—17:30**

**Title:** *Isometric actions on pseudo-Riemannian nilmanifolds*

**Abstract:** This work concerns the structure of the isometry group of pseudo-Riemannian 2-step nilmanifolds. We show that the action of the nilradical of the full isometry group does not need to be transitive, contrary to the Riemannian situation [3, 4]. Also, we study the action by isometries of several groups and we construct examples showing substantial differences with the Riemannian case. For a nilpotent Lie group endowed with a left-invariant pseudo-Riemannian metric we study conditions for which the subgroup of isometries fixing the identity element equals the subgroup of isometric automorphisms, improving the results in [2]. These conditions are satisfied by any pseudo-*H*-type Lie group (Lie groups introduced by Ciatti [1]) so the set equality holds in this family. This is a work in collaboration with Gabriela Ovando and a preliminar version of the article is available at arXiv:1303.4450v2 [math.DG].

**References**

- [1] P. Ciatti, *Scalar products on Clifford modules and pseudo-H-type Lie algebras*, Ann. Mat. Pura Appl., IV. Ser., 178 (2000), 1-31.
- [2] L. Cordero and P. Parker, *Isometry groups of pseudoriemannian 2-step nilpotent Lie groups*, Houston J. Math. 35 (2009) no.1, 49-72.
- [3] E. Wilson, *Isometry groups on homogeneous nilmanifolds*, Geom. Dedicata 12 (1982) no. 2, 337-345.
- [4] J. Wolf, *On Locally Symmetric Spaces of Non-negative Curvature and certain other Locally Homogeneous Spaces*, Comment. Math. Helv. 37 (1962-63) 266-295.

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**Zdenek Dusek**  
**Palacky University, Faculty of Science, Czech Republic**

**Monday 22<sup>nd</sup>, 16:00—16:30**

**Title:** *The existence of light-like homogeneous geodesics in homogeneous Lorentzian manifolds*

**Abstract:** In previous works, a fundamental affine method for studying homogeneous geodesics was developed. Using this method and elementary differential topology it was proved that any homogeneous affine manifold and in particular any homogeneous pseudo-Riemannian manifold admits a homogeneous geodesic through arbitrary point. In this talk I will show that this affine method can be refined and adapted to the pseudo-Riemannian case. Using this method and elementary topology it is proved that any homogeneous Lorentzian manifold of even dimension admits a light-like homogeneous geodesic. The method will be illustrated in detail with an example of the Lie group of dimension 3 with an invariant metric, which does not admit any light-like homogeneous geodesic. These are contained in Ref. [1].

**Reference**

[1] Z. Dusek, *The existence of light-like homogeneous geodesics in homogeneous Lorentzian manifolds*. arXiv:1203.6757v1.

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**José Luis Flores Dorado**  
**University of Malaga, Spain**

**Thursday 25<sup>th</sup>, 16:30—15:30**

**Title:** *Connections between the causal boundary and isocausality*

**Abstract:** Recently, a new viewpoint on the classical c-boundary in Mathematical Relativity has been developed, its relation with the conformal and other classical boundaries has been analyzed, and its computation in some classes of spacetimes, as the standard stationary ones, has been carried out. In this talk we consider the notion of isocausality given by García-Parrado and Senovilla, and explore its connections with the c-boundary within the framework of standard stationary spacetimes. As a consequence, the qualitative behavior of the c-boundary (at the three levels: point set, topological and chronological) of a wide class of spacetimes, is obtained.

## References

- [1] J.L.F., J. Herrera, M. Sánchez, *Isocausal spacetimes may have different causal boundaries*, Class. Quant. Grav., 28 (2011) 175016.
- [2] J.L.F., J. Herrera, M. Sánchez, *On the final definition of the causal boundary and its relation with the conformal boundary*, Adv. Theor. Math. Phys., 15 (2011), 991-1058.
- [3] J.L.F., J. Herrera, M. Sánchez, Gromov, *Cauchy and causal boundaries for Riemannian, Finslerian and Lorentzian manifolds*, Memoirs of the AMS, at press (arXiv:1011.1154).
- [4] J.L.F., J. Herrera, M. Sánchez, *Computability of the causal boundary by using isocausality*, Class. Quant. Grav., 30 (2013) 075009.
- [5] A. García-Parrado, J. M. M. Senovilla, *Causal relationship: a new tool for the causal characterization of Lorentzian manifolds*, Class. Quant. Grav., 22 (2003) 625-664.

