Ulisse Dini-type Helicoidal Hypersurface 4-Space

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ABSTRACT

We consider Ulisse Dini-type helicoidal hypersurface in Euclidean 4-space E^4. We give some basic notions of the four dimensional Euclidean geometry in section 2. In section 3, we consider Ulisse Dini helicoidal hypersurface. We obtain Ulisse Dini-type helicoidal hypersurface, and calculate its curvatures in the last section.

We calculate the first and second fundamental forms, matrix of the shape operator S, Gaussian curvature K, and the mean curvature H of hypersurface M=M(u,v,w) in Euclidean 4-space E^4.

We define the rotational hypersurface and helicoidal hypersurface in E^4. For an open interval $I\subset R$, let $\gamma: I\rightarrow \Pi$ be a curve in a plane Π in E^4, and let ℓ be a straight line in Π. A rotational hypersurface in E^4 is defined as a hypersurface rotating a curve γ around a line ℓ (these are called the profile curve and the axis, respectively). Suppose that when a profile curve γ rotates around the axis ℓ, it simultaneously displaces parallel lines orthogonal to the axis ℓ, so that the speed of displacement is proportional to the speed of rotation. Then the resulting hypersurface is called the helicoidal hypersurface with axis ℓ and pitches $a,b\in R\backslash\{0\}$. We may suppose that ℓ is the line spanned by the vector $(0,0,0,1)^t$.

Finally, we obtain and calculate its differential geometric properties of the Dini-type helicoidal hypersurface:

$$D(u,v) = \begin{pmatrix} \sin u \cos v \cos w \\ \sin u \sin v \cos w \\ \sin u \sin w \\ \varphi(u) + av + bw \end{pmatrix},$$

where $\varphi(u): I\subset R\rightarrow R$ is a differentiable function for all $u\in I\subset R\backslash\{0\}$, $0\leq v,w\leq 2\pi$ and $a,b\in R\backslash\{0\}$.

Additionally, we find some relations for the curvatures.

Key Words: Dini-type helicoidal hypersurface, Gauss map, Gaussian curvature, mean curvature.

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