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Article

Dini-Type Helicoidal Hypersurfaces with Timelike Axis in Minkowski 4-Space $\mathbb{E}_1^4$

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Abstract: We consider Ulisse Dini-type helicoidal hypersurfaces with timelike axis in Minkowski 4-space $\mathbb{E}_1^4$. Calculating the Gaussian and the mean curvatures of the hypersurfaces, we demonstrate some special symmetries for the curvatures when they are flat and minimal.

Keywords: Minkowski 4-space; Dini-type helicoidal hypersurface; Gauss map; timelike axis

1. Introduction

The concept of finite-type immersion of submanifolds of a Euclidean space has been known in classifying and characterizing Riemannian submanifolds [1]. Chen proposed the problem of classifying these kinds surfaces in the three-dimensional Euclidean space $\mathbb{E}^3$. A Euclidean submanifold is called Chen finite-type if its coordinate functions are a finite sum of eigenfunctions of its Laplacian Δ [1]. Hence, the idea of finite-type can be enlarged to any smooth functions on a submanifold of Euclidean or pseudo-Euclidean spaces.

Takahashi [2] obtained spheres and the minimal surfaces are the unique surfaces in $\mathbb{E}^3$ satisfying the condition $\Delta r = \lambda r$, where r is the position vector, $\lambda \in \mathbb{R}$. Ferrandez, Garay and Lucas [3] showed the surfaces of $\mathbb{E}^3$ providing $\Delta H = AH$. Here H is the mean curvature and $A \in Mat(3,3)$ are either of a right circular cylinder, or of an open piece of sphere, or minimal. Choi and Kim [4] worked the minimal helicoid with pointwise 1-type Gauss map of the first type.

Dillen, Pas, and Verstraelen [5] studied the unique surfaces in $\mathbb{E}^3$ providing $\Delta r = Ar + B$, $A \in Mat(3,3)$, $B \in Mat(3,1)$ are the spheres, the circular cylinder, the minimal surfaces. Senoussi and Bekkar [6] obtained helicoidal surfaces in $\mathbb{E}^3$ by using the fundamental forms I, II and III.

In classical surface geometry, it is well known a pair of the right helicoid and the catenoid is the unique ruled and rotational surface, which is minimal. When we look at ruled (i.e., helicoid) and rotational surfaces, we meet Bour's theorem in [7]. By using a result of Bour [7], Do Carmo and Dajczer [8] worked isometric helicoidal surfaces.

Lawson [9] defined the generalized Laplace-Beltrami operator. Magid, Scharlach and Vrancken [10] studied the affine umbilical surfaces in 4-space. Vlachos [11] introduced hypersurfaces with harmonic mean curvature in $\mathbb{E}^4$. Scharlach [12] gave the affine geometry of surfaces and hypersurfaces in 4-space. Cheng and Wan [13] studied complete hypersurfaces of 4-space with CMC. Arslan, Deszcz and Yaprak [14] obtained Weyl pseudosymmetric hypersurfaces. Arvanitoyeorgos, Kaimakamais and Magid [15] wrote that if the mean curvature vector field of M_1^3 satisfies the equation $\Delta H = \alpha H$ (α a constant), then M_1^3 has constant mean curvature in Minkowski 4-space $\mathbb{E}_1^4$. This equation is a natural generalization of the biharmonic submanifold equation $\Delta H = 0$.

General rotational surfaces in the four-dimensional Euclidean space were originated by Moore [16,17]. Ganchev and Milousheva [18] considered the analogue of these surfaces in $\mathbb{E}_1^4$. Verstraelen, Valrave, and Yaprak [19] studied the minimal translation surfaces in $\mathbb{E}^n$ for arbitrary dimension n. Kim and Turgay [20] studied surfaces with L_1-pointwise 1-type Gauss map in $\mathbb{E}^4$. Moruz