

On Bour's Minimal Surface

Erhan Güler

Dedicated to the memory of Jacques Edmond Émile BOUR (1832-1866)

ABSTRACT. In this work, we focus on the differential geometry of the Bour's minimal surface in three dimensional Euclidean space. We calculate the mean and the Gaussian curvatures, and show the Weierstrass representation of the surface. In addition, we reveal amazing minimal surfaces.

1. Introduction

The origins of minimal surface theory can be traced back to 1744 with the Swedish mathematician Leonhard Euler's paper, and to the 1760 French mathematician Joseph Louis Lagrange's paper.

A *minimal surface* in \mathbb{E}^3 is a regular surface for which the mean curvature vanishes identically. This is a definition of Lagrange, who first defined minimal surface in 1760.

In Riemannian geometry, early classical minimal surfaces are known by almost all the mathematicians, especially the geometers. A few of them, however, have very little knowledge about the Bour's minimal surface. We study on this surface and reveal it in this paper. Next, we look at the brief history of the classical minimal surfaces. Plane (trivial), Euler's Catenoid minimal surface (1740), Meusnier's Helicoid minimal surface (1776), Scherk's minimal surface (1835), Catalan's minimal surface (1855), Riemann's minimal surface (1860), *Bour's minimal surface (1862)*, Enneper's minimal surface (1864), Schwarz's minimal surface (1865), Heneberg's minimal surface (1875), Richmond's minimal surface (?), and almost a hundred years later Chen-Gackstatter's minimal surface (1981), Costa's minimal surface (1982), Jorge-Meeks's minimal surface (1983), 1990's-... Hoffman, Meeks, Karcher, Kusner, Rosenberg, Lopez, Ros, Rossman, Miyaoka, Sato, and 2000s – ... Fujimori, Shoda, Traizet, Weber, ...

In 1862, the French mathematician Edmond Bour used semigeodesic coordinates and found a number of new cases of deformations of surfaces. He gave a well

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To whom it may concern,

I hereby declare that Erhan Güler gave a talk in the Geometry Seminar of KU Leuven on February 10, 2012 at 3 p.m. The title of his talk was "On Bour's minimal surface".

Sincerely,

A handwritten signature in blue ink, appearing to read 'Joeri'.

Prof. dr. Joeri Van der Veken,
Head of the Geometry Section