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**Erhan Güler**

**Bartın University, Faculty of Science, Turkey**

**Tuesday 23<sup>rd</sup>, 17:30—18:00**

**Title:** *Bours minimal surface in three dimensional Lorentz-Minkowski space*

**Abstract:** In this talk, we focus on the differential geometry of the Bour's minimal surface. Bour's minimal surface has remarkable properties in three dimensional Lorentz- Minkowski space. We reveal the definite and indefinite cases of the Bour's surface using Weierstrass representations, and give some differential geometric properties of the astonishing maximal and minimal surfaces.

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**Atsufumi Honda**  
**Miyakonojo National College of Technology, Japan**

**Friday 26<sup>th</sup>, 14:00—14:30**

**Title:** *Global properties of wave fronts with one principal curvature constant*

**Abstract:** It is well known that any complete at surface in the Euclidean 3-space  $\mathbb{R}^3$  must be a cylinder over a plane curve. However, if one admits some singularities, there are many non-trivial complete surfaces in  $\mathbb{R}^3$ . Murata-Umehara [1] investigated global properties of surfaces with admissible singularities, called wave fronts. One of the most interesting and deepest results is that complete wave fronts must be orientable. On the other hand, Shiohama-Takagi [2] showed that any complete surface with one principal curvature a nonzero constant in  $\mathbb{R}^3$  must be a tube of a complete regular curve. In this talk, we introduce a generalization to wave fronts. Moreover, we prove that they must be orientable. We shall also introduce some related results for other ambient spaces.

**References**

- [1] S. Murata and M. Umehara, *Flat surfaces with singularities in Euclidean 3-space*, J. Differential Geom. 82 (2009), 279-316.
- [2] K. Shiohama and R. Takagi, *A characterization of a standard torus in  $E^3$* , J. Differential Geom. 4 (1970), 477-485.

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**James Isenberg**  
**University of Oregon, USA**

**Wednesday 24<sup>th</sup>, 11:00-12:00**

**Title:** *On the Nature of Singularities in Cosmological Solutions of Einstein's Equations*

**Abstract:** The Hawking-Penrose theorems tell us that cosmological solutions of Einstein's equations are generally singular, in the sense of causal geodesic incompleteness. These singularities might be marked by the blowup of curvature and tidal forces, or by the breakdown of physical determinism. Penrose has conjectured (in his "Strong Cosmic Censorship Conjecture") that it is generically unbounded curvature that causes singularities, rather than causal breakdown. The verification of Belinsky-Khalatnikov-Lifshitz behavior (the so-called BKL behavior) is generically present in a family of solutions has proven to be a useful tool for studying Strong Cosmic Censorship in that family. We discuss what is known about BKL behavior and Strong Cosmic Censorship in families of solutions defined by varying degrees of isometry, and discuss new results which we believe will extend this knowledge and provide new support for Strong Cosmic Censorship.

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**Miyuki Koiso**  
**Kyushu University, Japan**

**Thursday 25<sup>th</sup>, 10:00—11:00**

**Title:** *Bernstein-type theorems for surfaces with constant anisotropic mean curvature and CMC surfaces in the Lorentz-Minkowski space*

**Abstract:** A surface with constant anisotropic mean curvature (CAMC surface) is a stationary surface of a given anisotropic surface energy functional for volume-preserving variations. Surfaces with constant mean curvature (CMC surfaces) in the Lorentz-Minkowski space are regarded as CAMC surfaces for a certain special anisotropic surface energy. In this talk, we show that if a complete CAMC surface for a uniformly convex anisotropic surface energy in the euclidean three-space is a graph of a function in a whole plane, then it is a plane. Moreover, by using a similar method, we show that if a spacelike complete CMC surface in the Lorentz-Minkowski three-space satisfies a certain condition on the order of divergence of its Gauss map, then it is a plane.

