Astrohelicoidal Hypersurfaces in 4-space

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Abstract

We consider a new kind helicoidal hypersurface which its profile curve has astroid curve in the four dimensional Euclidean space $\mathbb{E}^4$.

$\gamma: I \longrightarrow \Pi$ be a space curve for an open interval $I \subset \mathbb{R}$, and let ℓ be a line in Π. A rotational hypersurface is defined as a hypersurface rotating a curve γ profile curve around axis ℓ in the four dimensional Euclidean space $\mathbb{E}^4$. When a profile curve γ rotates around the axis ℓ, it simultaneously displaces parallel lines which are orthogonal to the axis ℓ, so that the speed of displacement is proportional to the speed of rotation. Hence, obtaining surface is named the helicoidal hypersurface has axis ℓ and pitches a and b for positive real numbers. See some papers (Arslan et al. 2012, Ganchev and Milousheva 2014, Güler et al. 2018, Güler et al. 2019, Güler and Turgay 2019), and books (Eisenhart 1909, Hacısalihoğlu 1994, Nitsche 1989) about hypersurfaces

By using rotational matrix in $\mathbb{E}^4$, and profile curve γ with translation vector on axis x_4, we find helicoidal hypersurface which has astroid curve. Resulting hypersurface we called is astrohelicoidal hypersurface $\mathcal{A}(u, v, w)$. Considering function $\Phi(u)$ on the profile curve γ, we calculate the Gauss map of the hypersurface. Then we find Gaussian curvature and the mean curvature of the astrohelicoidal hypersurface.

We also draw some figures of the astrohelicoidal hypersurface, and its Gauss map with projection from four dimensional Euclidean space to the three dimensional Euclidean space.

Obtaining some second order differential equations, we have minimality and flatness conditions of the astrohelicoidal hypersurface.

Key Words: 4-space, astrohelicoidal hypersurface, Gauss map, Gaussian curvature, mean curvature.

Mathematics Subject Classification: 53, 65.

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