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Article

Isometric Deformation of (m,n)-Type Helicoidal Surface in the Three Dimensional Euclidean Space

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Abstract: We consider a new kind of helicoidal surface for natural numbers (m,n) in the three-dimensional Euclidean space. We study a helicoidal surface of value (m,n), which is locally isometric to a rotational surface of value (m,n). In addition, we calculate the Laplace–Beltrami operator of the rotational surface of value $(0,1)$.

Keywords: euclidean three-space; helicoidal surface of value (m,n); rotational surface of value (m,n); mean curvature; Gaussian curvature; Gauss map

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1. Introduction

The notion of the finite-type immersion of submanifolds of a Euclidean space has been used in classifying and characterizing well-known Riemannian submanifolds [1]. Chen posed the problem of classifying the finite-type surfaces in the three-dimensional Euclidean space $\mathbb{E}^3$. Then, the theory of submanifolds of a finite type was studied by many geometers [1–21].

Lawson [22] gave the general definition of the Laplace–Beltrami operator in his lecture notes. Takahashi [23] stated that minimal surfaces and spheres are the only surfaces in $\mathbb{E}^3$ satisfying the condition $\Delta r = \lambda r$, $\lambda \in \mathbb{R}$. Ferrandez, Garay and Lucas [10] proved that the surfaces of $\mathbb{E}^3$ satisfying $\Delta H = AH$, $A \in Mat(3,3)$ are either minimal, or an open piece of a sphere, or of a right circular cylinder. Choi and Kim [5] characterized the minimal helicoid in terms of a pointwise one-type Gauss map of the first kind.

Dillen, Pas and Verstraelen [7] proved that the only surfaces in $\mathbb{E}^3$ satisfying $\Delta r = Ar + B$, $A \in Mat(3,3)$, $B \in Mat(3,1)$ are the minimal surfaces, the spheres and the circular cylinders. Senoussi and Bekkar [24] studied helicoidal surfaces M^2 in $\mathbb{E}^3$, which are of the finite type in the sense of Chen with respect to the fundamental forms I, II and III.

The right helicoid (resp. catenoid) is the only ruled (resp. rotational) surface that is minimal in classical surface geometry in Euclidean space. If we focus on the ruled (helicoid) and rotational characters, we see Bour's theorem in [25]. The French mathematician Edmond Bour used the semi-geodesic coordinates and found a number of new cases of the deformation of surfaces in 1862. He also gave in [25] a well-known theorem about the helicoidal and rotational surfaces.

Kenmotsu [26] focused on the surfaces of revolution with the prescribed mean curvature. Regarding helicoidal surfaces, do Carmo and Dajczer [3] proved that, by using a result of Bour [25], there exists a two-parameter family of helicoidal surfaces isometric to a given helicoidal surface. Hitt and Roussos [27] also studied the helicoidal surfaces with constant mean curvature. Ikawa [14,15] determined pairs of surfaces by Bour's theorem. Güler [28] also studied the isometric helicoidal and rotational surfaces of value m. Güler and Yaylı [12] focused on the generalized Bour's theorem in three-space.

We consider a new kind of helicoidal surface of value (m,n) in Euclidean three-space $\mathbb{E}^3$ in this paper. We give some basic notions of the three-dimensional Euclidean geometry in Section 2. In