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**Henrique de Lima**  
**Federal University of Campina Grande, Brazil**

**Thursday 25<sup>th</sup>, 17:30—18:00**

**Title:** *On the Unicity of Complete Hypersurfaces Immersed in a Semi-Riemannian Warped Product*

**Abstract:** Our purpose is to apply appropriate generalized maximum principles in order to study the unicity of complete hypersurfaces immersed in a semi-Riemannian warped product, which is supposed to obey a suitable convergence condition. In this setting, by assuming a natural comparison inequality between the  $r$ -th mean curvatures of the hypersurface and the ones of the slices of the slab where the hypersurface is contained, we establish sufficient conditions to guarantee that such a hypersurface must be a slice.

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**Jorge Herbert de Lira**  
**Federal University of Ceará, Brazil.**

**Monday 22<sup>nd</sup>, 11:30—12:30**

**Title:** *Spacelike graphs with prescribed mean curvature in static Lorentzian spaces*

**Abstract:** We will discuss some recent results on existence and uniqueness of spacelike graphs with prescribed mean curvature in Lorentzian manifolds endowed with a timelike Killing vector field.

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**Francesco Mercuri**  
**State University of Campinas, Brazil**

**Friday 26<sup>th</sup>, 10:00—11:00**

**Title:** *Minimal surfaces in Lorentzian manifolds*

**Abstract:** We will discuss the Weierstrass representation formula for minimal surfaces in Lorentzian manifolds, and we give some applications to the construction of examples and to the Bjorling problem.

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**Gabriela Ovando**  
**National University of Rosario, Argentina.**

**Friday 26<sup>th</sup>, 15:30—16:00**

**Title:** *Lorentzian Solvmanifolds in dimension four*

**Abstract:** The purpose here is the study of a family of Lorentzian solvmanifolds of dimension four. Results in [2] show that the oscillator group  $G$ , that is, the solvable Lie group of dimension four equipped with the bi-invariant Lorentzian metric is isometric to a nilmanifold which corresponds to the trivial extension of the Heisenberg Lie group  $\mathbb{R} \times H_3$ . This can be seen by proving that there is a free and transitive action by isometries of both groups on the same simply connected manifold [2]. On the other hand  $G$  admits a family of cocompact subgroups  $\Gamma_i$ , giving the compact spaces  $M_i = G/\Gamma_i$ . Thus the bi-invariant Lorentzian metric of  $G$  can be induced to  $M_i$  and  $G$  acts by isometries on  $M_i$ . We study the geometry of these compact spaces.

- We prove that every null geodesic is closed.
- There are closed and nonclosed time and space-like geodesics.
- We compute the isometry group of  $G/\Gamma_i$  with help of that of  $G$ .

By considering the list of groups acting by isometries on these spaces [1] we shall explain how to realize the action of  $\mathbb{R} \times H_3$  in these spaces.

### References

- [1] S. Adams and Garrett Stuck, *The isometry group of a compact Lorentz manifold*, Invent. math. 129 (1997) 239-261.
- [2] V. del Barco and G. Ovando, *Isometric actions on pseudo-Riemannian nilmanifolds*, arXiv:1303.4450.
- [3] V. del Barco, G. Ovando and F. Vittone, *Naturally reductive pseudo-Riemannian Lie groups in low dimensions*, arXiv:1211.0884.
- [4] V. del Barco, G. Ovando and F. Vittone, *Lorentzian compact solvmanifolds in dimension four*, preprint.

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**Bennett Palmer**  
**Idaho State University, USA**

**Tuesday 23<sup>rd</sup>, 15:00—16:00**

**Title:** *Area Minimization in Lorentzian 4-manifolds*

**Abstract:** Space-like zero mean curvature surfaces in a four dimensional Lorentzian manifold can neither locally minimize nor maximize area. Nevertheless, we will discuss ways in which these surfaces can be considered from a variational standpoint. The idea is to compare the area of a space-like zero mean curvature surfaces with only the areas of marginally trapped surfaces having the same boundary values in some strong sense.

