

ON STATISTICAL CONVERGENCE WITH RESPECT TO MEASURE

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Abstract. Several notions of convergence for subsets of metric spaces appear in the literature. In this paper, for real valued measurable functions defined on a measurable space (X, \mathcal{M}, μ) , we obtain a statistical version of Lebesgue's bounded convergence theorem (when $\mu(X) < \infty$) and examine the validity of the classical theorems of Measure Theory for statistical convergences.

1. Introduction and background

Let us start with fundamental definitions from the literature. The natural density of a set K of positive integers is defined by

$$\delta(K) := \lim_{n \rightarrow \infty} \frac{1}{n} |\{k \leq n : k \in K\}|,$$

where $|\{k \leq n : k \in K\}|$ denotes the number of elements of K not exceeding n .

Statistical convergence of sequences of points was introduced by [6]. Schoenberg [17] established some basic properties of statistical convergence and also studied the concept as a summability method. Later, this concept has been generalized in many directions. More details on this matter and on applications of this concept can be found in [1].

A sequence $x = (x_k)$ is said to be statistically convergent to the number ξ if for every $\varepsilon > 0$,

$$\lim_{n \rightarrow \infty} \frac{1}{n} |\{k \leq n : |x_k - \xi| \geq \varepsilon\}| = 0.$$

In this case we write $st - \lim x_k = \xi$. Statistical convergence is a natural generalization of ordinary convergence. If $\lim x_k = \xi$, then $st - \lim x_k = \xi$. The converse does not hold in general.

Regarding statistical convergence of numerical sequences we have the following well-known proposition.

PROPOSITION 1. [15] *Let (x_n) be a sequence in \mathbb{R} , and $\xi \in \mathbb{R}$. Then*

$$(x_n) \xrightarrow{st} \xi \Leftrightarrow \exists K = \{k_1 < k_2 < \dots < k_n < \dots\} \subseteq \mathbb{N} :$$

$$d(K) = 1 \text{ and } (x_{n_k}) \rightarrow \xi$$

(By $(x_{n_k}) \rightarrow \xi$ we denote the usual convergence).

Mathematics subject classification (2010): 40G15, 40A35.

Keywords and phrases: Statistical convergence, measurable function.