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Astrohelicoidal Surfaces

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Abstract

We assume $\gamma: I \longrightarrow \Pi$ be a curve in a plane Π for an open interval $I \subset \mathbb{R}$, and ℓ be a line in Π. A rotational surface in $\mathbb{E}^3$ is defined as a surface rotating a curve γ profile curve around a line ie. axis ℓ. When a profile curve γ rotates around the axis ℓ, it simultaneously displaces parallel lines which are orthogonal to the axis ℓ, so that the speed of displacement is proportional to the speed of rotation. Hence, obtaining surface is named the helicoidal surface with axis ℓ and pitch $a \in \mathbb{R}^+$. See also (Eisenhart 1909, Forsyth 1920, Gray et al. 2006, Hacısalihoğlu 1982, Nitsche 1989, Spivak 1999) for details.

We construct a new kind helicoidal surface which its profile curve has astroid curve in the three dimensional Euclidean space $\mathbb{E}^3$.

Using rotational matrix in $\mathbb{E}^3$, profile curve γ, and adding translation vector on axis z, we obtain helicoidal surface which has astroid curve. We called resulting surface as astrohelicoidal surface. Taking function $\varphi(u)$ on the profile curve γ, we calculate the Gauss map of the surface. We also find Gaussian curvature and the mean curvature of the astrohelicoidal surface $A(u, v)$. We also draw some figures for the astrohelicoidal surface, and its Gauss map in the three dimensional Euclidean space.

Finally, calculating some differential equations, we give minimality and flatness conditions of the astrohelicoidal surface.

Key Words: astrohelicoidal surface, Gauss map, Gaussian curvature, mean curvature.

Mathematics Subject Classification: 53, 65.

References

[1] L.P. Eisenhart, A Treatise on the Differential Geometry of Curves and Surfaces. Dover Publications, N.Y. 1909.

[2] A.R. Forsyth, Lectures on the Differential Geometry of Curves and Surfaces, Cambridge Un. press, 2nd ed. 1920.

[3] A. Gray, S. Salamon, E. Abbena, Modern Differential Geometry of Curves and Surfaces with Mathematica, Third ed. Chapman & Hall/CRC Press, Boca Raton, 2006.

[4] H.H. Hacısalihoğlu, Diferensiyel Geometri I. Ankara Ün., Ankara, 1982.

[5] J.C.C. Nitsche, Lectures on Minimal Surfaces. Vol. 1. Introduction, Fundamentals, Geometry and Basic Boundary Value Problems. Cambridge Un. Press, Cambridge, 1989.

[6] M. Spivak, A Comprehensive Introduction to Differential Geometry, Vol. IV. Third edition. Publish or Perish, Inc., Houston, Texas, 1999.