

Article

# New Definitions about $A^{\mathcal{I}}$ -Statistical Convergence with Respect to a Sequence of Modulus Functions and Lacunary Sequences

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**Abstract:** In this paper, using an infinite matrix of complex numbers, a modulus function and a lacunary sequence, we generalize the concept of  $\mathcal{I}$ -statistical convergence, which is a recently introduced summability method. The names of our new methods are  $A^{\mathcal{I}}$ -lacunary statistical convergence and strongly  $A^{\mathcal{I}}$ -lacunary convergence with respect to a sequence of modulus functions. These spaces are denoted by  $S_{\theta}^A(\mathcal{I}, F)$  and  $N_{\theta}^A(\mathcal{I}, F)$ , respectively. We give some inclusion relations between  $S^A(\mathcal{I}, F)$ ,  $S_{\theta}^A(\mathcal{I}, F)$  and  $N_{\theta}^A(\mathcal{I}, F)$ . We also investigate Cesàro summability for  $A^{\mathcal{I}}$  and we obtain some basic results between  $A^{\mathcal{I}}$ -Cesàro summability, strongly  $A^{\mathcal{I}}$ -Cesàro summability and the spaces mentioned above.

**Keywords:** lacunary sequence; statistical convergence; ideal convergence; modulus function;  $\mathcal{I}$ -statistical convergence

**MSC:** 40A35, 40A05

## 1. Introduction

As is known, convergence is one of the most important notions in mathematics. Statistical convergence extends the notion. After giving the definition of statistical convergence, we can easily show that any convergent sequence is statistically convergent, but not conversely. Let  $E$  be a subset of  $\mathbb{N}$ , and the set of all natural numbers  $d(E) := \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \chi_E(j)$  is said to be a natural density of  $E$  whenever the limit exists. Here,  $\chi_E$  is the characteristic function of  $E$ .

In 1935, statistical convergence was given by Zygmund in the first edition of his monograph [1]. It was formally introduced by Fast [2], Fridy [3], Salat [4], Steinhaus [5] and later was reintroduced by Schoenberg [6]. It has become an active research area in recent years. This concept has applications in different fields of mathematics such as number theory [7], measure theory [8], trigonometric series [1], summability theory [9], etc.

Following this very important definition, the concept of lacunary statistical convergence was defined by Fridy and Orhan [10]. In addition, Fridy and Orhan gave the relationships between the lacunary statistical convergence and the Cesàro summability. Freedman and Sember [9] established the connection between the strongly Cesàro summable sequences space  $|\sigma_1|$  and the strongly lacunary summable sequence space  $N_{\theta}$ .