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**Raul Quiroga**  
**Center of Research in Mathematics, Mexico**

**Monday 22<sup>nd</sup>, 16:30—17:00**

**Title:** *Pseudo-Riemannian Manifolds and Isometric Actions of Simple Lie Groups*

**Abstract:** Let  $M$  be a compact pseudo-Riemannian manifold with a dense orbit of its isometry group; Gromov [1] has shown that such conditions imply the local homogeneity of  $M$  as pseudo-Riemannian manifold, which clearly imposes a restriction on  $M$ . Also, if we assume that there is an isometric  $G$ -action on  $M$  with a dense orbit, where  $G$  is a noncompact simple Lie group, then it has been shown that the possible manifolds  $M$  are quite restricted: e.g. their fundamental groups cannot be amenable (see [5]). Furthermore, based on Gromov-Zimmer machinery we have proved, under suitable restrictions, that manifolds admitting an isometric  $G$ -action as above must be of a very specific type: a double coset  $K/H/\Gamma$ , where  $H$  is a semisimple Lie group containing  $G$  as a subgroup,  $K \subset H$  a compact subgroup of  $H$  centralizing  $G$  and  $\Gamma \subset H$  a cocompact lattice (see for example [3, 4, 2]). In particular, this provides solutions to Zimmer's program in a pseudo-Riemannian setup. In this talk, we will explain some of the basic ideas of Gromov-Zimmer machinery, the way they are used to prove the results in [3, 4, 2] and some work under current development.

#### **References**

[1] M. Gromov, *Rigid transformations groups*, in *Geometrie differentielle, Colloquie Geometrie et Physique de 1986 en l'honneur de André Lichnerowicz* (D. Bernard and Y. Choquet-Bruhat, eds.), Hermann, 1988, 65-139.

- [2] G. Olafsson and R. Quiroga-Barranco, *On low-dimensional manifolds with isometric  $SO_0(p, q)$ -actions*. Transformation Groups 17 (2012), no. 3, 835-860.
- [3] R. Quiroga-Barranco, *Isometric actions of simple Lie groups on pseudo-Riemannian manifolds*, Ann. of Math. (2) 164 (2006), no. 6, 941-969.
- [4] R. Quiroga-Barranco, *Isometric actions of simple groups and transverse structures: the integrable normal case*, Geometry, Rigidity and Group Actions, 229-261, Chicago Lectures in Mathematics Series, The University of Chicago Press.
- [5] R. J. Zimmer, *Automorphism groups and fundamental groups of geometric manifolds*. Differential geometry: Riemannian geometry (Los Angeles, CA, 1990), 693-710, Proc. Sympos. Pure Math., 54, Part 3, Amer. Math. Soc., Providence, RI, 1993.

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**Neil Russell**  
**Northern Michigan University, USA**

**Thursday 25<sup>th</sup>, 15:00—16:00**

**Title:** *Pseudo-Riemann-Finsler geometry and Lorentz violation*

**Abstract:** In recent years, the possibility of Lorentz violation in nature has received much attention. The Standard-Model Extension (SME) is a framework incorporating such violations in effective field theory. In the SME, classical particles follow geodesics in (3 + 1)-dimensional pseudo-Riemann-Finsler spacetimes. The Lorentz-breaking Lagrange function that governs the motion of particles in the spacetime manifold plays a role analogous to that of the Finsler structure in Riemann-Finsler geometries. We discuss pseudo-Riemann-Finsler spacetimes that arise from studies of Lorentz violation, and aspects of the related Riemann-Finsler geometries.

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**Juan Jesús Salamanca Jurado**  
**University of Córdoba, Spain**

**Friday 26<sup>th</sup>, 14:30—15:00**

**Title:** *Parabolicity of complete spacelike hypersurfaces in certain GRW spacetimes. Applications to uniqueness of complete maximal hypersurfaces*

**Abstract:** In this talk, we will consider spacelike hypersurfaces in a Generalized Robertson- Walker spacetime whose fiber is a parabolic Riemannian manifold. Some physical reasons which support this choice of spacetime to model a relativistic universe model are explained. Under some reasonable extrinsic assumptions, it is shown that the parabolicity is inherited on a complete spacelike hypersurface in such spacetimes. Then, this is applied to obtain new uniqueness theorems of complete maximal hypersurfaces. Moreover, the corresponding Calabi-Bernstein type results for the associated maximal hypersurface equation are exposed. The contents of this talk is based on ref. [1].

**Reference**

[1] A. Romero, R.M. Rubio and J.J. Salamanca, *Uniqueness of complete maximal hypersurfaces in spatially parabolic Generalized Robertson-Walker spacetimes*, *Class. Quantum Grav.*, (2013), (to appear).

