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On $\mathscr{I}_\sigma$-convergence of folner sequence on amenable semigroups

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Abstract: In this paper, the concepts of σ-uniform density of subsets A of the set $\mathbb{N}$ of positive integers and corresponding $\mathscr{I}_\sigma$-convergence of functions defined on discrete countable amenable semigroups were introduced. Furthermore, for any Folner sequence inclusion relations between $\mathscr{I}_\sigma$-convergence and invariant convergence also $\mathscr{I}_\sigma$-convergence and $[V_\sigma]_p$-convergence were given. We introduce the concept of $\mathscr{I}_\sigma$-statistical convergence and $\mathscr{I}_\sigma$-lacunary statistical convergence of functions defined on discrete countable amenable semigroups. In addition to these definitions, we give some inclusion theorems. Also, we make a new approach to the notions of $[V,\lambda]$-summability, σ-convergence and λ-statistical convergence of Folner sequences by using ideals and introduce new notions, namely, $\mathscr{I}_\sigma$-$[V,\lambda]$-summability, $\mathscr{I}_\sigma$-λ-statistical convergence of Folner sequences. We mainly examine the relation between these two methods as also the relation between $\mathscr{I}_\sigma$-statistical convergence and $\mathscr{I}_\sigma$-λ-statistical convergence of Folner sequences introduced by the author recently.

Keywords: Folner sequence, amenable group, inferior, superior, $\mathscr{I}$-convergence.

1 Introduction

Statistical convergence of sequences of points was introduced by Fast [5]. Schoenberg [27] established some basic properties of statistical convergence and also studied the concept as a summability method.

The natural density of a set K of positive integers is defined by

$$\delta(K) := \lim_{n\to\infty} \frac{1}{n} |\{k \le n : k \in K\}|,$$

where $|k \le n : k \in K|$ denotes the number of elements of K not exceeding n.

A number sequence $x = (x_k)$ is said to be statistically convergent to the number L if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n} |\{k \le n : |x_k - L| \ge \varepsilon\}| = 0.$$

In this case we write $st - \lim x_k = L$. Statistical convergence is a natural generalization of ordinary convergence. If $\lim x_k = L$, then $st - \lim x_k = L$. The converse does not hold in general.

By a lacunary sequence we mean an increasing integer sequence $\theta = \{k_r\}$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. Throughout this paper the intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$.

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