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Article

Implicit Equations of the Henneberg-Type Minimal Surface in the Four-Dimensional Euclidean Space

Erhan Güler [1,*], **Ömer Kişi** [1] and **Christos Konaxis** [2]

[1] Department of Mathematics, Faculty of Sciences, Bartın University, 74100 Bartın, Turkey; okisi@bartin.edu.tr
[2] Department of Informatics and Telecommunications, National and Kapodistrian University of Athens, 15784 Athens, Greece; ckonaxis@di.uoa.gr
* Correspondence: eguler@bartin.edu.tr; Tel.: +90-378-5011000-1521

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Abstract: Considering the Weierstrass data as $(\psi, f, g) = (2, 1 - z^{-m}, z^n)$, we introduce a two-parameter family of Henneberg-type minimal surface that we call $\mathfrak{H}_{m,n}$ for positive integers (m, n) by using the Weierstrass representation in the four-dimensional Euclidean space $\mathbb{E}^4$. We define $\mathfrak{H}_{m,n}$ in (r, θ) coordinates for positive integers (m, n) with $m \neq 1, n \neq -1, -m + n \neq -1$, and also in (u, v) coordinates, and then we obtain implicit algebraic equations of the Henneberg-type minimal surface of values $(4, 2)$.

Keywords: Henneberg-type minimal surface; Weierstrass representation; four-dimensional space; implicit equation; degree

1. Introduction

The theory of surfaces has an important role in mathematics, physics, biology, architecture, see e.g., the classical books [1,2] and papers [3–9].

A minimal surface in the three-dimensional Euclidean space $\mathbb{E}^3$, also in higher dimensions, is a regular surface for which the mean curvature vanishes identically. See [10–27] for details. On the other hand, a Henneberg surface [4–6], also obtained by the Weierstrass representation [8,9] is well-known classical minimal surface in $\mathbb{E}^3$.

In the four-dimensional Euclidean space $\mathbb{E}^4$, a general definition of rotation surfaces was given by Moore in [28] as follows

$$X(u,t) = \begin{pmatrix} x_1(u)\cos(at) - x_2(u)\sin(at) \\ x_1(u)\cos(at) + x_2(u)\sin(at) \\ x_3(u)\cos(bt) - x_4(u)\sin(bt) \\ x_3(u)\cos(bt) + x_4(u)\sin(bt) \end{pmatrix}.$$

A more restricted case can be found in [29]:

$$W(u,t) = (x_1(u), x_2(u), r(u)\cos(t), r(u)\sin(t)).$$

It is a bit too general since the curve is not located in any subspace before rotation.

Güler and Kişi [30] studied the Weierstrass representation, the degree and the classes of surfaces in $\mathbb{E}^4$, see [31–38] for some previous work.

In this paper, we study a two-parameter family of Henneberg-type minimal surfaces using the Weierstrass representation in $\mathbb{E}^4$. We give the Weierstrass equations for a minimal surface in $\mathbb{E}^4$, and obtain two normals of the surface in Section 2.