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**José M. M. Senovilla**  
**University of the Basque Country, Spain**

**Monday 22<sup>nd</sup>, 14:00—15:00**

**Title:** *Kerr-Schild vector fields*

**Abstract:** The aim of this talk is to introduce a generalization of Killing vectors to the Lorentzian case when they are adapted to Kerr-Schild transformations. These results are contained in [1].

**References**

[1] B. Coll, S. R. Hildebrandt and J. M. M. Senovilla, *Kerr-Schild Symmetries*. *General Relativity and Gravitation* 33, no.4 (2001), 649-670.

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**Stefan Suhr**  
**University of Hamburg, Germany**

**Tuesday 23<sup>rd</sup>, 10:00—11:00**

**Title:** *Manifolds all of whose geodesics of at least one causal type are closed*

**Abstract:** In my talk I will give an introduction to the study of pseudo-Riemannian manifolds all of whose lightlike/timelike/spacelike geodesics are closed. In the first part I will be concentrating on the case of surfaces. The case of close lightlike geodesics on compact surfaces was noted in 1989 by V. Guillemin. I will explain a solution to the topological question, i.e. which diffeomorphism types of surfaces admit pseudo-Riemannian metrics all of whose spacelike/timelike geodesics are closed. The proof makes use of a pseudo-Riemannian version of Wadsley's theorem on geodesic foliations, well known from the study of the Riemannian problem. The connection to the above problem will be explained. Counterexamples to obvious generalizations will be given. Further I will explain a rigidity result on manifolds all of whose geodesics are closed. In the second part I will explain a construction of counterexamples to a conjecture by V. Guillemin on 3-manifolds all of whose lightlike geodesics are closed. The methods include a global version of the arrival time functional for stationary spacetimes and charged particle functionals on surfaces.

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**Laszlo Beno Szabados**  
**Hungarian Academy of Sciences, Budapest, Hungary**

**Monday 22<sup>nd</sup>, 10:30-11:30**

**Title:** *Total mass of closed universes*

**Abstract:** In this talk we review the key properties of a recently suggested notion of total mass (density) for closed universes. This mass  $M$  is a manifestly non-negative expression of the 3-surface twistor operator and the energy-momentum tensor of the matter fields satisfying the dominant energy condition. It has been shown that  $M$  can also be given as the smallest eigenvalue of the square of the SenWitten operator, it is zero if and only if the spacetime is flat with toroidal spatial topology, and Witten's gauge condition admits non-trivial solution if and only if this mass (density) is zero. In the present talk we report on an extension of these results to the case where there is a positive cosmological constant. Then the minimal mass configuration is the de Sitter spacetime.

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**SPECIAL SESSION**  
***XXI Century Singularity Theorems***  
**By José. M. M. Senovilla**  
**University of the Basque Country, Spain**

**Friday 26<sup>th</sup>, 16:30—17:30**

In this special session we will review some modern singularity theorems, including results on averaging in cosmological models, and recent results of myself with Galloway for cases with trapped submanifolds of arbitrary codimension.

**References**

- [1] G. J. Galloway and J. M. M. Senovilla, *Singularity theorems based on trapped submanifolds of arbitrary co-dimension*. Class. Quantum Grav. 27 (2010) 152002.
- [2] J. M. M. Senovilla, *A singularity theorem based on spatial averages*. Available in arXiv:gr-qc/0610127.
- [3] J. M. M. Senovilla, *A New Type of Singularity Theorem*. Available in arXiv:0712.1428 [gr-qc].
- [4] J. M. M. Senovilla, *Singularity Theorems in General Relativity: Achievements and Open Questions*. Available in arXiv:physics/0605007.

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## MINICOURSES

### Nicolas Ginoux

Regensburg University, Germany

Time: Tuesday 23<sup>rd</sup>, Wednesday 24<sup>th</sup> and Thursday 25<sup>th</sup>, 9:00—10:00

**Title:** *Linear wave equations on spacetimes*

**Abstract:** Starting with elementary features about Riesz distributions on the Minkowski-spacetime, we shall construct local and (in case the underlying spacetime is globally hyperbolic) global fundamental solutions for general linear wave operators. Along the way, we shall relate fundamental solutions with the solutions to the associated Cauchy problem. The lecture will be based on the book by Christian Bär, Nicolas Ginoux and Frank Pfäffle, *Wave Equations on Lorentzian Manifolds and Quantization*, published by the European Mathematical Society. The book is freely available on the arxiv, [click here](#).

