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$\mathscr{I}$-Cesàro Summability of a Sequence of Order α of Random Variables in Probability

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Abstract

In this paper, we define four types of convergence of a sequence of random variables, namely, $\mathscr{I}$-statistical convergence of order α, $\mathscr{I}$-lacunary statistical convergence of order α, strongly $\mathscr{I}$-lacunary convergence of order α and strongly $\mathscr{I}$-Cesàro summability of order α in probability where $0<\alpha<1$. We establish the connection between these notions.

1. Introduction and background

Theory of statistical convergence was firstly originated by Fast [1]. After Fridy [2] and Šalát [3] statistical convergence became a notable topic in summability theory. Lacunary statistical convergence was defined by using lacunary sequences in [4]. $\mathscr{I}$-convergence was fistly considered by Kostyrko et al. [5]. Also, Das et al. [6] gave new definitions by using ideal, such as $\mathscr{I}$-statistical convergence, $\mathscr{I}$-lacunary statistical convergence. Ulusu et al. [7] also studied asymptotically $\mathscr{I}$-Cesaro equivalence of sequences of sets.
Statistical convergence of order α $(0<\alpha<1)$ was introduced using the notion of natural density of order α where n is replaced by n^{α} in [8]. This new type convergence was different in many ways from statistical convergence. Lacunary statistical convergence of order α is studied by Sengöl and M. Et [9], $\mathscr{I}$-statistical and $\mathscr{I}$-lacunary statistical convergence of order α is studied by Das and Savas [10].
In probability theory, if for $n>0$, a random variable X_n given on space S, a probability function $P: X\to\mathbb{R}$, then we say that $X_1, X_2, \ldots, X_n, \ldots$ is a sequence of random variables and it is demonstrated by $\{X_n\}_{n\in\mathbb{N}}$.
It is important that if there exists $c\in\mathbb{R}$ for which $P(|X-c|<\varepsilon)=1$, where $\varepsilon>0$ is sufficiently small, that is, it is means that values of X lie in a very small neighbourhood of c.
New concepts have begun to be studied in probability theory by Das et al. [6], and others ([11]-[15]).

2. Main results

Definition 2.1. *$\{X_k\}_{k\in\mathbb{N}}$ is said to be $\mathscr{I}$-statistically convergent of order α in probability to a random variable X if for any ε, δ, $\gamma>0$*

$$\left\{n\in\mathbb{N}:\frac{1}{n^{\alpha}}\left|\{k\leq n: P\left(|X_k-X|\geq\varepsilon\right)\geq\delta\}\right|\geq\gamma\right\}\in\mathscr{I},$$

and demonstrated by $X_k \overset{PS(\mathscr{I})^{\alpha}}{\to} X$.

Definition 2.2. *$\{X_n\}_{n\in\mathbb{N}}$ is said to be $\mathscr{I}$-lacunary statistically convergent of order α in probability to a random variable X if for any $\varepsilon,\delta,\gamma>0$*

$$\left\{r\in\mathbb{N}:\frac{1}{h_r^{\alpha}}\left|\{k\in I_r: P\left(|X_k-X|\geq\varepsilon\right)\geq\delta\}\right|\geq\gamma\right\}\in\mathscr{I},$$

and it is demonstrated by $X_k \overset{PS_{\theta}(\mathscr{I})^{\alpha}}{\to} X$.

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