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Article

Helical Hypersurfaces in Minkowski Geometry $\mathbb{E}_1^4$

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Abstract: We define helical (i.e., helicoidal) hypersurfaces depending on the axis of rotation in Minkowski four-space $\mathbb{E}_1^4$. There are three types of helicoidal hypersurfaces. We derive equations for the curvatures (i.e., Gaussian and mean) and give some examples of these hypersurfaces. Finally, we obtain a theorem classifying the helicoidal hypersurface with timelike axes satisfying $\Delta^I \mathbf{H} = A\mathbf{H}$.

Keywords: helicoidal hypersurface; Laplace–Beltrami operator; Gaussian curvature; mean curvature; Minkowski four-space

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1. Introduction

Chen [1] served the problem of classifying finite type surfaces in the 3-dimensional Euclidean space $\mathbb{E}^3$. If its coordinate functions are a finite sum of eigenfunctions of its Laplacian Δ, a Euclidean submanifold is called of Chen finite type.

Moreover, the notion of finite type may be extended to any smooth function on a submanifold of a Euclidean space or a pseudo-Euclidean space. The submanifolds theory of finite type has been discussed by mathematicians.

Takahashi [2] obtained that minimal surfaces and spheres are the only surfaces in $\mathbb{E}^3$ satisfying the condition $\Delta r = \lambda r$, $\lambda \in \mathbb{R}$. Ferrandez, Garay, and Lucas [3] introduced the surfaces of $\mathbb{E}^3$ satisfying $\Delta H = AH$, $A \in Mat(3,3)$ are either minimal, or an open piece of sphere or of a right circular cylinder. Choi and Kim [4] worked the minimal helicoid in terms of pointwise 1-type Gauss map of the first kind.

Dillen, Pas, and Verstraelen [5] gave the only surfaces in $\mathbb{E}^3$ satisfying $\Delta r = Ar + B$, $A \in Mat(3,3)$, $B \in Mat(3,1)$ are the minimal surfaces, the spheres and the circular cylinders. Dillen, Fastenakels, and Van der Veken [6] studied rotation hypersurfaces of $\mathbb{S}^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$. Beneki, Kaimakamis, and Papantoniou [7] worked helicoidal surfaces with spacelike, timelike and lightlike axis in three-dimensional Minkowski space. Senoussi and Bekkar [8] focused helicoidal surfaces in $\mathbb{E}^3$ which are of finite type in the sense of Chen with respect to the fundamental forms I, II and III.

The right helicoid (resp. catenoid) is the only ruled (resp. rotational) surface which is minimal. Hence, we meet Bour's theorem in [9]. Do Carmo and Dajczer [10] proved that, by using Bour [9], there exists a two-parameter family of helicoidal surfaces isometric to a given helicoidal surface. Güler and Vanlı [11] worked Bour's theorem in Minkowski three-space. Using Bour's theorem in Minkowski geometry, Güler [12] investigated helicoidal surface with lightlike profile curve. Mira and Pastor [13] studied helicoidal maximal surfaces in Lorentz–Minkowski three-space.

Lawson [14] gave the general definition of the Laplace–Beltrami operator. Magid, Scharlach, and Vrancken [15] introduced the affine umbilical surfaces in $\mathbb{E}^4$. Hasanis and Vlachos [16] considered hypersurfaces in 4-space with harmonic mean curvature vector field. Scharlach [17] studied the affine geometry of surfaces and hypersurfaces in $\mathbb{E}^4$. Cheng and Wan [18] considered complete hypersurfaces of four-space with CMC. Arslan, Deszcz, and Yaprak [19] studied Weyl pseudosymmetric hypersurfaces. Turgay and Upadhyay [20] considered biconservative hypersurfaces in 4-dimensional Riemannian space forms.