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### Levi Lopes de Lima

Universidade Federal do Ceará, Fortaleza, Brasil

Time: Tuesday 23<sup>rd</sup> 14:00—15:00, Wednesday 24<sup>th</sup> 12:00—13:00, and Thursday 25<sup>th</sup> 14:00—15:00

**Title:** *An introduction to the Riemannian Penrose inequality*

**Abstract:** We present a survey of recent results on the Riemannian version of the Penrose inequality in General Relativity, which relates the mass of asymptotically flat (or hyperbolic) initial data sets to the total area of the black hole horizon. We shall focus on results which can be obtained by elementary methods, notably the graph and conformally flat cases.

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## POSTERS

**Marco Antonio Lázaro Velásquez**

**Federal University of Campina Grande, Brazil**

*On the Geometry of Linear Weingarten Spacelike Hypersurfaces in the de Sitter Space*

**Abstract:** Our purpose is to study the geometry of linear Weingarten spacelike hypersurfaces

immersed in the de Sitter space  $S_1^{n+1}$ . In this setting, by using as main analytical tool a suitable maximum principle for complete noncompact Riemannian manifolds, we establish new characterizations of the hyperbolic cylinders of  $S_1^{n+1}$ . In the compact case, we obtain a rigidity result concerning to a such hypersurface according to the length of its second fundamental form. These results are contained in in H. F. de Lima and M. A. L. Velásquez, On the Geometry of Linear Weingarten Spacelike Hypersurfaces in the de Sitter Space, Bull Braz Math Soc, New Series 44(1), 1-17.

**Hector Fabian Ramirez Ospina**

**Universidad de Murcia, Spain**

*Hypersurfaces in pseudo-Euclidean spaces satisfying a linear condition in the linearized operator of a higher order mean curvature.*

**Abstract:** We study hypersurfaces  $M$  immersed in  $(n + 1)$ -dimensional Euclidean space of index  $t$  whose position vector  $x$  satisfies the condition  $L_k x = Ax + b$ , where  $L_k$  is the linearized operator of the  $(k + 1)$ -th mean curvature of the hypersurface for a fixed  $k = 0, \dots, n-1$ ,  $A$  is a constant matrix  $(n + 1) \times (n + 1)$  and  $b$  is a constant vector. For every  $k$ , we prove that the only hypersurfaces satisfying such condition are hypersurfaces with zero  $(k+1)$ -th mean curvature, open pieces of totally umbilical hypersurfaces of the De Sitter space of index  $t$ , or the hyperbolic space of index  $t-1$ , and open pieces of generalized cylinders in the Euclidean space of index  $t$ .

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**Eraldo Lima**

**Federal Universidade of Ceará, Brazil**

*Bernstein Type Results on Complete Spacelike Hypersurfaces in Lorentzian Product Spaces*

**Abstract:** In this poster we present that under appropriated L1 bounds on the gradient of the height function, we establish uniqueness results concerning to complete spacelike hypersurfaces immersed in a Lorentzian product space. We make use of the so-called Omori-Yau generalized maximum principle, and another maximum principle at the infinity also due to S.T. Yau. We also give examples of graphs on the edges of the estimates.

**Eli Roblero**

**Center of Research in Mathematics, Mexico**

*Characterizing the exceptional Lie group  $G_2(2)$  through its isometries.*

Abstract: We analyze the structure of a finite volume pseudo-Riemannian analytic manifold  $M$  that admits an isometric  $SL(3, \mathbb{R})$ -action with a dense orbit. We prove under some conditions that there exists a finite covering from the quotient by a lattice of the simple Lie group  $G_2(2)$  to the manifold  $M$ .

**References**

[OQ] G. Olafsson, R. Quiroga-Barranco, *On Low Dimensional Manifolds with Isometric  $SO_0(p, q)$ -Actions*, Transformation Groups 17, 2012, no. 3, 835-860.

[Q] R. Quiroga-Barranco, *Isometric actions of simple lie groups and transverse structures: the integrable normal case*, Geometry, Rigidity and Group Actions, 229-261, Chicago Lectures in Math., Univ. Chicago Press, Chicago, IL, 2011.

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**Francisco Vittone**

**National University of Rosario, Argentina**

*The normal holonomy of CR submanifolds*

**Abstract:** In [OW01] it was proved that the normal holonomy group of a Riemannian submanifold of a Lorentzian space acts on the normal space as the isotropy representation of a symmetric space. It is a generalization of the analogous result for real space forms [OI90]. Both Theorems have been used as a tool to study important problems both in submanifold geometry and in intrinsic Riemannian geometry. The proof of the normal holonomy theorem is based in the existence of an algebraic curvature tensor  $\mathcal{R}^\top$  on the normal bundle with non-vanishing sectional curvature, which carries the same geometric information as the normal curvature tensor  $R^\top$  of the submanifolds. The fact of the tangent space being Riemannian plays a fundamental role in the proof and it can not be therefore adapted to an arbitrary submanifold of a Lorentzian manifold. In this work we discuss these limitations and prove that the Normal Holonomy Theorem is valid for an important family of Lorentzian submanifolds of the pseudo-hyperbolic space  $H_1^n$  (also called anti-De Sitter space). We finally use this to study the action of the normal holonomy group of a family of CR-submanifolds of a complex space form (via the Hopf fiberings). This is a joint work with Antonio J. Di Scala, from Politecnico di Torino, Italy.

### **References**

[OI90] Olmos, C. *The normal holonomy group*. Proc. Am. Math. Soc. 110 (2004), 813-818.

[OW01] Olmos, C. and Will, A. *Normal holonomy in Lorentzian space and submanifold geometry*. Indiana Univ. Math. J. 50 no.4 (2001), 1777-1788.

**Olaf Müller**

TBA

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Prof. Paolo Piccione  
Departamento de Matemática  
Universidade de São Paulo  
email: [piccione@ime.usp.br](mailto:piccione@ime.usp.br)

**To:** Professor Erhan Güler,  
Bartın University, Faculty of Science  
Department of Mathematics  
75100 Bartın, Turkey

São Paulo, June 11<sup>th</sup> 2013

Dear Professor Erhan Güler,

I am happy to inform you that your proposal of talk

**Bour's minimal surface in three dimensional Lorentz-Minkowski space**

at the GeLoSP2013, 7th International Meeting on Lorentzian Geometry, has been accepted by the Scientific Committee, according to reviewers' comments.

On behalf of the organizing committee, I have the pleasure to invite you to participate to the VII INTERNATIONAL MEETING ON LORENTZIAN GEOMETRY, that will be held at the University of São Paulo (Brazil) from July 22nd to July 26th of 2013.

The aim for these biennial meetings, that started in 2001 in Benalmadena (Spain), is to gather together researchers working on topics such as Lorentzian or Pseudo-Riemannian Geometry, General Relativity and Mathematical Relativity.

Your talk will be of 30 minutes. The complete program of the conference will be available soon through our webpage.

My best regards,

Paolo Piccione  
Chairman of the Scientific Committee  
GeLoSP2013  
<http://www.ime.usp.br/~gelosp2013/>



*Departamento de Matemática*

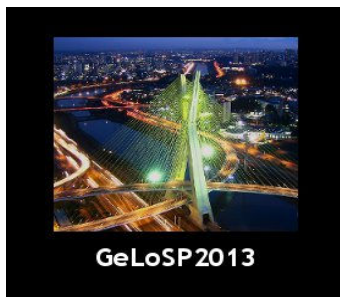
UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE MATEMÁTICA E ESTATÍSTICA

Cidade Universitária  
Rua do Matão, 1010  
05508-090 - São Paulo, Brasil

Caixa Postal 66281  
05311-970 - São Paulo, SP  
Brasil

Tels: (011) 3091-6194/6193  
Fax: (011) 3091-6183/3814-4135  
e-mail: [secmat@ime.usp.br](mailto:secmat@ime.usp.br)



# GeLoSP2013

## 7th International Meeting on Lorentzian Geometry

São Paulo, July 22 - 26, 2013



# C E R T I F I C A T E

We hereby certify that Erhan Güler participated in the GeLoSP2013 - 7th International Meeting on Lorentzian Geometry, held at University of São Paulo, Brazil, from July 22 to 26, 2013, and presented the following:

Talk: Bours minimal surface in three dimensional Lorentz-Minkowski space

São Paulo, July 26, 2013

Prof. Dr. Paolo Piccione  
Chairman of the Scientific Committee

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